Modal Logic

Exercises and preliminary exam question(s)

Exercises for the first day:

1. Prove in K: $\Diamond(\phi \vee \psi) \to (\Diamond\phi \vee \Diamond\psi)$
2. Prove in K: $\Diamond(\phi \wedge \psi) \to \Diamond\phi$
3. Prove in K: $\Box(\phi \leftrightarrow \psi) \to (\Box\phi \leftrightarrow \Box\psi)$
4. Prove validity of axiom K for arbitrary ϕ, ψ: $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$
5. Generalise definition of derivability in K from an empty set of premises to derivability from a set Γ.
6. For which semantic relation $\models_i$ holds: $\Gamma \vdash_K \phi \;\Rightarrow \Gamma \models_i \phi$:
   (a) $\models_w$: for every M, w, if $M, w \models \Gamma$ then $M, w \models \phi$
   (b) $\models_M$: for every M, if $M \models \Gamma$ then $M, w \models \phi$
   (c) $\models_g$: if $\models \Gamma$ then $\models \phi$
7. Find as many omissions, imprecisions and bugs in the lecture as you can and let me know.

Exercises for the second day:

1. In the lecture, I hope to describe Tarjan's algorithm for deciding whether two graphs are bisimilar. I will ignore propositional variables. Write the full version of the algorithm which does not ignore propositional variables.

Preliminary exam questions (please wait for a confirmation on `http://www.cs.nott.ac.uk/~nza/modal.html`. I will add more details for the last question after I talk to Thorsten about how he defined intuitionistic Kripke models.)

Choose ONE of the following three questions:

1. Submit all the exercises above (don't forget bugs in the lectures for both days)
2. Write an algorithm which given a finite Kripke structure $M = (W, R, V)$ and a formula of basic modal logic ϕ annotates each world $w \in W$ with subformulas of ϕ which are true at w. Give an inductive definition of $Subf(\phi)$ while you are at it.
3. Define a translation function t from the language of propositional intuitionistic logic into the language of basic modal logic so that for every intuitionistic formula ϕ, ϕ satisfiable in an intuitionistic Kripke model iff $t(\phi)$ is satisfiable in a Kripke structure with a reflexive and transitive accessibility relation. Prove that your t is satisfiability preserving.

Recommended literature:

1. P. Blackburn, M. de Rijke, and Y. Venema. *Modal Logic*, volume 53 of *Cambridge Tracts in Theoretical Computer Science*. Cambridge University Press, 2001. **For most systems and logical background.**

2. Sally Popcorn. *First Steps in Modal Logic*. Cambridge University Press, 1994. **For very precise introduction to basic multimodal logic.**

3. M. Huth and M. Ryan. *Logic in Computer Science: Modelling and reasoning about systems*. Cambridge University Press, 2000. **For exposition aimed at computer scientists, easy introduction to model checking.**

4. R. Paige and R. Tarjan. Three partition refinement algorithms. In *SIAM Journal of Comput.* **16**(6), 1987, 973–989. **Bisimilarity checking algorithm.**