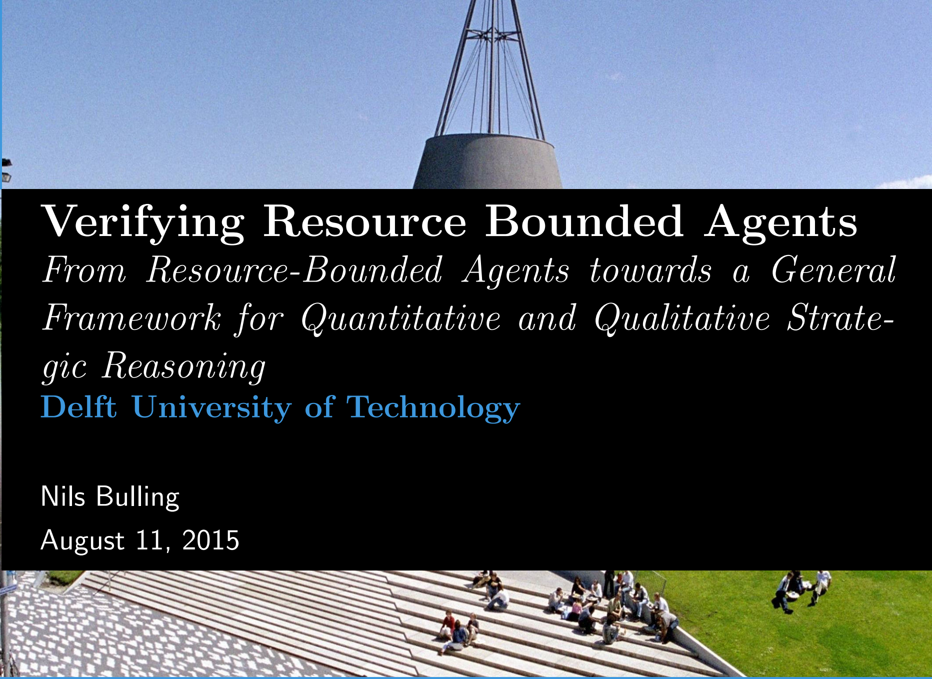

Verifying Resource Bounded Agents

From Resource-Bounded Agents towards a General Framework for Quantitative and Qualitative Strategic Reasoning

Delft University of Technology

Nils Bulling

August 11, 2015

Outline

Next Section

Motivation

Logics for MAS: specification and verification

- **Strategic logics**
  - What can teams of agents achieve?
  - Can a set of interacting processes ensure correct functioning?

⇝ **alternating-time temporal logic** (ATL)[Alur et al., 2002]

- **Resources**

  present in and crucial for many multi-agent systems
  - Do agents have sufficient energy to achieve a task?
  - Can a team of robots defend the base with the given energy status?
  - Do agents have enough resources and capabilities to complete a task?

⇝ many variants with resources: **Resource Agent Logics** (RAL)

Be careful with resources:

- RAL + unbounded production/consumption of resources
  - (model checking over) Petri nets
  - (model checking over) vector addition systems
- rule of thumb: often undecidability if **zero-test** can be encoded
- but: **decidable model checking** possible. . . when?

Today's talk:

1. introduce general resource-bounded framework
2. review some undecidablity results
3. review some decidable cases
4. motivate general quantitative, game theoretic setting ⇝ Valentin

focus of talk: **key concepts and techniques**

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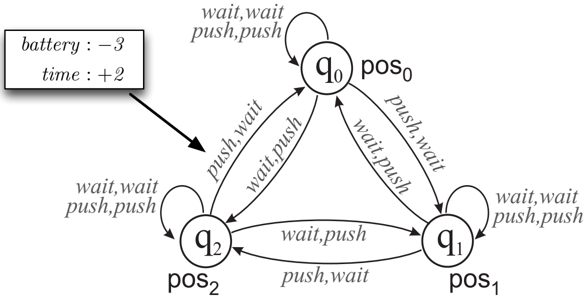
Resource-Bounded Models

- Computational systems often need a notion of resource.
- **Resource-bounded agents**
- Actions **consume** / **produce** resources.
- Non-empty set $\mathcal{R} = \{r_1, \ldots, r_\rho\}$ of **resources**.

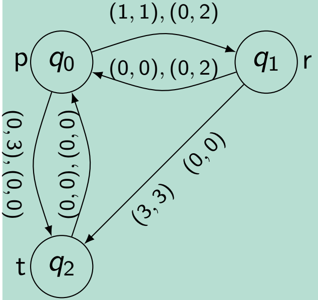


A Single Agent Example

Example (Resource-Bounded Tree Logic [Bulling and Farwer, 2010a])

RTL replaces CTL's path operator: $\mathsf{E}\gamma \rightsquigarrow \langle\rho\rangle\gamma$

$\mathfrak{M}, q \models \langle\rho\rangle\varphi$ iff $\exists$ ρ-**feasible path** λ such that $\mathfrak{M}, \lambda \models \varphi$



- feasible path: $(q_0, (\infty, 1))(q_1, (\infty, 2))(q_0, (\infty, 4))\ldots$
- resources ≥ 0
- $\mathfrak{M}, q_0 \models \langle(\infty, 1)\rangle \mathbf{G}\top$
- $\mathfrak{M}, q_0 \models \langle(1, \infty)\rangle \mathbf{G}(\mathsf{p} \vee \mathsf{t})$
- Note: nested operators re-set resources $\langle\rho_1\rangle\mathbf{F}\langle\rho_2\rangle\mathbf{F}\mathsf{p}$.

Main result: **Model checking RTL is decidable** (open for RTL*) (reduction to Petri net reachability)

Related Work on Resource Agent Logics

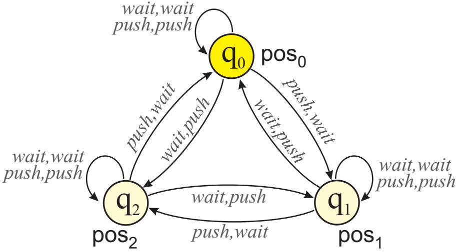
- Resource-Bounded Coalition Logic [Alechina et al., 2009]
  $\rightsquigarrow$ only consumption, Coalition Logic
- Resource-Bounded Alternating Time Temporal Logic
  [Alechina et al., 2014, Alechina et al., 2010]
  $\rightsquigarrow$ only consumption (**RB-ATL**), axiomatization, model checking, consumption & production, resource flat, proponent restricted, (**RB+-ATL**) ATL -based
- Resource Agent Logic
  [Bulling and Farwer, 2010b, Alechina et al., 2015]
  $\rightsquigarrow$ consumption & production RAL, undecidability & decidability, shared resources, ATL -based
- Resources and money [Della Monica et al., 2011]
  $\rightsquigarrow$ decidability, bounded shared resources, ATL -based

What makes settings (un)decidable?

Concurrent Game Structures and ATL

Agents:

- execute actions
- cooperate
- model: **concurrent game structure**



Strategic logic ATL (Alur et al. 1997-2002):

- $\langle\!\langle A \rangle\!\rangle\gamma$ “Group A has a strategy to **guarantee** γ”
- **ATL**: $\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\!\langle A \rangle\!\rangle \mathbf{X}\varphi \mid \langle\!\langle A \rangle\!\rangle \mathbf{G}\varphi \mid \langle\!\langle A \rangle\!\rangle \varphi \mathbf{U} \varphi$
- **ATL***: Allows arbitrary combinations of cooperation and temporal modalities (e.g. $\langle\!\langle A \rangle\!\rangle \mathbf{GF}\varphi$).

Example: $\mathfrak{M}, q_0 \models \langle\!\langle 1 \rangle\!\rangle \mathbf{G}\neg \mathrm{pos}_1$ $\mathfrak{M}, q_0 \not\models \langle\!\langle 1 \rangle\!\rangle \mathbf{F}\mathrm{pos}_1$

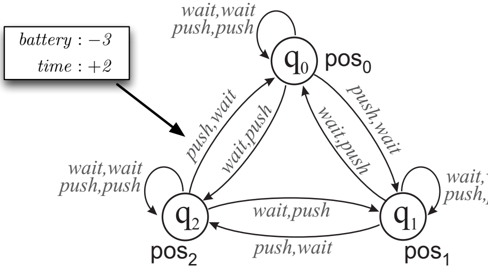
Variants of Resource agent logics

- transitions have costs (or rewards) and the syntax can **express resource requirements for a strategy**, e.g.:

  agents A can enforce outcome φ if they have at most b_1 units of resource r_1 and b_2 units of resource r_2

In the following:

- consumption & production
- unbounded resources
- all agents may act under resource constraints



Definition (Resource Agent Logic RAL [Bulling and Farwer, 2010b])

RAL-formulae are defined by:

$$\phi ::= \mathsf{p} \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle A\rangle\rangle_B^{\downarrow}\mathbf{X}\varphi \mid \langle\langle A\rangle\rangle_B^{\eta}\mathbf{X}\varphi \mid \langle\langle A\rangle\rangle_B^{\downarrow}\varphi\mathbf{U}\psi \mid \langle\langle A\rangle\rangle_B^{\eta}\varphi\mathbf{U}\psi \mid \langle\langle A\rangle\rangle_B^{\downarrow}\mathbf{G}\varphi \mid \langle\langle A\rangle\rangle_B^{\eta}\mathbf{G}\varphi$$

where $\mathsf{p} \in \Pi$ is a proposition, $A, B \subseteq \mathbb{A}\mathrm{gt}$ are sets of agents, and η is a resource endowment.

$\langle\langle A\rangle\rangle_B^{\eta}\varphi$: agents A have a strategy compatible with the endowment η to enforce φ whatever the opponent agents do (opponents in B also act under resource bound η)

$\langle\langle A\rangle\rangle_B^{\downarrow}\varphi$: agents A have a strategy compatible with the current resource endowment to enforce φ whatever the opponent agents do (opponents in B also act under the current resource bound)

Computational costs: $\rightsquigarrow$ $\boxed{\langle\langle A\rangle\rangle^{\eta_1}\mathbf{X}\langle\langle A\rangle\rangle^{\eta_2}\gamma \text{ vs. } \langle\langle A\rangle\rangle^{\eta_1}\mathbf{X}\langle\langle \mathbf{A}\rangle\rangle^{\downarrow}\gamma}$

Important fragments

rfRAL: **resource-flat RAL**, each nested ATL operator has a fresh assignment of resources ($\langle\langle A\rangle\rangle_B^{\downarrow}\varphi$ is not allowed):

given their initial fuel, rescue robots A can safely get to a position from which they can refuel and perform rescue while in visual contact with the base

$$\langle\langle A\rangle\rangle_A^{\eta_{\text{init}}}(\text{safe}\,\mathbf{U}\,(\langle\langle A\rangle\rangle_A^{\eta_{\text{refuel}}}(\text{visual}\,\mathbf{U}\,\text{rescue})))$$

contrast: $\langle\langle A\rangle\rangle_A^{\eta_{\text{init}}}(\text{safe}\,\mathbf{U}\,(\langle\langle \mathbf{A}\rangle\rangle_{\mathbf{A}}^{\downarrow}(\text{visual}\,\mathbf{U}\,\text{rescue})))$

prRAL: **proponent-restricted RAL**, only the strategy of the proponent agents is resource bounded—the opponent agents have no resource bound $\langle\langle A\rangle\rangle_A^{\eta}\varphi$, $\langle\langle A\rangle\rangle_A^{\downarrow}\varphi$

rfprRAL: combination

Strategies and Their Outcome

- Perfect information perfect recall strategy for agent a (**IR-strategy**):

$$s_a : Q^+ \rightarrow Act \ .$$

- Perfect information memoryless strategy for agent a (**Ir-strategy**):

$$s_a : Q \rightarrow Act$$

- ATL: it is known that memory does not matter [Alur et al., 2002]

if agents can win with memory they can also do so without!

- RAL: **memory does (usually) matter**!

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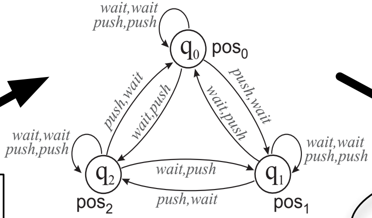
Model Checking

Formal model



Problem
(e.g. mobile phone)
+
(Safety) Property
(e.g. deadlock free)

$$\mathfrak{M} \overset{?}{\models} \langle\langle\{1,2\}\rangle\rangle \Box \bigcirc \top$$

Let's model ckeck...

Computational Complexity?

$$\varphi = \langle\langle\{1,2\}\rangle\rangle \Box \bigcirc \top$$

Logical (formal) specification

Overview: (Un)Decidability

- variants of RTL: language, memory, models
- unbounded production ⇒ mostly undecidable
- overview results of [Bulling and Farwer, 2010b]:

	$\mathcal{L}_{RAL^*}$	$\mathcal{L}_{RAL^+}$	$\mathcal{L}_{RAL}$	$pr\text{-}\mathcal{L}_{RAL^*}$	$pr\text{-}\mathcal{L}_{RAL^+}$	$pr\text{-}\mathcal{L}_{RAL}$
$\models_R$	U^1	U^1	U^1	U^1	U^1	U^1
$\models_r$	U^1	U^1	U^1	U^2	U^2	U^2
$rf+\models_R$ / $\models_R^\infty$	U^2	U^2	U^2	U^2/ U^2_∞	**?** / U^2_∞	**?** / U^2_∞
$rf+\models_r$	**?**	**?**	**?**	**?**	**?**	**?**
$\models_R^k$, $\models_r^k$	**D**	**D**	**D**	**D**	**D**	**D**

- Decidability with **unbounded production**:
  - **RB±ATL** [Alechina et al., 2014]:
    (1) resource-flat, (2) proponent restricted, (3) idle action
  - prRAL^r [Alechina et al., 2015]:
    (1) proponent restricted, (2) idle action, (3) positive fragment
  - 1-shared unbounded resource [Bulling and Nguyen, 2015]

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Undecidability of rfRAL over **iRBMs**

Models	RAL	rfRAL	prRAL	rfprRAL
RBM	U [1]	U [1]	U [1]	U [1]
iRBM	U [1]	**U[3]**	**U** [1, 3]	D [2]

RBM Resource Bounded Models (infinite semantics)
iRBM Resource Bounded Models with **idle actions**

We also show undecidability wrt. 1 resource type

[1] Bulling & Farwer 2010
[2] Alechina et al. 2014
[3] Alechina et al. 2015

An aside: **RBM** + finatary semantics = **iRBM** + std. semantics

High-Level Idea of Reduction

1. reduce halting problem for **two counter machines** (pushdown automaton with two stacks)

2. encode transition table as an **iRBM**
   two counters simulated by two resource types

3. two agents:
   - (1) **simulator agent** selects transitions of the automaton
   - (2) the **spoiler** agent is used to ensure that only valid transitions are selected by the simulator agent

   spoiler agent used to encode **zero-test**

Observation

Proponent restrictedness is essential for decidability, even over **iRBM**s

Two-counter automaton[Hopcroft and Ullman, 1979]

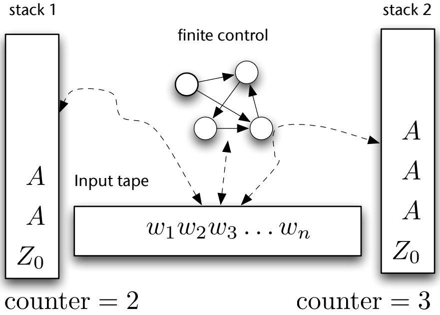
Two-counter automaton is essentially a PDA with 2 stacks.

Transitions depend on

- state,
- symbol read,
- counters **zero or non-zero**.

Counters:

- $+1$
- -1



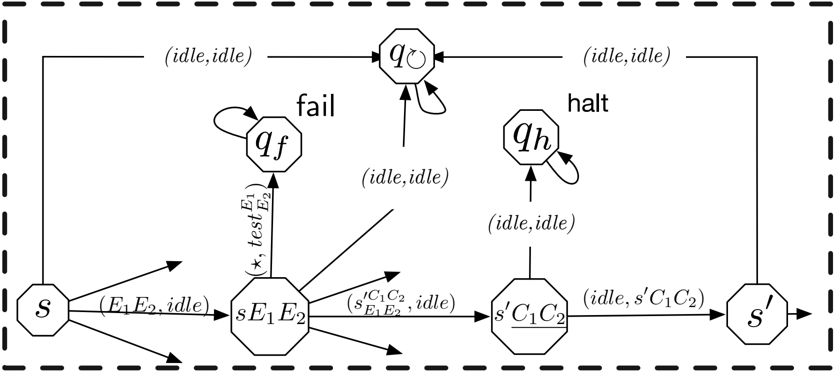
Crucial: The logic is used to implement the **zero/emptiness test**

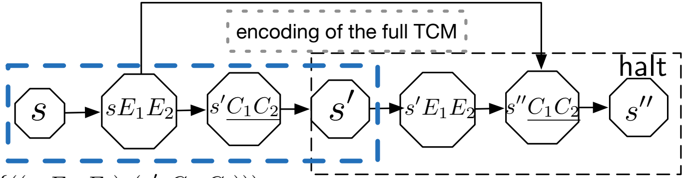
Does $\mathcal{A}$ **halt on empty input**? $\rightsquigarrow$ **undecidable**

Counters	⇝	**Resource** types
Transitions	⇝	**Actions**
Runs	⇝	**strategies**/paths + **validity condition**
Accepting run	⇝	**strategies** which **ensure F**halt

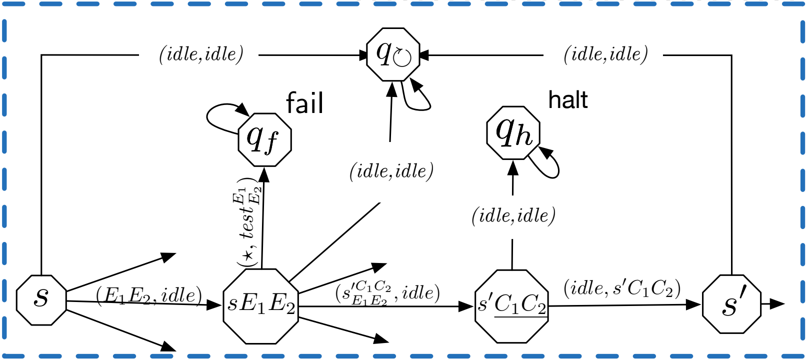
Transition relation: $(s, E_1, E_2)\Delta(s', C_1, C_2)$

$E_i \in \{0,1\} \quad C_i \in \{-1,0,+1\}$





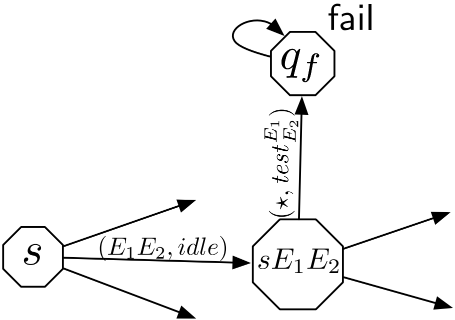
$$\begin{aligned}\Delta = \{&((s,E_1,E_2),(s',C_1,C_2)),\\ &((s,E_1,E_2),(s'',C_1,C_2)),\\ &((s',E_1,E_2),(s'',C_1,C_2))\}\end{aligned}$$



Two agents:

- 1: simulate transitions
- 2: ‘spoil” execution in states sE_1E_2

$\mathcal{A}$ **halts on** ε iff
$\mathfrak{M}^{\mathcal{A}}, s^{\textbf{init}}, \eta \models_R \langle\langle 1\rangle\rangle^{\bar{0}}_{\{1,2\}} \mathbf{F}\mathsf{p}$

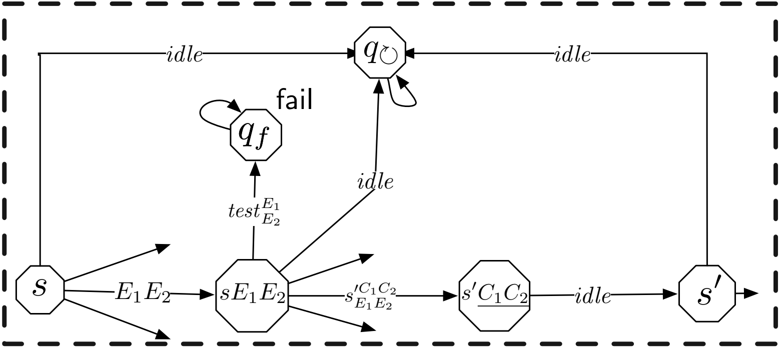


$(s, 0, 1)\Delta(s', C_1, C_2)$:
$test^0_1$ requires unit of R_1

Theorem

Model checking rfRAL **over iRBMs is undecidable** *even with 2 agent and 2 resource types.*

What about the **single agent** case?



$\mathcal{A}$ **halts on** ε iff $\mathfrak{M}_1^{\mathcal{A}}, s_{\text{init}}, \eta_0 \models_R \langle\langle\{1\}\rangle\rangle^{\bar{0}}(\underbrace{(\neg\langle\langle\{\mathbf{1}\}\rangle\rangle^{\downarrow}\mathbf{X}\,\text{fail})}_{\text{Test in error state}}\,\mathbf{U}\,\text{halt})$

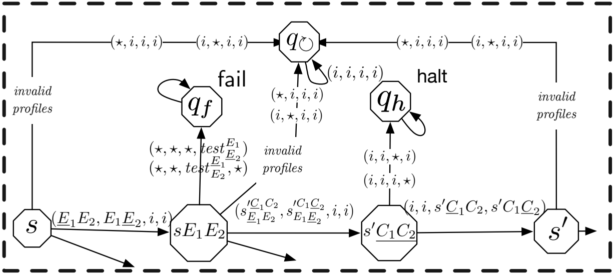
Theorem

Model checking prRAL **over iRBM***s* **is undecidable** *even with 1 agent and 2 resource types.*

Single Resource Setting

We can adapt the reduction to work with **1 resource type only**.

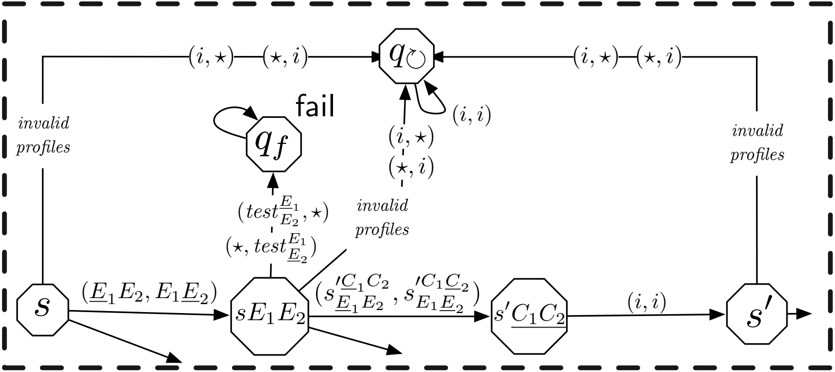
Introduce **more agents** and coordinate their actions.



$\mathcal{A}$ **halts on** ε iff $\mathfrak{M}_2^{\mathcal{A}}, s_{\mathbf{init}}, \bar{0} \models_R \langle\langle 1,2\rangle\rangle^{\bar{0}}_{\{1,2,3,4\}}$ **Fhalt**

Theorem (forthcoming)

Model checking rfRAL **over iRBM*s* is undecidable** *even with 4 agent and 1 resource type.*



$\mathcal{A}$ **halts on** ε iff $\mathfrak{M}_1^{\mathcal{A}}, s_{\mathsf{init}}, \bar{0} \models \langle\langle\{1,2\}\rangle\rangle^{\bar{0}}((\neg\langle\langle\{1,2\}\rangle\rangle^{\downarrow}\mathbf{X}\,\mathsf{fail})\mathbf{U}\,\mathsf{halt})$

Theorem (forthcoming)

Model checking prRAL **over iRBM*s* is undecidable** *even with 2 agent and 1 resource type.*

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Decidable Fragments

Formula used in the reduction of prRAL:

$$\mathfrak{M}_1^{\mathcal{A}}, s_{\text{init}}, \bar{0} \models \langle\langle\{1,2\}\rangle\rangle^{\bar{0}}((\neg\langle\langle\{1,2\}\rangle\rangle^{\downarrow}\mathbf{X}\,\text{fail})\,\mathbf{U}\,\text{halt})$$

Definition (prRAL^r)

prRAL^r is the positive fragment of prRAL, more precisely, at no coalition modality is under the scope of a negation.

Models	RAL	rfRAL	prRAL	rfprRAL	prRAL^r	1 shared
RBM	U	U	U	U	U	D [4]
iRBM	U	U	U	D [2]	**D [3]**	D [4]

[2] Alechina et al. 2014

[3] Alechina et al. 2015 [4] Bulling & Nguyen 2015

prRAL^r vs rfprRAL

- given their initial battery charge, rescue robots A can safely get to a position from which they can perform rescue while in visual contact with the base

$$\langle\langle A\rangle\rangle^{\eta_{\mathrm{init}}}(\mathsf{safe}\,\mathbf{U}\,(\langle\langle \mathbf{A}\rangle\rangle^{\downarrow}(\mathsf{visual}\,\mathbf{U}\,\mathsf{rescue})))$$

i.e., the robots cannot recharge their batteries after reaching the position from which they can perform rescue

- given their initial fuel and battery, booster (1) & satellite (2) can safely reach a position from which satellite can monitor indefinitely

$$\langle\langle 1,2\rangle\rangle^{\eta_{\mathrm{init}}}(\mathsf{safe}\,\mathbf{U}\,(\langle\langle \mathbf{2}\rangle\rangle^{\downarrow}\mathbf{G}\,\mathsf{monitor}))$$

i.e., satellite has an action to recharge its batteries

Decidability of prRAL^{r} over iRBMs

The algorithm requires as input $\mathfrak{M}$, q, η, ϕ and returns true or false

1. algorithm performs an and-or search of the model
2. $\langle\langle A\rangle\rangle^{\downarrow}\varphi$: propagate the current endowment to the nested search
3. $\langle\langle A\rangle\rangle^{\eta}\varphi$: start a new search with endowment η
4. termination: check for loops with comparable endowments
   introduce *arb* if there is a productive loop, finite but arbitrary amount of resources
   - important that no negation is allowed

     $\langle\langle 1\rangle\rangle\mathbf{F}\neg\langle\langle 1\rangle\rangle^{\downarrow}\mathbf{F}\mathrm{p}$: if *arb* is introduced, 1 has too much power
   - important that only proponent restricted

     $\langle\langle A\rangle\rangle_{B}\mathbf{F}\mathrm{p}$: interplay between A and B tricky when introducing *arb*
   - important that **iRBM**s are used

     introduction of *arb* not sufficient $\rightsquigarrow$ existence of infinite path

Shared Resources

- we consider **shared resources**: common pool
- opponents always have priority (similar to [Della Monica et al., 2011])

Example

Departmental travel budget. All agents compete for the same resources.

Theorem ([Bulling and Nguyen, 2015])

RAL *over* k-**unbounded iRBMs** *is decidable for* $k \leq 1$ *and undecidable otherwise.*

Reduction to CTL over alternating Büchi pushdown systems.

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The following topics are related (conceptually or technically):

- resource logics
- Petri nets
- vector addition systems
- (infinite) games (with quantitative aspects)
- quantitative reasoning tools

Can a **unified framework** help to understand such systems?

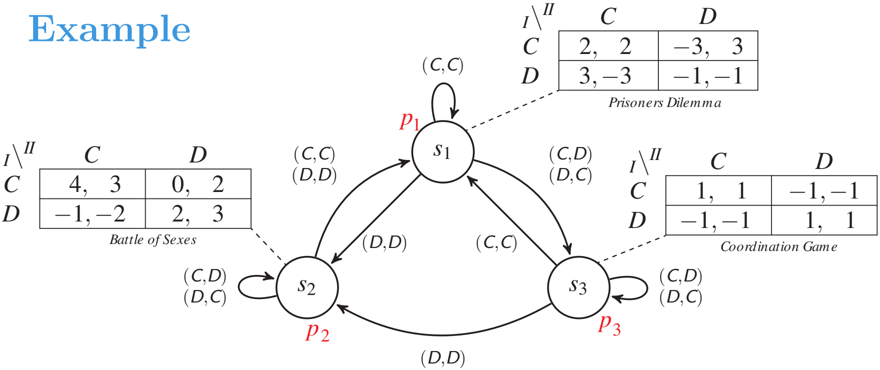
- also: resource consumption/production may depend on action profiles$\rightsquigarrow$ closer to game theory

Quantitative Reasoning

Expressing specifications in QATL*[Bulling and Goranko, 2013]:

- QATL* extends **ATL***, **qualitative properties**: $\langle\!\langle A\rangle\!\rangle(\mathbf{G}p \land q\mathbf{U}r)$
- Purely **quantitative properties**:
  - $\langle\!\langle\{\mathbf{a}\}\rangle\!\rangle\mathbf{G}(v_{\mathbf{a}} > 0)$ "*Player* **a** *has a strategy to maintain his accumulated payoff positive*",
  - $\langle\!\langle A\rangle\!\rangle(w_a \geq 3)$ "*The coalition A has a strategy to guarantee the value (i.t., limit payoff) of the play for player* **a** *to be at least 3*".
- Combined **qualitative and quantitative properties**:
  - $\langle\!\langle\{\mathbf{a}\}\rangle\!\rangle((\mathbf{a}\ \texttt{is happy})\ \mathbf{U}\ (v_{\mathbf{a}} \geq 100))$
  - $\langle\!\langle\{\mathbf{a},\mathbf{b}\}\rangle\!\rangle((v_{\mathbf{a}} + v_{\mathbf{b}} > v_{\mathbf{c}})\ \mathbf{U}\ \mathbf{G}(\mathbf{a}\ \texttt{is happy})))$
- In general easily undecidable

Example



I\II	C	D
C	2, 2	−3, 3
D	3, −3	−1, −1

Prisoners Dilemma

I\II	C	D
C	4, 3	0, 2
D	−1, −2	2, 3

Battle of Sexes

I\II	C	D
C	1, 1	−1, −1
D	−1, −1	1, 1

Coordination Game

$u > 0 \Rightarrow$ any action $\quad u = 0 \Rightarrow C \quad u < 0 \Rightarrow$ max min payoff

1. $\langle\langle\{I, II\}\rangle\rangle \mathbf{F}(p_1 \wedge v_I > 100 \wedge v_{II} > 100) \rightsquigarrow$
   $(s_1,(0,0)),(s_1,(2,2)),(s_1(4,4))\ldots$
2. $\langle\langle\{I, II\}\rangle\rangle \mathbf{XXX} \langle\langle\{II\}\rangle\rangle(\mathbf{G}(p_2 \wedge v_I = 0) \wedge \mathbf{F}\ v_{II} > 100) \rightsquigarrow$
   $(s_1,(0,0)),(s_1,(2,2)),(s_2,(1,1)),(s_2,(0,-1)),(s_2,(0,1)),(s_2,(0,3))$

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Conclusions

- Extensions of **ATL**
- Main Interest: **What can be verified**?
- decidability depends on many design choices

Future work:

- **implementation** of prRAL^r in MCMAS
- practical settings
- other decidable fragments of RAL
- computational complexity

Thank you for your attention.

Questions?

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