Model checking invariants for resource-bounded MAS with discounting

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LRBA 2015

Introduction

- Existing logics for resouce-bounded MAS concentrate on modelling costs of action wrt private and/or shared resources:
  - Private: each agent has a local storage for resources not accessible by other agents;
  - Shared: all agents can access a public storage for resources.
- Costs of computations (runs) are defined as the sum of costs of individual actions along the computations.
- However, costs can also be computed in other ways similar to calculating *payoff profiles* in Repeated Game Theory:
  - Discounting: action costs are gradually reduced by a discount factor ≤ 1.
- Our aim: using discounting in formulating and model checking invariant properties.

Syntax of RB$\pm$ATL with discounting

Resources and bounds

- Resources: $Res = \{res_1, \ldots, res_r\}$ for some $r \geq 1$
- Resource bounds: $B = (\mathbb{N} \cup \infty)^r$

Example: a system of autonomous satelites.

- $Res = \{electricity, fuel, oxygen\}$ with number of resources $r = 3$.
- bound $(2000, 100, 50)$ means not to spend spending more than 2000 units of electricity, 100 of fuel and 50 of oxygen.
- bound $(2000, \infty, 50)$ means not to spend spending more than 2000 units of electricity, 50 of oxygen but there is no limitation on spending fuel.

Syntax

dRB±ATL

$$\varphi \leftarrow p \mid \neg\varphi \mid \varphi_1 \vee \varphi_2$$

$\langle\langle A^b \rangle\rangle \bigcirc \varphi$ A can enforce φ next w/o spending $> b$

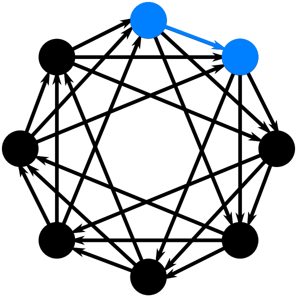
$\langle\langle A^b \rangle\rangle \varphi \mathcal{U} \psi$ A can maintain φ until ψ w/o spending $> b$

$\langle\langle A^b \rangle\rangle \Box_\beta \varphi$ A can maintain φ forever w/o spending $> b$ while cost is reduced by factor $\beta \in [0, 1]$

Semantics of dRB$\pm$ATL

Resource-bounded CGS

$$(Agt, \boxed{Res,} S, \Pi, \pi, Act, d, \boxed{c,} \sigma)$$



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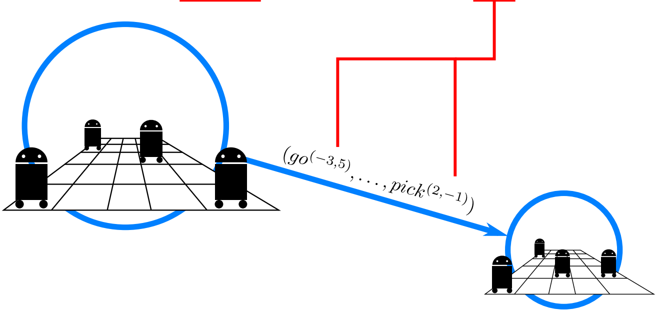

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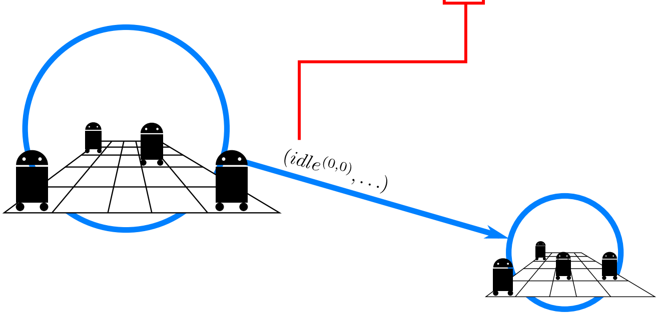
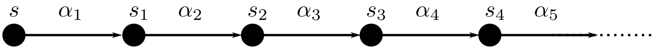


Strategies, Computations and Costs

- Strategy: $F_A : S^+ \to Act^{|A|}$ st $\forall s : F_A(\lambda s) \in D_A(s)$
- Consistent computation from a state s:

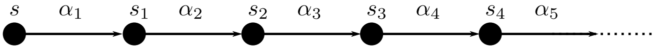


- Cost of computation:

$$\sum \begin{array}{|ccccc|c} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & \cdots \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\ c_{k1} & c_{k2} & c_{k3} & c_{k4} & c_{k5} & \cdots \end{array}$$

b-consistent computations

- Consistent computation:



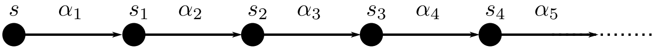
- b-consistent computation:

$$
\sum \boxed{\begin{array}{ccccc} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{k1} & c_{k2} & c_{k3} & c_{k4} & c_{k5} \end{array}} \begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \end{array}
$$

$$\leq b$$

b-consistent computations wrt discount factor

- Consistent computation:



- *b*-consistent computation wrt discount factor β:

$$\sum \boxed{\begin{array}{ccccc} c_{11} & \beta^1 c_{12} & \beta^2 c_{13} & \beta^3 c_{14} & \beta^4 c_{15} \\ c_{21} & \beta^1 c_{22} & \beta^2 c_{23} & \beta^3 c_{24} & \beta^4 c_{25} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{k1} & \beta^1 c_{k2} & \beta^2 c_{k3} & \beta^3 c_{k4} & \beta^4 c_{k5} \end{array}} \begin{array}{c} \dots \\ \dots \\ \dots \\ \dots \end{array}$$

$$\leq b$$

Truth evaluation: dRB$\pm$ATL

- Boolean connectivities are standard;
- $M, s \models \langle\langle A \rangle\rangle \bigcirc \varphi$ iff there exists F_A st for any consistent λ from s : λ is b-consistent and $M, \lambda[1] \models \varphi$;
- $M, s \models \langle\langle A \rangle\rangle \varphi \mathcal{U} \psi$ iff there exists F_A st for any consistent λ from s : λ is b-consistent, $\exists i \geq 0 : M, \lambda[i] \models \psi$ and $\forall 0 \leq j \leq i : M, \lambda[j] \models \varphi$;
- $M, s \models \langle\langle A \rangle\rangle \Box_\beta \varphi$ iff there exists F_A st $\forall$ consistent λ from s : λ is b-consistent wrt discount factor β and $\forall i \geq 0 : M, \lambda[i] \models \varphi$.

Model checking of dRB±ATL

Problem statement

- Input: M, s, φ.
- Output: *Yes* (if $M, s \models \varphi$) or *No* (otherwise).

Outermost algorithm

function $\textsc{RB}\pm\textsc{ATL-LABEL}(M, \phi)$

for $\phi' \in Sub(\phi)$ **do**

case $\phi' = p,\ \neg\psi,\ \psi_1 \wedge \psi_2,$
$\langle\langle A\rangle\rangle \bigcirc \psi,\ \langle\langle A\rangle\rangle \psi_1 \,\mathcal{U}\, \psi_2,\ \langle\langle A\rangle\rangle \Box \psi$

inherited from ATL

case $\phi' = \langle\langle A^b\rangle\rangle \bigcirc \psi, \langle\langle A^b\rangle\rangle \psi_1 \,\mathcal{U}\, \psi_2, \langle\langle A^b\rangle\rangle \Box_1 \psi$

inherited from RB±ATL

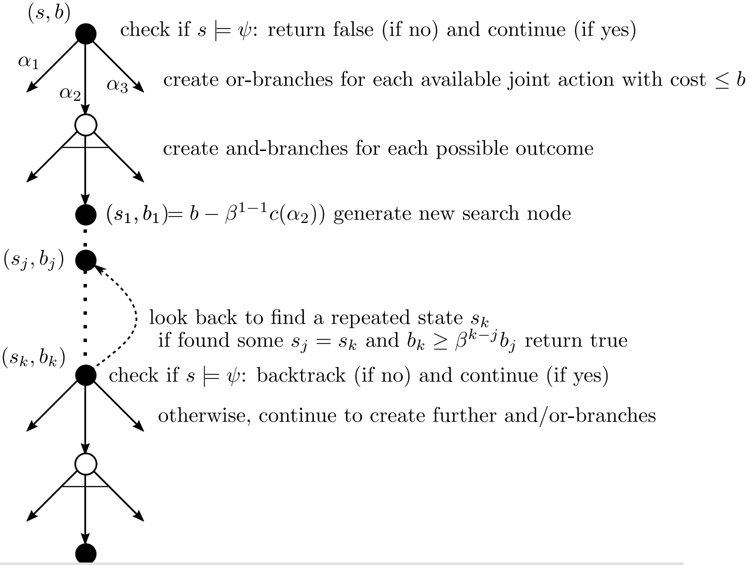
case $\phi' = \langle\langle A^b\rangle\rangle \Box_\beta \psi$ where $\beta < 1$

$[\phi']_M \leftarrow \{\ s \mid s \in S \wedge$
$\textsc{BOX-STRATEGY}(node_0(s, b), \langle\langle A^b\rangle\rangle \Box_\beta \psi)\}$

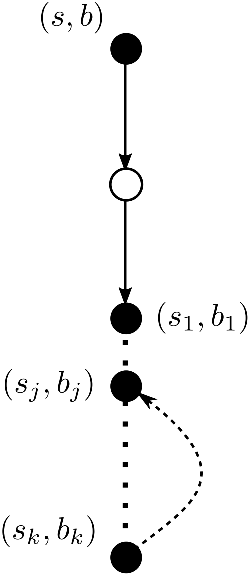
return $[\phi]_M$

Illustrating BOX-STRATEGY

And/or Search for a feasible strategy:



Towards correctness



- Essentially, BOX-STRATEGY looks for every and branch a loop, e.g., from $s_k \to s_j$.
- To carry out this loop forever, we need to check if the resource bound b_k is large enough, i.e.,

$$b_k \geq \frac{\beta^{k-j}(b_j - b_k)}{1 - \beta^{k-j}} \text{ iff } b_k \geq \beta^{k-j} b_j$$

Summary

- Introduced discounting to resource-bounded logic RB$\pm$ATL: on both syntactic and semantics level:
  - In the syntax, discounting factor is attached to box operator;
  - In the semantics, discounting factor is used for computing cost of infinite computations.
- Furture work:
  - Formalise the model checking algorithm, show correctness and termination;
  - Extend discounting to until operator.

Thank You!