The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 3 MODULE, AUTUMN SEMESTER 2011-2012

KNOWLEDGE REPRESENTATION AND REASONING

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR out of SIX questions

Only silent, self contained calculators with a Single-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

1. (a) Define what it means for a set of sentences Γ to logically entail a sentence α. (3 marks)

   (b) Prove that the following set of sentences does not entail $\exists x(P(x) \wedge \neg Q(x))$: (12 marks)

   S1 $\forall x(P(x) \supset \neg Q(x))$

   S2 $\exists x \neg Q(x)$

   S3 $\exists x(P(x) \vee Q(x))$

   (c) Translate the following sentences into first order logic and show that they cannot be satisfied in the same interpretation: (10 marks)

   S1 There is a person who is loved by everybody.

   S2 It is not true that everybody loves somebody.

2. (a) Reduce the following sentences to clausal form: (6 marks)

   S1 $\forall x \forall y(R(x,y) \supset (R(y,x) \wedge Q(y)))$

   S2 $\forall x \exists y \forall z(P(x,y,z) \supset \exists u R(x,u,z))$

   S3 $\forall x(\neg \exists y P(x,y) \wedge \neg(Q(x) \wedge \neg R(x)))$

   (b) Derive by resolution an empty clause from the following clauses: (9 marks)

   C1 $[\neg P(x_1), Q(x_1)]$

   C2 $[P(x_2), \neg Q(x_2)]$

   C3 $[\neg Q(x_3), Q(f(x_3))]$

   C4 $[\neg P(x_4), \neg P(f(x_4))]$

   C5 $[P(a)]$ where a is a constant.

   (c) Use resolution with answer extraction to prove that S4 below follows from S1-S3 and to extract the answer (a substitution for x which makes $(Like(anne,x) \wedge Student(x)$ true). (10 marks)

   S1 $\forall x \forall y(Friend(x,y) \supset Like(x,y))$

   S2 $Friend(anne, ben)$

   S3 $Student(ben)$

   S4 $\exists x(Like(anne,x) \wedge Student(x))$

3. Recall the description logic $\mathcal{DL}$ given in the textbook:
Concepts:

- atomic concept is a concept
- if r is a role and b is a concept, then [**ALL** r b] is a concept (e.g. [**ALL** $:Child\ Girl$] describes someone all of whose children are girls).
- if r is a role and n is a positive integer, then [**EXISTS** n r] is a concept (e.g. [**EXISTS** 2 $:Child$] describes someone who has at least 2 children)
- if r is a role and c is a constant, then [**FILLS** r c] is a concept (e.g. [**FILLS** $:Child\ john$] describes someone whose child is John).
- if $b_1, \ldots, b_n$ are concepts, [**AND** $b_1, \ldots, b_n$] is a concept.

Sentences:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
- if c is a constant and b a concept then $c \rightarrow b$ is a sentence (the individual denoted by c satisfies the description expressed by b).

(a) Express the following concepts in $\mathcal{DL}$ using atomic concepts $School$ and $Female$, roles $:Pupil$ and $:Employee$, and a constant $anne$:

i. A school which has at least 30 pupils. (2 marks)

ii. A school which has at least 30 pupils and 5 employees. (2 marks)

iii. A school where all the pupils are girls. (2 marks)

iv. A school where one of the pupils is Anne. (2 marks)

(b) Express the following sentences in $\mathcal{DL}$ using the atomic concepts $School$, $Female$, $GirlsSchool$, the roles $:Pupil$ and $:Employee$:

i. A girls school is defined as a school where all pupils are girls. (3 marks)

ii. In girls schools all employees are female. (4 marks)

(c) Prove that sentences S1-S3 below do not entail S4. (10 marks)

S1 $a \rightarrow$ [**EXISTS** 1 $:Friend$]

S2 $a \rightarrow$ [**ALL** $:Friend\ Female$]

S3 [**ALL** $:Friend\ Female$] $\sqsubseteq$ [**ALL** $:Friend$ [**ALL** $:Friend\ Female$]]

S4 $a \rightarrow [Female]$

4. (a) Consider the following Horn clauses. Draw a goal tree for the goal $HasFootballClub$. (5 marks)

   C1 $[\neg Nottingham, InEngland]$
   C2 $[\neg Nottingham, City]$
   C3 $[\neg InEngland, \neg City, EnglishCity]$
   C4 $[\neg EnglishCity, HasFootballClub]$
   C5 $[Nottingham]$

   (b) Give the backward chaining procedure for propositional Horn clauses. (5 marks)

   (c) Trace this procedure for the example in part (a). (5 marks)

   (d) Define an SLD resolution derivation of a clause c from a set of clauses S. (4 marks)

   (e) Give an SLD resolution derivation of an empty clause from the clauses C1–C5 in part (a) and $[\neg HasFootballClub]$. (6 marks)

5. (a) Describe the process of knowledge acquisition for a production rule system using decision tables. Show a decision table for a production rule system making medical insurance reimbursement decisions based on the following policy:

   Charges below \$10 are not reimbursed. If the charges are above \$10, the amount to be reimbursed depends on whether or not the doctor or hospital is approved by the insurance company. Visits to an approved doctor are reimbursed at 70 % and visits to approved hospitals are reimbursed at 90%. Otherwise, doctor's visits are reimbursed at 60% and hospital visits at 80 %. (15 marks)

   (b) Give the forward chaining procedure for propositional Horn clauses. (5 marks)

   (c) Trace it on the following example, for the input $\{HasFootballClub\}$: (5 marks)

   C1 $[\neg Nottingham, InEngland]$
   C2 $[\neg Nottingham, City]$
   C3 $[\neg InEngland, \neg City, EnglishCity]$
   C4 $[\neg EnglishCity, HasFootballClub]$
   C5 $[Nottingham]$

6. (a) What are the main differences between subjective probabilities (degrees of belief) and fuzzy truth values? Give examples of typical uses of subjective probabilities and fuzzy truth values. (5 marks)

(b) Consider the following example. Denote the proposition that a patient A has a symptom S by s, and the proposition that patient A has disease D by d. Suppose we want to use subjective probabilities to calculate the degree of belief that A has D given that A has S ($Pr(d|s)$): (6 marks)

i. which formula do we use?

ii. Assume that D always causes S. Which a priori probabilities do we need?

iii. How can we arrive at those a priori probabilities?

(c) Consider the following Bayesian network:

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fire ---> smoke
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  \
   \
    v
   fire engine
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$Pr(fire) = 0.01$
$Pr(smoke|fire) = 0.9$
$Pr(smoke|\neg fire) = 0$
$Pr(engine|fire) = 1$
$Pr(engine|\neg fire) = 0.01$

i. What is the probability that there is a fire without smoke? (3 marks)

ii. What is the probabilility that there is no fire but a fire engine turns up? (3 marks)

iii. What is the probability that there is a fire given that there is smoke? (4 marks)

iv. What is the probability that there is a fire given that there is smoke and a fire engine has arrived? (4 marks)