MODEL ANSWER TO INFORMAL COURSEWORK 2

Question 1

To show that n^2+7n+4 is in $O(n^2)$, we have to prove that there are positive numbers n_0 and c such that, for all $n > n_0$, $n^2+7n+4 \leq cn^2$.

Let's note that when $n > 1$, $n < n^2$ and $1 < n^2$. Therefore, with $n > 1$, $7n < 7n^2$ and $4 < 4n^2$, and hence $n^2+7n+4 \leq n^2+7n^2+4n^2$. Consequently, for all $n > 1$, $n^2+7n+4 \leq 12x^2$; thus, $n_0 = 1$ and $c = 12$.

Question 2

We have to find such n_0 and c that, for all $n > n_0$, $3log_4 n^9 + 5 \leq clog_2 n$.

First, let's note that, for any n, $3log_4 n^3 = 9log_4 n = \frac{9}{2}log_2 n$. Secondly, when $n > 2$, $1 < log_2 n$ and hence $5 < 5log_2 n$. Therefore, for $n > 2$, $3log_4 n^3 + 5 \leq \frac{9}{2}log_2 n + 5log_2 n = 9\frac{1}{2}log_2 n$. Thus, $n_0 = 2$ and $c = 9\frac{1}{2}$.

Question 3

We have to find such n_0 and c that, for all $n > n_0$, $7n^2 \leq cn^3$.

Let's note that when $n > 7$, $7n^2 < n^3$. Thus, we can take $n_0 = 7$ and $c = 1$.

Question 4

No, x^3 is not in $O(7x^2)$. This can be proven by *reductio ad absurdum*. Let's suppose that x^3 *is* in $O(7x^2)$. Then, by definition of O, there exist positive numbers n_0 and c such that, for all $n > n_0$, $n^3 \leq c \times 7n^2$. Since n, and hence n^2, is a positive number, we can devide both sides of the above inequality by n^2, again obtaining a valid inequality. Thus, $n \leq c \times 7$, for all $n > n_0$. This last assertion is, however, absurd since n, being an arbitrarily large number, may clearly exceed $c \times 7$, no matter what c is. Thus, our assumption that x^3 is in $O(7x^2)$ has led to absurd, hence it was false. Therefore, x^3 is not in $O(7x^2)$.