G53KRR: propositional resolution

1. Main idea: to check whether $KB \models \alpha$, we check whether $KB \cup \{\neg\alpha\} \models \bot$, where $\bot$ is a contradiction.

2. We do the check for $KB \cup \{\neg\alpha\} \models \bot$ by first rewriting KB and $\neg\alpha$ to *clausal form* and then applying the inference rule of *resolution*.

3. A sentence is in CNF (conjunctive normal form) if it is a conjunction of disjunctions of literals, where a literal is an atomic sentence or its negation. So it is something like
$$(p \vee \neg q) \wedge (r \vee q)$$

4. Clausal form is obtained from CNF by representing disjunctions as *clauses* (sets of literals) and the whole formula as a set of clauses:
$$\{[p, \neg q], [r, q]\}$$

5. The resolution rule is basically:
$$\frac{p \vee A, \quad \neg p \vee B}{A \vee B}$$
In clause notation:
$$\frac{\{p\} \cup c_1 \quad \{\neg p\} \cup c_2}{c_1 \cup c_2}$$

6. $\bot$ is the empty clause $[\,]$.

7. How to translate a propositional sentence to CNF: some useful equivalences:

   definition 1 $(\alpha \supset \beta) \equiv (\neg\alpha \vee \beta)$

   definition 2 $(\alpha \equiv \beta) \equiv ((\alpha \supset \beta) \wedge (\beta \supset \alpha))$

   double negation $\neg\neg\alpha \equiv \alpha$

   de Morgan 1 $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$

   de Morgan 2 $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$

   distributivity 1 $\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

   distributivity 2 $\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$

   collect terms 1 $(\alpha \wedge \alpha) \equiv \alpha$

   collect terms 2 $(\alpha \vee \alpha) \equiv \alpha$