Brachman and Levesque, Chapter 4 exercise 1. Determine whether the following sentence is valid using resolution:

$$\exists x \forall y \forall z ((P(y) \supset Q(z)) \supset (P(x) \supset Q(x)))$$

Answer To do this we need to check if from the negation of the sentence we can derive an empty clause (a contradiction).

First transfer the negation into clausal form:

$$\neg \exists x \forall y \forall z ((P(y) \supset Q(z)) \supset (P(x) \supset Q(x)))$$

$$\neg \exists x \forall y \forall z (\neg(\neg P(y) \vee Q(z)) \vee (\neg P(x) \vee Q(x)))$$

$$\forall x \exists y \exists z \neg(\neg(\neg P(y) \vee Q(z)) \vee (\neg P(x) \vee Q(x)))$$

$$\forall x \exists y \exists z (\neg\neg(\neg P(y) \vee Q(z)) \wedge \neg(\neg P(x) \vee Q(x)))$$

$$\forall x \exists y \exists z ((\neg P(y) \vee Q(z)) \wedge (\neg\neg P(x) \wedge \neg Q(x)))$$

$$\forall x \exists y \exists z ((\neg P(y) \vee Q(z)) \wedge P(x) \wedge \neg Q(x))$$

$$\forall x ((\neg P(f(x)) \vee Q(g(x))) \wedge P(x) \wedge \neg Q(x))$$

Clauses:

C1 $[\neg P(f(x)), Q(g(x))]$

C2 $[P(x)]$

C3 $[\neg Q(x)]$

Note that the definition of the resolution rule (p.58 of the textbook) presupposes that all clauses have distinct variables. (We can do this without loss of generality because variables are universally quantified, and $\forall x P(x)$ is equivalent to $\forall v P(v)$, similarly $\forall x \neg Q(x)$ is equivalent to $\forall w \neg Q(w)$.) So the clauses we actually are going to work with are

C1 $[\neg P(f(x)), Q(g(x))]$

C2 $[P(v)]$

C3 $[\neg Q(w)]$

Proof:

(1) $[Q(g(x))]$ from C1 and C2, $v/f(x)$

(2) [] from (1) and C3, $w/g(x)$

Brachman and Levesque, Chapter 4 exercise 2. This is a follow-up to Exercise 1 of Chapter 3, which is on

`http://www.cs.nott.ac.uk/~nza/G53KRR/ch3.pdf`

Use resolution to prove that there exists a member of the Alpine club who is a climber but not a skier.
Answer. 1. Translation into first order logic.

S1 $Member(tony)$

S2 $Member(mike)$

S3 $Member(john)$

S4 $\forall x(Member(x) \wedge \neg Skier(x) \supset Climber(x))$

S5 $\forall x(Climber(x) \supset \neg Like(x, rain))$

S6 $\forall x(\neg Like(x, snow) \supset \neg Skier(x))$

S7 $\forall x(Like(tony, x) \supset \neg Like(mike, x))$

S8 $\forall x(\neg Like(tony, x) \supset Like(mike, x))$

S9 $Like(tony, rain)$

S10 $Like(tony, snow)$

S11 $\exists x(Member(x) \wedge Climber(x) \wedge \neg Skier(x))$.

2. Same in clausal form:

C1 $[Member(tony)]$

C2 $[Member(mike)]$

C3 $[Member(john)]$

C4 $[\neg Member(x), Skier(x), Climber(x)]$

C5 $[\neg Climber(x), \neg Like(x, rain)]$

C6 $[Like(x, snow), \neg Skier(x)]$

C7 $[\neg Like(tony, x), \neg Like(mike, x)]$

C8 $[Like(tony, x), Like(mike, x)]$

C9 $[Like(tony, rain)]$

C10 $[Like(tony, snow)]$

C11 $[\neg Member(x), \neg Climber(x), Skier(x)]$ [1]

3. Proof that together C1-C11 are inconsistent:
(1) $[\neg Like(mike, snow)]$ from C10 and C7
(2) $[\neg Skier(mike)]$ from (1) and C6
(3) $[\neg Member(mike), Climber(mike)]$ from (2) and C4
(4) $[Climber(mike)]$ from (3) and C2
(5) $[\neg Member(mike), Skier(mike)]$ from (4) and C11
(6) $[Skier(mike)]$ from (5) and C2
(7) [] from (6) and (2).

[1] negation of S11: $\forall x(\neg Member(x) \vee \neg Climber(x) \vee Skier(x))$