G53KRR handout on description logic.

Variety of description logics/ontology languages. This lecture (based on Brachman and Levesque chapter 9) is only one particular flavour of description logic. There are very many, with different syntax and expressive power.

Basic idea description logic talks about relationships between *concepts* (noun phrases).

Precise definition of the syntax of DL :

Logical symbols (apart from brackets etc.):

- concept-forming operators: **ALL**, **EXISTS**, **FILLS**, **AND**
- connectives: $\sqsubseteq$, $\doteq$, $\rightarrow$
- symbols for numbers (1,2,3,...),

Non-logical symbols:

- Atomic concepts: $Person$, $Thing$,.... Correspond to unary predicates in FOL.
- Roles: $:Age$, $:Employer$, $:Child$, $:Arm$,.... Correspond to binary predicates in FOL.
- Constants: $john, mary, roomA7, \ldots$. Correspond to constants (0-ary functional symbols) in FOL.

Concepts:

- atomic concept is a concept
- if r is a role and b is a concept, then [**ALL** r b] is a concept (e.g. [**ALL** $:Child$ $Girl$] describes someone all of whose children are girls).
- if r is a role and n is a positive integer, then [**EXISTS** n r] is a concept (e.g. [**EXISTS** 2 $:Child$] describes someone who has at least 2 children)
- if r is a role and c is a constant, then [**FILLS** r c] is a concept (e.g. [**FILLS** $:Child$ $john$] describes someone whose child is John).
- if $b_1, \ldots, b_n$ are concepts, [**AND** $b_1 \ldots b_n$] is a concept.

Sentences:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s, b_1 is *subsumed* by b_2)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
- if c is a constant and b a concept then $c \rightarrow b$ is a sentence (the individual denoted by c satisfies the description expressed by b).

DL knowledge base (or an *ontology*) is a set of DL sentences.

T box and A box this is not in Brachman and Levesque, but you are likely to come across this distinction if you read up on description logics and ontologies. A DL knowledge base is usually split into terminological part or T box which describes general relationships between concepts, e.g. $Surgeon \sqsubseteq Doctor$, and assertions about individuals or A box (e.g. $mary \rightarrow Doctor$).

Interpretations for DL same as for FOL: a set of individuals D and an interpretation mapping I such that

- for a constant c, $I(c) \in D$
- for an atomic concept a, $I(a) \subseteq D$
- for a role r, $I(r) \subseteq D \times D$
- $I([\mathbf{A}LL\ r\ b]) = \{x \in D :$ for any y, if $(x, y) \in I(r)$, then $y \in I(b)\}$. Same as

$$\forall y(R(x, y) \supset B(y))$$

- $I([\textbf{EXISTS}\ n\ r]) = \{x \in D :$ there are at least n distinct y such that $(x, y) \in I(r)$. Same as

$$\exists y_1 \ldots \exists y_n(\neg(y_1 = y_2) \wedge \ldots \wedge \neg(y_{n-1} = y_n) \wedge R(x, y_1) \wedge \ldots \wedge R(x, y_n))$$

- $I([\textbf{FILLS}\ r\ c]) = \{x \in D : (x, I(c)) \in I(r)\}$. Same as: $R(x, c)$.
- $I([\textbf{AND}\ b_1 \ldots b_n]) = I(b_1) \cap \ldots \cap I(b_n)$. Same as

$$B_1(x) \wedge \ldots \wedge B_n(x)$$

Finally, for sentences:

- $(D, I) \models c \rightarrow b$ iff $I(c) \in I(b)$. Same as $B(c)$.
- $(D, I) \models b_1 \sqsubseteq b_2$ iff $I(b_1) \subseteq I(b_2)$. Same as

$$\forall x(B_1(x) \supset B_2(x))$$

- $(D, I) \models b_1 \doteq b_2$ iff $I(b_1) = I(b_2)$. Same as

$$\forall x(B_1(x) \equiv B_2(x))$$

Reasoning Entailment is defined exactly like in FOL: a set of sentences Γ entails a sentence ϕ (in symbols $\Gamma \models \phi$) if and only if ϕ is true in every interpretation where all of the sentences in Γ are true.

Since (this particular) DL is a fragment of first order logic, reasoning in it is more efficient (it is decidable whether a finite set of sentences entails another sentence). This holds for many other description logics, although not all of them.

Exercise

- Define the following concept: Attendee (of some work-life balance workshop) is a working mother employed by the University of Nottingham. Assume that you have an atomic concept Woman, roles Child and Employer, and a constant uon for the University of Nottingham.
-
    - Do $d_1 \sqsubseteq d_2$ and $d_2 \sqsubseteq d_3$ entail $d_1 \sqsubseteq d_3$?
    - Do $c \rightarrow d_1$ and $d_2 \sqsubseteq d_1$ entail $c \rightarrow d_2$?
    - Do $c \rightarrow d_1$ and $d_1 \sqsubseteq d_2$ entail $c \rightarrow d_2$?