G53KRR

Revision 2

Plan of the lecture

- circumscription
- defaults
- any other questions

Circumscription

- The main idea: formalise common sense rules which admit exceptions.
- Rules like ‘Birds fly’ formalised as

$$\forall x(Bird(x) \land \neg Ab(x) \supset Flies(x))$$

- To check whether something is entailed by a knowledge base which contains such rules, we only check if it is entailed under the assumption that the set of exceptions $I(Ab)$ is as small as possible
- This is called circumscription or minimal entailment

Example

$$KB = \{Bird(tweety), \forall x(Bird(x) \wedge \neg Ab(x) \supset Flies(x))\}$$

- Classically, $KB \not\models Flies(tweety)$ because there are interpretations of KB where Tweety is an exceptional bird (it is in $I(Ab)$) and it does not fly
- But such interpretations do not minimise the set of exceptions: nothing which is said in KB forces us to think that Tweety is exceptional, so it does not have to be in $I(Ab)$
- If we only consider interpretation which satisfy KB and where the set of exceptions is as small as possible, Tweety is not in this set, so $Bird(tweety) \wedge \neg Ab(tweety)$ holds and hence $Flies(tweety)$ holds
- KB entails $Flies(tweety)$ on 'minimal' interpretations where $I(Ab)$ is circumscribed (made as small as possible)

Definition of minimal entailment

- Let $M_1 = (D, I_1)$ and $M_2 = (D, I_2)$ be two interpretations over the same domain such that every constant and function are interpreted the same way.
- $M_1 \leq M_2 \;\Leftrightarrow\; I_1(Ab) \subseteq I_2(Ab)$
- $M_1 < M_2$ if $M_1 \leq M_2$ but not $M_2 \leq M_1$. (There are strictly fewer abnormal things in M_1).
- *Minimal entailment:* $KB \models_{\leq} \alpha$ iff for all interpretations M which make KB true, either $M \models \alpha$ or M is not minimal (exists M' such that $M' < M$ and $M' \models KB$).

Back to the example

- $KB = \{Bird(tweety), \forall x(Bird(x) \wedge \neg Ab(x) \supset Flies(x))\}$
- $KB \models_{\leq} Flies(tweety)$ because for every interpretation M which makes KB true and $Flies(tweety)$ false, it has to be that $I(tweety) \in I(Ab)$.
- So for for every such interpretation there is an interpretation M' which is just like M, but $I'(tweety) \notin I'(Ab)$ and $I'(tweety) \in I'(Flies)$, and M' still makes KB true and $M' < M$.

Defaults

- A *default rule* consists of a *prerequisite* α, *justification* β, *conclusion* γ and says 'if α holds and it is consistent to believe β, then believe γ': $\frac{\alpha : \beta}{\gamma}$
- For example:

$$\frac{Bird(x) : Flies(x)}{Flies(x)}$$

- Default rules where justification and conclusion are the same are called *normal default rules* and are writted $Bird(x) \Rightarrow Flies(x)$.

Default theories and extensions

- A default theory *KB* consists of a normal first-order knowledge base *F* and a set of default rules *D*
- A set of reasonable beliefs given a default theory $KB = \{F, D\}$ is called an *extension* of *KB*
- *E* is an *extension* of (F, D) iff for every sentence π,

$$\pi \in E \;\Leftrightarrow\; F \cup \{\gamma \mid \frac{\alpha : \beta}{\gamma} \in D, \alpha \in E, \neg\beta \notin E\} \models \pi$$

How one could construct an extension

$$\pi \in E \;\Leftrightarrow\; F \cup \{\gamma \mid \frac{\alpha : \beta}{\gamma} \in D, \alpha \in E, \neg\beta \notin E\} \models \pi$$

1. $E := F$;
2. close E under classical entailment: $E := \{\pi : E \models \pi\}$
3. choose some (substitution instance of) $\frac{\alpha:\beta}{\gamma} \in D$
4. if $\alpha \in E$, and $\neg\beta \notin E$ (meaning, β is consistent with E), $E := E \cup \{\gamma\}$
5. go back to 2

Example

$$F = \{Bird(tweety)\}, D = \{\frac{Bird(x) : Flies(x)}{Flies(x)}\}$$

- $E := \{Bird(tweety)\}$
- close E under classical entailment: $E := \{\pi : Bird(tweety) \models \pi\}$
- $\frac{Bird(tweety):Flies(tweety)}{Flies(tweety)} \in D$
- $Bird(tweety) \in E$, and $\neg Flies(tweety) \notin E$
  $E := E \cup \{Flies(tweety)\}$
- $E := \{\pi : Bird(tweety), Flies(tweety) \models \pi\}$
- there are no more rules to apply

Example from 2008 exam, Q6e

$$F = \{Dutchman(peter),\ Dutchman(hans),\ Dutchman(johan),$$

$$peter \neq hans,\ hans \neq johan,\ peter \neq johan,$$

$$\neg Tall(peter) \vee \neg Tall(hans)\}$$

$$D = \{\frac{Dutchman(x) : Tall(x)}{Tall(x)}\}$$

Three instances of the default rule:

$$\frac{Dutchman(peter) : Tall(peter)}{Tall(peter)} \quad \frac{Dutchman(hans) : Tall(hans)}{Tall(hans)}$$

$$\frac{Dutchman(johan) : Tall(johan)}{Tall(johan)}$$

Exam 2008 example continued

- Suppose we start constructing E_1 with the first rule, for Peter. Since $\neg Tall(peter) \notin E_1$, we can add $Tall(peter)$ to E_1.
- After we close E_1 under consequence, from $Tall(peter)$ and $\neg Tall(peter) \vee \neg Tall(hans)$ we get $\neg Tall(hans) \in E_1$.
- So now the second rule for Hans is not applicable.
- The third rule is applicable, since $\neg Tall(johan) \notin E_1$, we can add $Tall(johan)$ to E_1
- Another possible extension is E_2: we use the second rule first, and add $Tall(hans)$ to E_2.
- Now the first rule is not applicable, because E_2 contains $\neg Tall(peter)$
- The third rule is applicable, since $\neg Tall(johan) \notin E_2$, we can add $Tall(johan)$ to E_2

Another example with two extensions

- Facts: $F = \{Republican(dick), Quaker(dick)\}$
- Default rules: $Republican(x) \Rightarrow \neg Pacifist(x)$, $Quaker(x) \Rightarrow Pacifist(x)$.
- Extension E_1 (pick the rule $Republican(x) \Rightarrow \neg Pacifist(x)$ first) is all consequences of $\{Republican(dick), Quaker(dick), \neg Pacifist(dick)\}$. Because we start with $E_1 = \{Republican(dick), Quaker(dick)\}$, $\neg\neg Pacifist(dick) \notin E_1$, so we can add $\neg Pacifist(dick)$ to E_1.
- Extension E_2 (pick the rule $Quaker(x) \Rightarrow Pacifist(x)$ first) is all consequences of $\{Republican(dick), Quaker(dick), Pacifist(dick)\}$. Because we start with $E_2 = \{Republican(dick), Quaker(dick)\}$, $\neg Pacifist(dick) \notin E_2$, so we can add $Pacifist(dick)$ to E_2.

Example with one extension

- Facts: $F = \{Republican(dick), Quaker(dick), \forall x(Republican(x) \supset MemberOfPoliticalParty(x))\}$
- Default rules: $Republican(x) \Rightarrow \neg Pacifist(x)$,

$$\frac{Quaker(x) : Pacifist(x) \wedge \neg MemberOfPoliticalParty(x)}{Pacifist(x)}$$

- Closure of F under consequence includes: $\{Republican(dick), Quaker(dick), \forall x(Republican(x) \supset MemberOfPoliticalParty(x)), MemberOfPoliticalParty(dick)\}$
- The second default rule is not applicable, because $\neg\neg MemberOfPoliticalParty(dick) \in E$
- only the first rule is applicable, since $\neg\neg Pacifist(dick) \notin E$, so $\neg Pacifist(dick)$ is added.