G53KRR 2016-17
Exercise on syntax and semantics of FOL
Set in lecture 4, 13 October 2016.

In the exercises below, P is a unary predicate symbol, R is a binary predicate symbol, and f is a unary function symbol.

1. Consider an interpretation $J = (D, I)$ where $D = \{1, 2, 3\}$, $I(P) = \{1, 2\}$ and $I(R) = \{\langle 2, 2\rangle, \langle 2, 3\rangle\}$. Which of the following formulas are true in J under the variable assignment μ which assigns 1 to x and 2 to y:
   (a) $P(x)$
   (b) $R(x, y)$
   (c) $\exists x R(x, x)$
   (d) $\exists x \exists y(\neg(x = y) \wedge R(x, y))$
   (e) $\exists x \forall y \neg R(x, y)$
2. Construct some interpretation where $\forall x \forall y(R(x, y) \supset R(y, x))$ is true.
3. Construct some interpretation where $\forall x \forall y(R(x, y) \supset R(y, x))$ is false.
4. Let $J_1 = (D_1, I_1)$, where $D_1 = \{a, b\}$, $I_1(f)$ is the identity function ($I_1(f)(a) = a$ and $I_1(f)(b) = b$), and $I_1(R) = \{\langle a, a\rangle, \langle b, a\rangle\}$. An assignment μ_1 is such that $\mu_1(x) = a$.

   Does it hold that $J_1, \mu_1 \models \exists x R(x, f(x))$?
5. Come up with an interpretation which makes $\forall x \exists y R(x, f(y))$ true.
6. Come up with an interpretation which makes $\forall x \exists y R(x, f(y))$ false.
7. (difficult - only do this if you actually like it). Find an interpretation where the three sentences below are true together. Is there a finite interpretation (one with a finite domain D) where they are all true?
   (a) $\forall x \neg R(x, x)$
   (b) $\forall x \exists y R(x, y)$
   (c) $\forall x \forall y \forall z(R(x, y) \wedge R(y, z) \supset R(x, z))$