G53KRR: unification, resolution with equality

A unifier of two literals ρ_1 and ρ_2 is a substitution θ such that $\rho_1\theta = \rho_2\theta$.

There are many possible unifiers, and some of them are too specific. For example, $P(x, y)$ and $P(a, z)$ can be unified by $\theta_1 = x/a, y/z$ and by $\theta_2 = x/a, y/b, z/b$ (the second one is too specific).

A most general unifier (mgu) for ρ_1 and ρ_2 is a unifier θ such that for any other unifier θ', there is a further substitution $\theta*$ such that $\theta' = \theta\theta*$. Basically, θ' is obtained from θ by doing some extra substitutions.

An mgu for ρ_1 and ρ_2 (assuming ρ_1 and ρ_2 do not have common variables to start with) can be computed as follows:

1. start with $\theta = \{\ \}$
2. exit if $\rho_1\theta = \rho_2\theta$
3. set DS to be the pair of terms at the first place where $\rho_1\theta$ and $\rho_2\theta$ disagree
4. find a variable v in DS and a term t in DS not containing v; if none exist, fail
5. otherwise set θ to $\theta\{v/t\}$ and go to step 2.

If we have to deal with a set of clauses containing equality, we need to add to KB the following axioms:

reflexivity $\forall x(x = x)$

symmetry $\forall x \forall y(x = y \supset y = x)$

transitivity $\forall x \forall y \forall z(x = y \land y = z \supset x = z)$

substitution for functions for every function f of arity n in the set of clauses, $\forall x_1 \forall y_1 \ldots \forall x_n \forall y_n (x_1 = y_1 \land \ldots \land x_n = y_n \supset f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n))$

substitution for predicates for every predicate P of arity n in the set of clauses,

$$\forall x_1 \forall y_1 \ldots \forall x_n \forall y_n (x_1 = y_1 \land \ldots \land x_n = y_n \supset P(x_1, \ldots, x_n) \supset P(y_1, \ldots, y_n))$$

With these axioms, resolution is sound and complete for refutation for first-order logic with equality.

Example: $\forall x(x = a \lor x = b)$, $\exists x P(x)$, $\neg P(a)$, $\neg P(b)$ should be inconsistent. Clauses:

C1 $[x = a, x = b]$

C2 $[P(c)]$

C3 $[\neg P(a)]$

C4 $[\neg P(b)]$

We will also use the following instance of substitution for predicates:

C5 $[\neg(x_1 = y_1), \neg P(x_1), P(y_1)]$

Proof:

(1) $[\neg(c = y_1), P(y_1)]$ from C5 and C2, x_1/c

(2) $[\neg(c = a)]$ from C3 and (1), y_1/a

(3) $[c = b]$ from (2) and C1, x/c

(4) $[P(b)]$ from (3) and (1), y_1/b

(5) $[]$ from (4) and C4.