The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 3 MODULE, AUTUMN SEMESTER 2010-2011

KNOWLEDGE REPRESENTATION AND REASONING

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR out of SIX questions

Only silent, self contained calculators with a Single-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

1. (a) Express the following sentences in first-order logic, using the binary predicates $StudyAt$, $WorkAt$, $LocatedIn$, $=$, the unary predicate $University$, and the constants uon (for the University of Nottingham) uob (for the University of Birmingham), and n (for Nottingham):

S1 Everybody who studies at the University of Nottingham, does not study at the University of Birmingham. (2 marks)

S2 There are exactly two universities in Nottingham. (2 marks)

S3 There are at least two people who both work at the University of Nottingham and study there. (2 marks)

(b) Prove that the following argument is not logically sound (that the premises do not logically entail the conclusion). (10 marks)

$$\frac{\begin{array}{c}\text{Some people are clever.}\\ \text{Some people are greedy.}\end{array}}{\text{Some people are clever and greedy.}}$$

(c) Prove that the propositional resolution rule below is sound (the premises logically entail the conclusion). (9 marks)

$$\frac{\alpha_1 \vee p \qquad \alpha_2 \vee \neg p}{\alpha_1 \vee \alpha_2}$$

2. (a) Reduce the following sentences to clausal form:

S1 $\forall x \forall y (P(x, y) \supset \exists z R(x, y, z))$ (2 marks)

S2 $\forall x \exists y R(x, y) \vee \forall x \exists y \neg R(x, y)$ (2 marks)

S3 $\exists x \exists y \forall z \neg (P(x, z) \vee P(y, z))$ (2 marks)

(b) Show by resolution that clauses **C1**–**C3** below entail $\exists x Q(x, a)$.

C1 $[\neg P(x), R(x, f(x))]$

C2 $[\neg R(y, z), Q(z, y)]$

C3 $[P(a)]$ (10 marks)

(c) Give a substitution which unifies $P(f(x), g(f(x), y), z)$ and $\neg P(x_1, g(x_1, f(y_1)), f(z_1))$. (3 marks)

(d) Define the notion of *most general unifier* (mgu). Then give an example of an mgu substitution and a non-mgu substitution. (3 marks)

(e) Is it possible to unify $[P(g(x))]$ and $[\neg P(f(x))]$? Explain your answer. (3 marks)

3. Recall the description logic $\mathcal{DL}$ given in the textbook:
Concepts:

- atomic concept is a concept
- if r is a role and b is a concept, then [**ALL** r b] is a concept (e.g. [**ALL** $:Child\ Girl$] describes someone all of whose children are girls).
- if r is a role and n is a positive integer, then [**EXISTS** n r] is a concept (e.g. [**EXISTS** 2 $:Child$] describes someone who has at least 2 children)
- if r is a role and c is a constant, then [**FILLS** r c] is a concept (e.g. [**FILLS** $:Child\ john$] describes someone whose child is John).
- if $b_1, \ldots, b_n$ are concepts, [**AND** $b_1, \ldots, b_n$] is a concept.

Sentences:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s, b_1 is *subsumed* by b_2)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
- if c is a constant and b a concept then $c \rightarrow b$ is a sentence (the individual denoted by c satisfies the description expressed by b).

(a) Express the following concepts in $\mathcal{DL}$ using the atomic concepts $Animal$, and $Fish$, and the roles $:Tail$, $:Leg$, and $:Eat$.

C1 An animal that has a tail (2 marks)

C2 An animal that has a tail and four legs (2 marks)

C3 An animal that eats only fish (2 marks)

C4 An animal that eats only things that themselves eat only fish (2 marks)

(b) Express the following sentences in $\mathcal{DL}$ using the atomic concepts Cat, $Fish$, and $Animal$, the roles $:Leg$, and $:Eat$, and the constant $tiddles$:

S1 Tiddles is a cat who eats only fish (4 marks)

S2 Cats are animals that have four legs (3 marks)

(c) Show that $john \rightarrow$ [**ALL** $:Child\ Girl$] and $john \rightarrow$ [**FILLS** $:Child\ mary$] entail $john \rightarrow$ [**EXISTS** 1 $:Child$] in $\mathcal{DL}$. (5 marks)

(d) Show that the sentence $john \rightarrow$ [**ALL** $:Child\ Girl$] does not entail $john \rightarrow$ [**EXISTS** 1 $:Child$] in description logic. (5 marks)

4. (a) Give the backward chaining procedure for propositional Horn clauses. (5 marks)

   (b) Trace it on the following example: $KB = \{[r, \neg p, \neg q], [t, \neg r, \neg s], [p], [q], [s]\}$, goal: t. (5 marks)

   (c) Define SLD derivation and explain why backward chaining is a special case of SLD derivation. (5 marks)

   (d) Show that there is an SLD derivation of $[]$ from the KB in part(b) and $[\neg t]$. (5 marks)

   (e) Is backward chaining for propositional Horn clauses guaranteed to terminate? If not, give an example of a case where backward chaining does not terminate and explain why. (5 marks)

5. (a) Define the closed world assumption and the entailment relation $\models_{CWA}$. (3 marks)

   (b) Why is the closed world assumption used in knowledge representation (i.e., why not store negative data in the knowledge base)? (2 marks)

   (c) What does monotonicity of an entailment relation mean? Give an example to show that $\models_{CWA}$ is non-monotonic. (5 marks)

   (d) How are rules of the form 'Normally, As are Bs' formalised in circumscription theory? Formalise 'Normally, scientists are intelligent'. (3 marks)

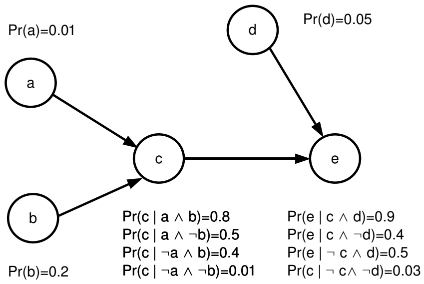
   (e) Give definitions of minimal models and minimal entailment in circumscription theory. (5 marks)

   (f) Show that 'John is intelligent' follows by circumscription from a knowledge base which contains sentences 'John is a scientist' and 'Normally, scientists are intelligent'. (5 marks)

   (g) Show that minimal entailment is not monotonic. (2 marks)

6. (a) What is a Bayesian network? What kind of information does it represent and how? Explain what an independence assumption is. (5 marks)

   (b) Give an example of an independence assumption implicit in the following network: (3 marks)



   (c) What is the advantage of using belief networks compared to explicitly giving a joint probability distribution? (2 marks)

   (d) Give Bayes's conditional probability rule (i.e., define $Pr(a|b)$ in terms of $Pr(a \wedge b)$ and $Pr(b)$). (1 marks)

   (e) Give the chain rule for computing the probability of the conjunction $Pr(a_1 \wedge \ldots \wedge a_n)$. (1 marks)

   (f) Suppose that we have events a, b and c, and we know that $Pr(a)$ is 1/3, $Pr(b|\neg a)$ is 1/2 and $Pr(c)$ is 1/2. We also know that c is conditionally independent of a and b. Compute $Pr(\neg a \wedge b \wedge c)$. (3 marks)

   (g) Represent the following scenario as a Bayesian network: (5 marks)

   > Disease d is caused by exposure to chemical c. The probability of c is 0.03. The probability of having d after exposure to c is 0.8. d almost never occurs without exposure to c (probability of d given no exposure is 0.001). The disease d may cause complication a. However a may also be caused by another disease b. The probability of b is 0.1. The probability of a given d but not b is 0.6, which is the same as the probability of a given b but not d. The probability of a given both b and d is 0.9, the probability of a without either b or d is 0.02.

   (h) What is the probability of $d \wedge \neg c \wedge b$ given the scenario in part (g) above? (5 marks)