Complexity of simple sorting algorithms

- Bubble sort
- Selection sort
- Insertion sort

Bubble sort

23 10 12 5

Bubble sort

23 *10* 12 5

Bubble sort

10 *23* 12 5

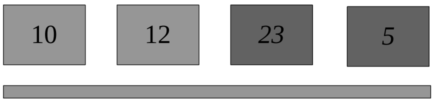
Bubble sort

10 *23* *12* 5

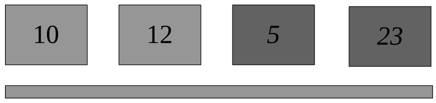
Bubble sort

10 *12* *23* 5

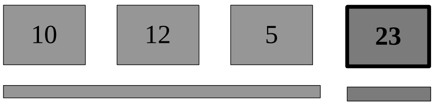
Bubble sort



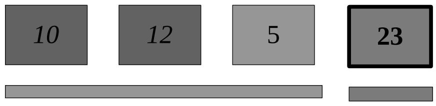
Bubble sort



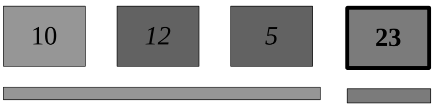
Bubble sort



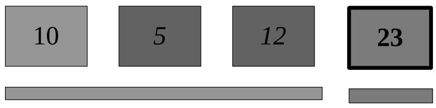
Bubble sort



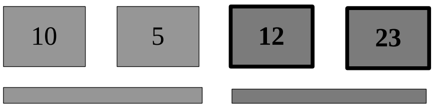
Bubble sort



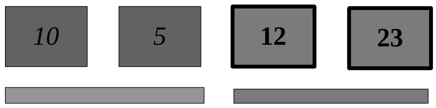
Bubble sort



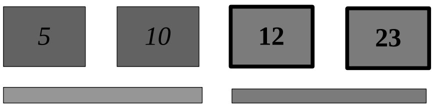
Bubble sort



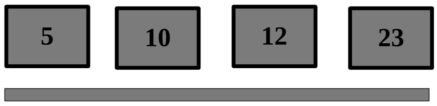
Bubble sort



Bubble sort



Bubble sort



Complexity of bubble sort

- For an array of size N, in the worst case:
  1st passage through the inner loop: N-1 comparisons and N-1 swaps
- ...
- (N-1)st passage through the inner loop: 1 comparison and 1 swap
  All together: c ((N-1) + (N-2) + ... + 1) +k where c is the time required to do one comparison and one swap

Complexity of bubble sort

c ((N-1) + (N-2) + ... + 1) +k

(N-1) + (N-2) + ... + 1
+
1 + 2 +… + (N-1) =
= N + N + … + N = N * (N-1)

so our function equals

c N*(N-1)/2 + k = 1/2c (N^2-N) +k

complexity O(N^2).

Complexity of bubble sort

?

$1/2c\,(N^2-N) + k \le K^* N^2$

for which values of K and N is this inequality true?

For example, K = c+ k and N > 0 (provided N can only take integer values).

Selection sort

23	10	12	5

pos_greatest=0

Selection sort

23	*10*	12	5

pos_greatest=0

Selection sort

23	10	*12*	5

pos_greatest=0

Selection sort

23	10	12	5

pos_greatest=0

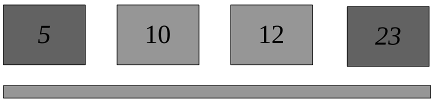
Selection sort

23	10	12	*5*

pos_greatest=0

swap(0,3)

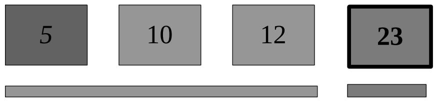
Selection sort



pos_greatest=0

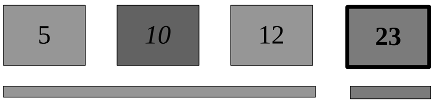
swap(0,3)

Selection sort



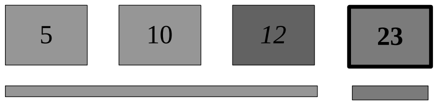
pos_greatest=0

Selection sort



pos_greatest=1

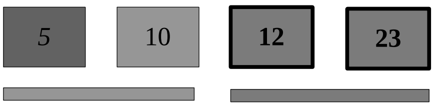
Selection sort



pos_greatest=2

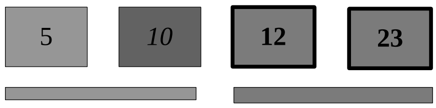
swap(2,2)

Selection sort



pos_greatest=0

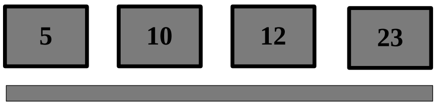
Selection sort



pos_greatest=1

swap(1,1)

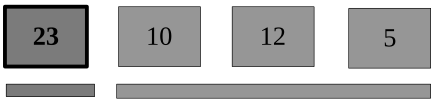
Selection sort



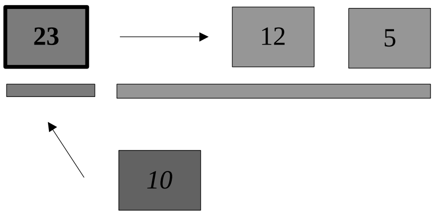
Complexity of selection sort

- Same number of iterations in the worst case
- for some different constant c', c' $(N^2 - N) + k$
- also $O(N^2)$

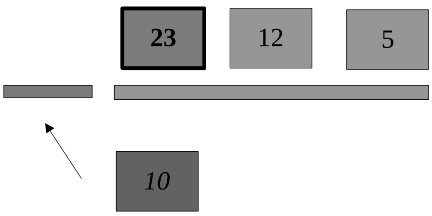
Insertion Sort



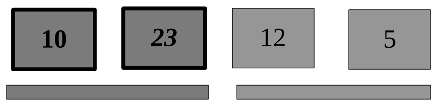
Insertion Sort



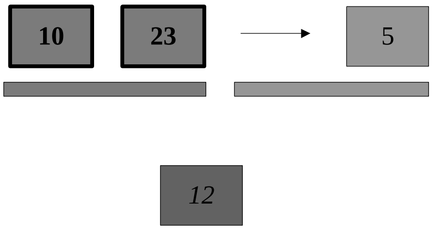
Insertion Sort



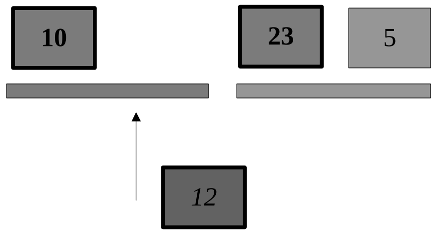
Insertion Sort



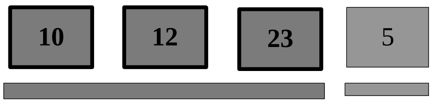
Insertion Sort



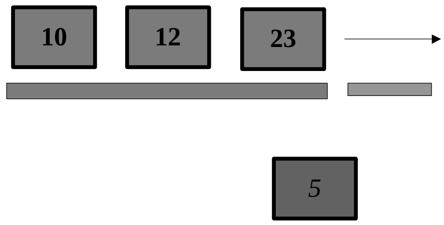
Insertion Sort



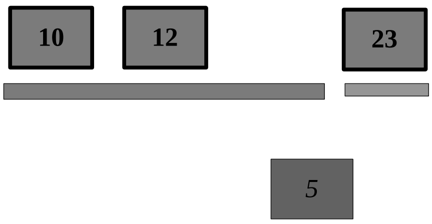
Insertion Sort



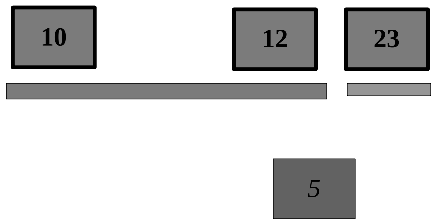
Insertion Sort



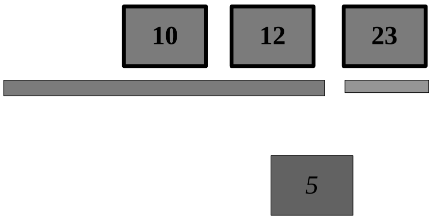
Insertion Sort



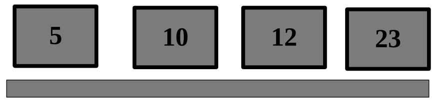
Insertion Sort



Insertion Sort



Insertion Sort



Complexity of insertion sort

- In the worst case, has to make N(N-1)/2 comparisons and shifts to the right
- also O(N^2) worst case complexity
- best case: array already sorted, no shifts.

Reading and informal coursework

- Shaffer, Chapter 8.1, 8.2.
- Informal coursework: prove that the simple sorting algorithms above are not in O(N).