Exercises on defaults Consider the following knowledge base:

S1 Cats don't attack people

S2 Wild cats are cats

S3 Wild cats when threatened attack people

S4 a is a cat

S5 b is a wild cat and is different from a

S6 b is threatened

1. Translate this knowledge base into first-order logic, using the circumscription approach to translating the default rule **S1**. Translate **S2** and **S3** as normal first order implications, which are true without exceptions. Use unary predicates C for cat, W for wild cat, A for attack people, T for being threatened.

2. Does this knowledge base minimally entail $\neg A(a)$ (a does not attack people)?

3. Does this knowledge base minimally enatil $\neg A(b)$ (b does not attack people)?

Answer

1. **S1** $\forall x(C(x) \wedge \neg Ab(x) \supset \neg A(x))$

   S2 $\forall x(W(x) \supset C(x))$

   S3 $\forall x(W(x) \wedge T(x) \supset A(x))$

   S4 $C(a)$

   S5 $W(b) \wedge b \neq a$

   S6 $T(b)$

2. Yes, $KB \models_{\leq} \neg A(a)$. We need to show that for any M, if $M \models KB$, then either $M \models \neg A(a)$, or we can find $M' < M$ - a model with a strictly smaller extension of Ab - such that $M' \models KB$.

   Note that in order to satisfy KB, any interpretation $M = (D, I)$ should have the following properties:

   S1' all elements of D which are in $I(C)$ and not in I(Ab) should not be in $I(A)$

   S2' $I(W) \subseteq I(C)$

   S3' $I(W) \cap I(T) \subseteq I(A)$

   S4' $I(a) \in I(C)$

   S5' $I(b) \in I(W)$, $I(a) \neq I(b)$

S6' $I(b) \in I(T)$

From S5' and S6', $I(b) \in I(W) \cap I(T)$, so by S3', $I(b) \in I(A)$. From S5' and S2', $I(b) \in I(C)$. So from S1', $I(b) \in I(Ab)$. In other words, in any interpretation which satisfies KB, the element denoted by b has to be abnormal. On the other hand, the element denoted by a, $I(a)$, is different from $I(b)$ by S5' and can always be removed from $I(Ab)$ (provided that we also remove it from $I(A)$), and KB will still be satisfied in the resulting interpretation. So any interpretation where $I(a) \in I(Ab)$ is not minimal and can be ignored for the purposes of minimal entailment.

If $I(a) \notin I(Ab)$, then from S4' and S1', $I(a)$ is not in $I(A)$, in other words all such interpretations satisfy $\neg A(a)$.

3. No, $KB \not\models_{\leq} \neg A(b)$. In fact, $KB \models A(b)$ in the usual classical sense of entailment which considers all possible interpretations, so it also holds that $KB \models_{\leq} A(b)$ (if something is classically entailed, it is also minimally entailed).