Model checking coalitional games in shortage resource scenarios

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Joint work with Margherita Napoli and Mimmo Parente

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Outline

1. Context
   - Multi-Agent Systems (MAS) — ***ATL***
   - MAS + resource constraints — ***RB-ATL / RAL***
2. Our proposal: *Priced* RB-ATL — ***PRB-ATL***
   - Model checking (lower bound)
   - Optimization problem
3. Conclusions and future work

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Multi-Agent Systems (MAS)

- Several agents
- Intelligent (take decisions, moves)
- Independent
- Global state (union of single states)
- Next state univocally identified by moves

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COALITION - modeling collective behaviors/strategies

Agents and coalitions

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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

Theorem (Goranko, TARK 2001)

CL *can be embedded into* ATL

ATL: syntax and semantics

Formulae of ATL are given by the grammar:

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Formulae of ATL predicate about abilities of coalitions of agents

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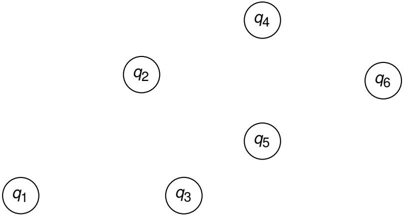
Formulae of ATL predicate about abilities of coalitions of agents

Formulae of ATL are evaluated wrt:

- a game structure (or game arena) G
- a location q of G

The arena of ATL

A game structure G is a state transition graph:



- vertices labeled by atomic propositions
- in vertices agents choose actions
- possible combinations $\rightarrow$ transitions (edges of the graph)

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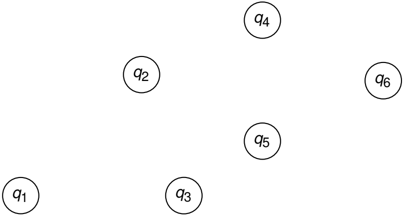

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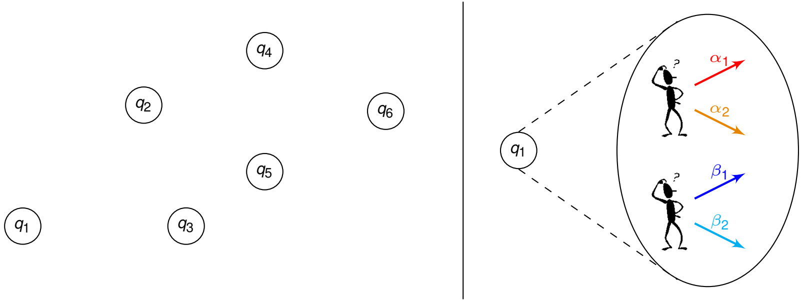

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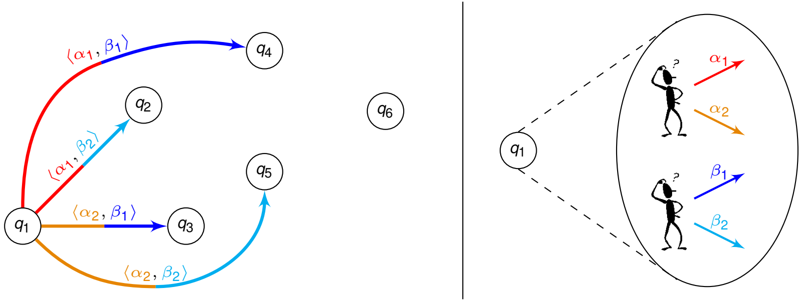

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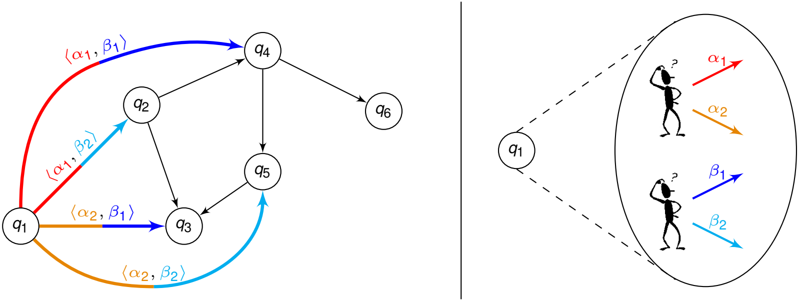

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Becoming friendly with ATL

Collective strategy for the proponent team to guarantee p holds

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$\langle\langle A\rangle\rangle \Box p$	always
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regardless of actions performed by other agents (opponent)

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Addition of bounds on resources to ATL

Resources are bounded

Extensions of ATL with bounds on resources

$$\langle\langle A^{\eta} \rangle\rangle \Box p$$

Endowment: $\eta : A \rightarrow \mathbb{N}^r$ (r = number of resources)

The literature about Resource Bounded ATL (RB-ATL)

RB-ATL [Alechina, Logan, Nga, Rakib, AAMAS 2010]

Model checking RB-ATL is decidable in $O(|\varphi|^{2 \cdot r+1} \times |G|)$

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Several logic variants, exploration of the (un)decidability border

E.g., if actions produce resources, Model Checking is generally UNDECIDABLE

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Under some conditions, RB$\pm$ATL (with production) is decidable

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Unification [Alechina, Bulling, Logan, Nga, IJCAI 2015]

Unifying several approaches and exploring decidability borders

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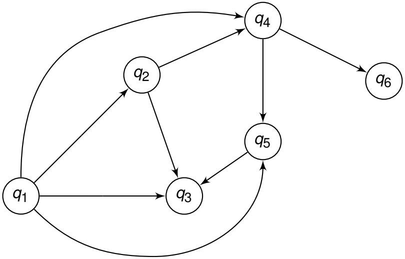

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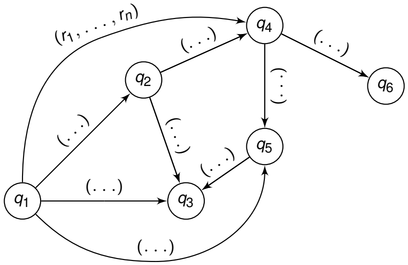

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team A, equipped with endowment η, can force the next state to be s.t. team A itself, equipped with the new endowment η', can guarantee that p always holds

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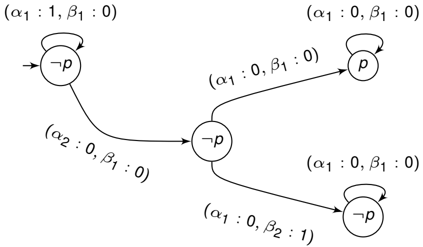
Shared resources: Example

2 agents: **a** and **b**
1 resource type: $\mathbf{r_1}$

$$G, q_0 \Vdash \langle\langle a^\eta \rangle\rangle \Diamond p$$



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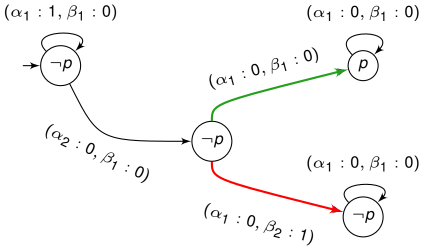

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$\eta(a) = any$
$\eta(b) > 0$

true with shared resources

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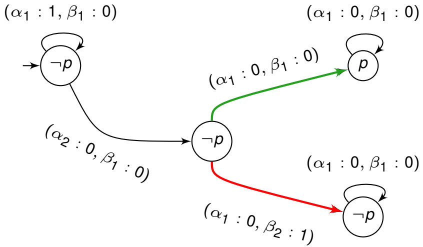

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proponent has the ability of consuming all resources to make opponent weak

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What we want

Public/shared resources + private ones (money)

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$\Rightarrow$ global availability of resources on the market

- a semantic component (part of the arena)
- evolves depending on agents' actions (also opponent)
- affects the choice of the actions (also opponent)

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$\Rightarrow$ price of resources

- agents equipped with money (private resources)
- money for getting resources
- price of resources function of several components (take into account the history of the system)

Resources vs. money

Resources

- part of the model
- represent the market (nature)
- public: agents draw on resources from a shared pool
- known
- availability checked for all agents

Money

- inside the formula
- assigned to agents
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Money is a *meta-resource*

- buy resources
- unit of measurement

Resource production and decidability

Alechina, Logan, Nga, Rakib, AAMAS 2010

Actions can **only consume** resources

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If actions produce resources,
Model Checking is generally UNDECIDABLE

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Actions may produce resources... but ***not so much***!!!

- model checking decidable
- several models fit

  (e.g. memory usage, leasing a car, releasing resources previously acquired)

Syntax and semantics

Formulae of PRB-ATL are given by the grammar:

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Priced game structure

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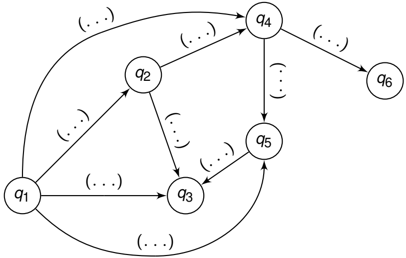

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- resources have a variable prices
- transition guards: also opponent

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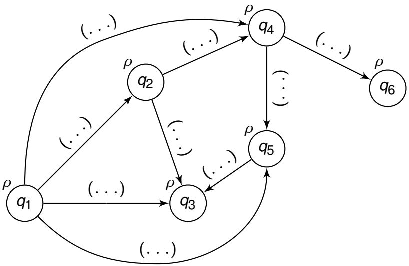

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Model checking complexity

Theorem

The model checking problem for PRB-ATL *is EXPTIME-complete*

- *membership (upper bound)* [LAMAS 2011]
- *hardness (lower bound)* [GandALF 2013]

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Reduction from the acceptance problem for Linearly-Bounded Alternating Turing Machine

Parametrized reduction

The algorithm runs in time $O(|\varphi| \cdot |G| \cdot M^{r+n})$

Model checking is exponential in

- n: number of agents
- r: number of resources
- size of M: max. component in resource/money vectors
  (when represented in binary)

1st reduction: parametric in the size of M (n and r are constant)

2nd reduction: parametric in r (n and M are constant)

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3rd reduction: parametric in n (r and M are constant)
OPEN PROBLEM

Linearly-Bounded Alternating Turing Machines

LB-ATM are Turing Machines

- linearly-bounded: tape length is bounded by a linear function of the size of the input word *w*
- alternating: existential and universal states

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Finite control:

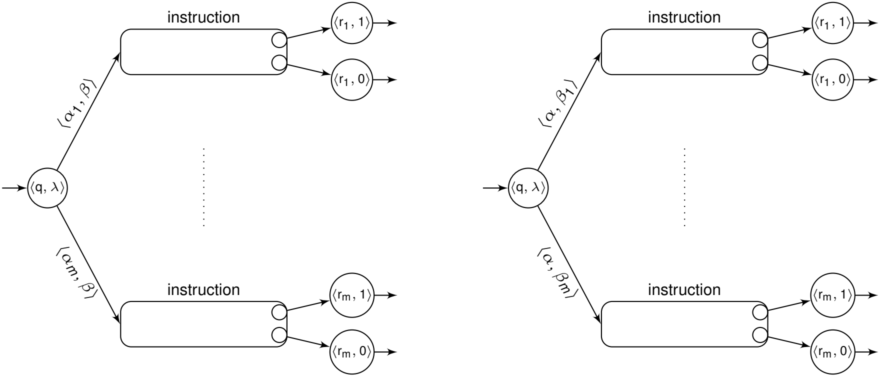
$$\langle q, \lambda \rangle \to \langle r_1, \nu_1, \sim_1 \rangle$$
$$\langle q, \lambda \rangle \to \langle r_2, \nu_2, \sim_2 \rangle$$
$$\ldots$$
$$\langle s, \gamma \rangle \to \ldots$$
$$\ldots$$

$q, s, r_i \in Q$: internal states
$\lambda, \gamma, \nu_i \in \Sigma$: alphabet symbols
$\sim_i \in \{\leftarrow, \rightarrow\}$: head movements

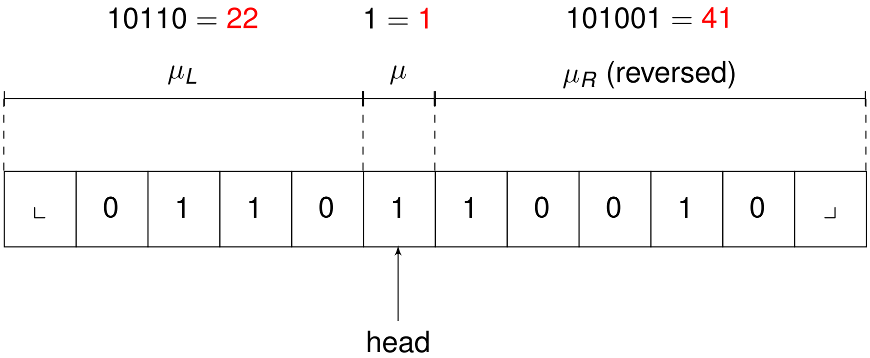
Sketch of the 1st reduction (I)

Encoding of instructions $\langle q, \lambda \rangle \to \langle r_i, \nu_i, \sim_i \rangle$ matching a full state $\langle q, \lambda \rangle$

q existential state　　　　*q* universal state

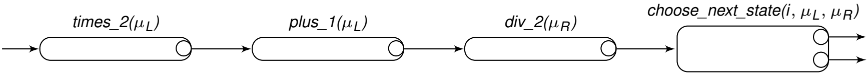


Encoding of the tape

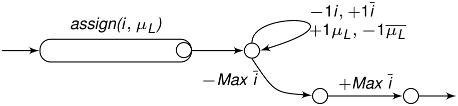


Sketch of the 1st reduction (II)

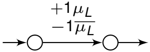
Module *shift_right_with_inc*



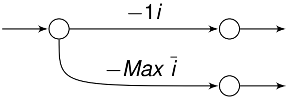
Module *times_2*(μ_L)



Module *plus_1*(μ_L)



Module *choose_next_state*(i, μ_L, μ_R)



Theorem

For each PRB-ATL formula φ, and each priced game structure G:

$$[\varphi] = [\varphi]^{ml}.$$

Outline

1. Context
   - Multi-Agent Systems (MAS) — ***ATL***
   - MAS + resource constraints — ***RB-ATL / RAL***
2. Our proposal: *Priced* RB-ATL — ***PRB-ATL***
   - Model checking (lower bound)
   - Optimization problem
3. Conclusions and future work

Parametric PRB-ATL formulae

- PRB-ATL: $\varphi = \langle\langle A_1^{\$_1} \rangle\rangle \Diamond (\langle\langle A_2^{\$_2} \rangle\rangle \bigcirc p \vee \langle\langle A_3^{\$_3} \rangle\rangle q \mathcal{U} p)$

Definition (Cost of a PRB-ATL formula)

$$f\_cost(\varphi) = \$_1(A_1) + \$_2(A_2) + \$_3(A_3)$$

- parametric PRB-ATL: $\varphi_{\vec{X}} = \langle\langle X_1^{\$_1} \rangle\rangle \Diamond (\langle\langle X_2^{\$_2} \rangle\rangle \bigcirc p \vee \langle\langle A_3^{\$_3} \rangle\rangle q \mathcal{U} p)$

The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

To determine minimal-cost coalitions that satisfy a PRB-ATL formula

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Theorem

The Optimal Coalition problem is EXPTIME-complete

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Conclusions and future works

Conclusions

- Theorem: Model checking PRB-ATL is EXPTIME-complete
  Reachability for PRB-ATL is EXPTIME-complete

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Future works

- 3rd reduction: parametric in n (r and M are constant)
- Exact complexity when actions cannot produce resources
  - Reachability is NP-hard
  - Model checking is PSPACE-hard
- Expressiveness comparative analysis wrt. other existing formalisms
- Resource-bounded extensions of other classical formalisms
  - μ-calculus [Della Monica, Lenzi - ICAART 2012]
  - ATL* ???
  - . . .

Conclusions and future works

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  Reachability for PRB-ATL is EXPTIME-complete

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The end

Thank you!