The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 3 MODULE, AUTUMN SEMESTER 2008-2009

KNOWLEDGE REPRESENTATION AND REASONING

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR out of SIX questions

Only silent, self contained calculators with a Single-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

1. (a) Give an inductive truth definition for first order logic, that is, define when an interpretation $M = (D, I)$ and an assignment μ satisfy a formula ϕ. Assume that ϕ is defined by the following grammar:

$$\phi \quad = \quad P(t_1, \ldots, t_n) \mid \neg\phi \mid \phi \wedge \phi \mid \forall x \phi$$

where the terms t_i are either variables or constants. (7 marks)

(b) Consider the following set of sentences:

S1 Andrew is the father of Bob.

S2 Bob is the father of Chris.

S3 Every grandfather is someone's father.

S4 Andrew is a grandfather of Chris.

Translate these sentences into first-order logic, using binary predicates $Father$ and $Grandfather$ and constants a, b, c for Andrew, Bob and Chris. Show semantically (by reasoning about interpretations) that **S1**–**S3** do not logically entail **S4**. (8 marks)

(c) Write in first-order logic an additional sentence that defines a general property of grandfathers, and show that **S1**–**S3** together with this new sentence entail $Grandfather(a, c)$. (10 marks)

2. (a) Reduce the following sentences to clausal form:

S1 $\exists x \forall y Less(x, y)$

S2 $\forall x \exists y Less(x, y)$

S3 $\forall x \forall y \forall z (Less(x, y) \wedge Less(y, z) \supset Less(x, z))$ (5 marks)

(b) Show by resolution that clauses **C1**–**C3** below entail $\exists x Q(x)$.

C1 $[\neg P(x), \neg R(x, y), Q(x)]$

C2 $[\neg P(x), R(x, f(x))]$

C3 $[P(a)]$ (10 marks)

(c) Show by resolution that the following set of clauses is inconsistent: (8 marks)

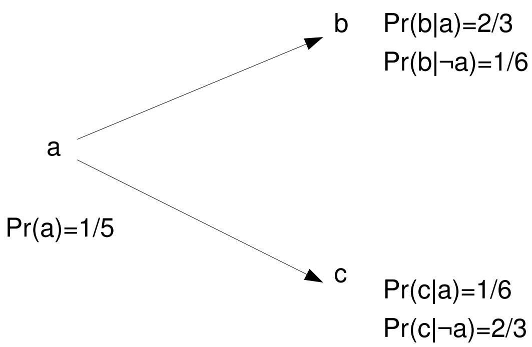
$$[A, B, C],\ [A, B, \neg C],\ [A, \neg B, C],\ [A, \neg B, \neg C],$$

$$[\neg A, B, C],\ [\neg A, B, \neg C], [\neg A, \neg B, C],\ [\neg A, \neg B, \neg C]$$

(d) Say whether your proof in part (c) is an SLD resolution proof. Explain your answer. (2 marks)

3. (a) Explain what the nodes and arcs represent in a belief network or Bayesian network, and how to compute a joint probability distribution given a belief network. What is the advantage of using belief networks compared to explicitly giving a joint probability distribution? (5 marks)

(b) Given the following belief network:



i. Give an example of an independence assumption that is implicit in this network. (3 marks)

ii. What is the probability that a, b and c are all true? (4 marks)

iii. What is the probability that a, b and c are all false? (4 marks)

iv. What is the probablity of $b \wedge c$? (4 marks)

v. Is $b \wedge c$ more probable when a is true or when a is false? (5 marks)

4. (a) Give a forward-chaining procedure for propositional Horn clauses. (5 marks)

(b) Explain the notions of rule matching, rule instance, conflict set, and conflict resolution strategy in rule-based systems. Give two examples of common conflict resolution strategies. Illustrate your answers on the following example of rules and working memory elements. State what the conflict set is for the current state of the working memory and which rules will be fired first under each conflict resolution strategy. You can also refer to the conflict set at the next cycle, after the selected rules are fired.

F1 $animal(tiger)$

F2 $animal(cat)$

F3 $large(tiger)$

F4 $eatsMeat(tiger)$

F4 $eatsMeat(cat)$

R1 $\forall x(animal(x) \wedge large(x) \wedge eatsMeat(x) \supset dangerous(x))$

R2 $\forall x(animal(x) \supset breathesOxygen(x))$

R3 $\forall x(dangerous(x) \supset runAwayNow)$ (10 marks)

(c) Explain how decision tables can be used for knowledge elicitation and designing an expert system. (3 marks)

(d) Suppose all you have to work with in designing an expert system for recognising spam email is the following set of correctly classified messages. Produce a decision table based on this set of examples. For full marks, do not include irrelevant checks in the rules. (7 marks)

Message 1 Properties: has an attachment, does not contain images, sender is in the receiver's address book, subject line contains "Prize". **Decision**: spam.

Message 2 Properties: no attachments, contains images, sender is not in the receiver's address book, subject line contains "Goods". **Decision**: spam.

Message 3 Properties: has an attachment, contains images, sender is in the receiver's address book, subject line contains "Prize". **Decision**: spam.

Message 4 Properties: no attachments, does not contain images, sender is not in the receiver's address book, subject line does not contain "Prize" or "Goods". **Decision**: not spam.

Message 5 Properties: has an attachment, does not contain images, sender is not in the receiver's address book, subject line contains "Prize". **Decision**: spam.

Message 6 Properties: has no attachments, contains images, sender is in the receiver's address book, subject line contains "Goods". **Decision**: not spam.

Message 7 Properties: has no attachments, does not contain images, sender is not in the receiver's address book, subject line contains "Goods". **Decision**: spam.

Message 8 Properties: has no attachments, contains images, sender is not in the receiver's address book, subject line does not contain "Prize" or "Goods". **Decision**: not spam.

5. Consider a description logic with the following definition of a concept (note that it is slightly different from the one in the textbook, namely the first concept constructor is new and the forth concept constructor is different from [**EXISTS** n r]):

- $\top$ is a special atomic concept which describes any object (it is a property which is trivially true for everything)
- an atomic concept is a concept
- if r is a role and b is a concept, then [**ALL** r b] is a concept (describing objects all of whose r-successors are described by b)
- if r is a role and b is a concept, then [**EXISTS** r b] is a concept (describing objects which have at least one r-successor which is described by b)
- if r is a role and c is a constant, then [**FILLS** r c] is a concept (describing objects which have an r-successor denoted by c)
- if $b_1, \ldots, b_n$ are concepts, [**AND** $b_1 \ldots b_n$] is a concept (describing objects which are described by all of $b_1, \ldots, b_n$)

and the following definition of a sentence:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
- if c is a constant and b a concept then $c \rightarrow b$ is a sentence (the individual denoted by c satisfies the description expressed by b).

(a) Define interpretations (D, I) for this description logic and give an inductive definition of the meaning of concepts (extend the interpretation mapping I to complex concepts). Give conditions for the truth of sentences in an interpretation. (10 marks)

(b) Given the atomic concepts $Female$, $Male$, roles $:Child$, $:Sibling$ and constant $alice$, define in the description logic above the following concepts:

i. "Mother of Alice" (someone female whose child is Alice) (3 marks)

ii. "Parent" (someone who has a child) [Hint: use $\top$ to describe the child] (3 marks)

iii. "Uncle" (someone male who has a sibling who has a child) (3 marks)

(c) Using the same atomic concepts as in part (b), translate the following sentences in description logic:

i. Every grandparent is a parent (3 marks)

ii. Alice is a grandmother (3 marks)

6. (a) Give the definitions of minimal models and minimal entailment in circumscription theory. (5 marks)

(b) For the following KB:

$$KB = \{Dutchman(peter),\ Dutchman(hans),\ Dutchman(johan),$$

$$peter \neq hans,\ hans \neq johan,\ peter \neq johan,$$

$$\neg Tall(peter) \vee \neg Tall(hans),\ \forall x(Dutchman(x) \wedge \neg Ab(x) \supset Tall(x))\}$$

state whether the following sentences are minimally entailed, and explain why:

i. $Tall(johan)$ (4 marks)

ii. $Tall(peter) \vee Tall(hans)$ (4 marks)

(c) Explain what a default rule $\frac{\alpha : \beta}{\delta}$ in Reiter's default logic means. (3 marks)

(d) What is an extension of a default theory (F, D) (where F is a set of first order sentences and D a set of default rules)? (5 marks)

(e) Is there an extension of the following default theory (F, D) which contains $Tall(peter)$:

$$F = \{Dutchman(peter),\ Dutchman(hans),\ Dutchman(johan),\ peter \neq hans,$$

$$hans \neq johan,\ peter \neq johan,\ \neg Tall(peter) \vee \neg Tall(hans)\}$$

and

$$D = \{\frac{Dutchman(x) : Tall(x)}{Tall(x)}\}$$

If yes, list applicable assumptions for this extension. (4 marks)