Reversal-Bounded Counter Machines

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Workshop on Logics for Resource-Bounded Agents,
Barcelona, August 2015

Overview

Presburger Counter Machines

Reversal-Bounded Counter Machines

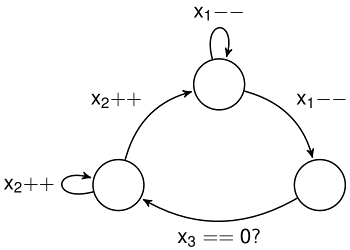
Verifying Temporal Properties

The Reversal-Boundedness Detection Problem

Presburger Counter Machines

Integer programs

- Finite-state automaton with counters interpreted by non-negative integers.



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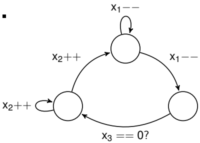

- Many applications:
  - Broadcast protocols, Petri nets, ...
  - Programs with pointer variables. [Bouajjani et al., CAV'06]
  - Replicated finite-state programs. [Kaiser & Kroening & Wahl, CAV'10]
  - Relationships with data logics. [Bojańczyk et al., TOCL 11]

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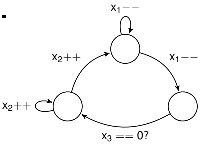

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  - Relationships with data logics. [Bojańczyk et al., TOCL 11]
- Techniques for model-checking infinite-state systems are required for formal verification.
- But, integer programs can simulate Turing machines.
- Checking safety or liveness properties is undecidable.

Taming verification of counter machines

- Design of subclasses with decidable reachability problems

  - Vector addition systems ($\approx$ Petri nets) [Kosaraju, STOC'82]
  - Flat relational counter machines. [Comon & Jurski, CAV'98]
  - Reversal-bounded counter machines. [Ibarra, JACM 78]
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- Decision procedures
  - Translation into Presburger arithmetic. [Fribourg & Olsén, CONCUR'97; Finkel & Leroux, FSTTCS'02]
  - Direct analysis on runs. [Rackoff, TCS 78]
  - Approximating reachability sets. [Karp & Miller, JCSS 69]
  - Well-structured transition systems. [Finkel & Schnoebelen, TCS 01]

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- Tools: FAST, LASH, TREX, FLATA, ...

A fundamental decidable theory

- First-order theory of $\langle \mathbb{N}, +, \leq \rangle$ introduced by Mojzesz Presburger (1929).
- Many properties: decidability, quantifier elimination, quantifier-free fragment in NP, ...

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- First-order theory of $\langle \mathbb{N}, +, \leq \rangle$ introduced by Mojzesz Presburger (1929).
- Many properties: decidability, quantifier elimination, quantifier-free fragment in NP, ...
- Terms $\mathtt{t} = a_1 \mathrm{x}_1 + \cdots + a_n \mathrm{x}_n + k$ where $a_1, \ldots, a_n \in \mathbb{N}$, k is in $\mathbb{N}$ and the x_i's are variables.
- Presburger formulae: $\phi ::= \ \mathtt{t} \leq \mathtt{t}' \mid \neg\phi \ \mid \ \phi \wedge \phi \ \mid \ \exists \ \mathrm{x} \ \phi$

Presburger arithmetic

- Valuation $\mathfrak{v} : \mathrm{VAR} \to \mathbb{N}$ + extension to all terms with

$$\mathfrak{v}(a_1 \mathsf{x}_1 + \cdots + a_n \mathsf{x}_n + k) \stackrel{\text{def}}{=} a_1 \mathfrak{v}(\mathsf{x}_1) + \cdots + a_n \mathfrak{v}(\mathsf{x}_n) + k$$

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- $\mathfrak{v} \models \mathtt{t} \leq \mathtt{t}'$ iff $\mathfrak{v}(\mathtt{t}) \leq \mathfrak{v}(\mathtt{t}')$; $\mathfrak{v} \models \phi \wedge \phi'$ iff $\mathfrak{v} \models \phi$ and $\mathfrak{v} \models \phi'$,

- $\mathfrak{v} \models \exists \mathsf{x}\ \phi \stackrel{\text{def}}{\Leftrightarrow}$ there is $n \in \mathbb{N}$ such that $\mathfrak{v}[\mathsf{x} \mapsto n] \models \phi$.

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- $\mathfrak{v} \models \exists \mathrm{x}\ \phi \ \stackrel{\text{def}}{\Leftrightarrow}$ there is $n \in \mathbb{N}$ such that $\mathfrak{v}[\mathrm{x} \mapsto n] \models \phi$.
- Formula $\phi(\mathrm{x}_1, \ldots, \mathrm{x}_n)$ with $n \geq 1$ free variables:

$$[\![\phi(\mathrm{x}_1, \ldots, \mathrm{x}_n)]\!] \stackrel{\text{def}}{=} \{\langle \mathfrak{v}(\mathrm{x}_1), \ldots, \mathfrak{v}(\mathrm{x}_n) \rangle \in \mathbb{N}^n : \mathfrak{v} \models \phi\}.$$

- ϕ is satisfiable $\stackrel{\text{def}}{\Leftrightarrow}$ there is $\mathfrak{v}$ such that $\mathfrak{v} \models \phi$.

Decision procedures and tools

- Quantifier elimination and refinements

  [Cooper, ML 72; Reddy & Loveland, STOC'78]

- Tools dealing with quantifier-free PA, full PA or quantifier elimination: Z3, CVC4, Alt-Ergo, Yices2, Omega test.

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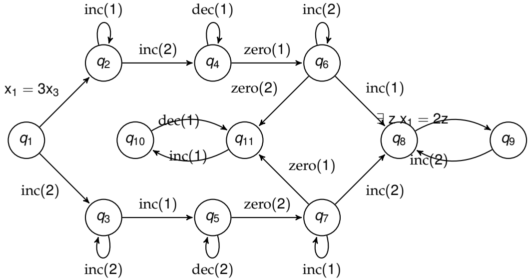
- Automata-based approach.

  [Büchi, ZML 60; Boudet & Comon, CAAP'96]

- Automata-based tools for Presburger arithmetic: LIRA, suite of libraries TAPAS, MONA, and LASH.

Presburger counter machines

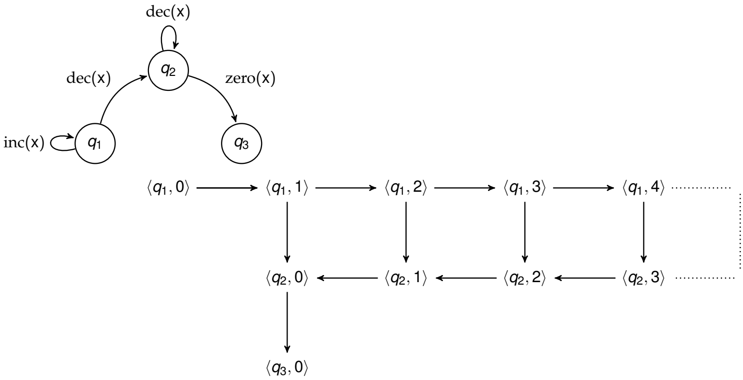
- Presburger counter machine $\mathtt{M} = \langle Q, T, C \rangle$:
  - Q is a nonempty finite set of control states.
  - C is a finite set counters $\{\mathsf{x}_1, \ldots, \mathsf{x}_d\}$ for some $d \geq 1$,
  - $d \geq 1$ is the dimension.
  - T = finite set of transitions of the form $t = \langle q, \phi, q' \rangle$ where $q, q' \in Q$ and ϕ is a Presburger formula with free variables $\mathsf{x}_1, \ldots, \mathsf{x}_d, \mathsf{x}'_1, \ldots, \mathsf{x}'_d$.



- Configuration $\langle q, \mathbf{x} \rangle \in \mathfrak{S} = Q \times \mathbb{N}^d$.

Transition system $\mathfrak{T}(\mathrm{C})$

- Transition system $\mathfrak{T}(\mathrm{C}) = \langle \mathfrak{S}, \rightarrow \rangle$:
  - $\langle q, \mathbf{x} \rangle \rightarrow \langle q', \mathbf{x}' \rangle \overset{\text{def}}{\Leftrightarrow}$ there is $t = \langle q, \phi, q' \rangle$ such that $\mathfrak{v}[\overline{\mathrm{x}} \leftarrow \mathbf{x}, \overline{\mathrm{x}'} \leftarrow \mathbf{x}'] \models \phi$



- $\xrightarrow{*}$: reflexive and transitive closure of $\rightarrow$.

Decision problems

- Reachability problem:

  Input: PCM $\mathbb{C}$, $\langle q_0, \mathbf{x}_0 \rangle$ and $\langle q_f, \mathbf{x}_f \rangle$.

  Question: $\langle q_0, \mathbf{x}_0 \rangle \xrightarrow{*} \langle q_f, \mathbf{x}_f \rangle$?

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- Control state reachability problem:

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- Control state repeated reachability problem:

  Input: PCM $\mathbb{C}$, $\langle q_0, \mathbf{x}_0 \rangle$ and q_f.

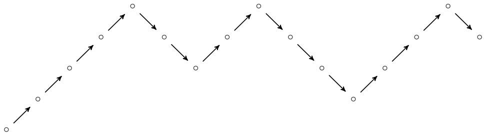
  Question: is there an infinite run starting from $\langle q_0, \mathbf{x}_0 \rangle$ such that the control state q_f is repeated infinitely often?

Subclasses of Presburger counter machines

- Counter machines (CM): transitions $q \xrightarrow{\phi_g \wedge \phi_u} q' \in T$ s.t.
  - ϕ_g is a Boolean combination of atomic formulae of the form $\mathsf{x} \geq k$,
  - $\phi_u = \bigwedge_{i \in [1,d]} \mathsf{x}'_i = \mathsf{x}_i + \mathbf{b}(i)$ where $\mathbf{b} \in \mathbb{Z}^d$.
- Minsky machines are counter machines.
- Vector addition systems with states (VASS): all the transitions are of the form $q \xrightarrow{\top \wedge \phi_u} q'$.
  ($\approx$ Minsky machines without tests)

Reversal-bounded counter machines

- Reversal: Alternation from nonincreasing mode to nondecreasing mode and vice-versa.



- Sequence with 3 reversals:

$$0011223334444\overline{3}33222\overline{3}334444555555\overline{4}$$

- A run is *r*-reversal-bounded whenever the number of reversals of each counter is less or equal to *r*.

Semilinearity

- Let $\langle \mathtt{M}, \langle q_0, \mathbf{x}_0 \rangle \rangle$ be r-reversal-bounded for some $r \geq 0$. For each control state q_f, the set

$$R = \{\mathbf{y} \in \mathbb{N}^d : \exists \text{ run } \langle q_0, \mathbf{x}_0 \rangle \xrightarrow{*} \langle q_f, \mathbf{y} \rangle\}$$

  is effectively semilinear [Ibarra, JACM 78].

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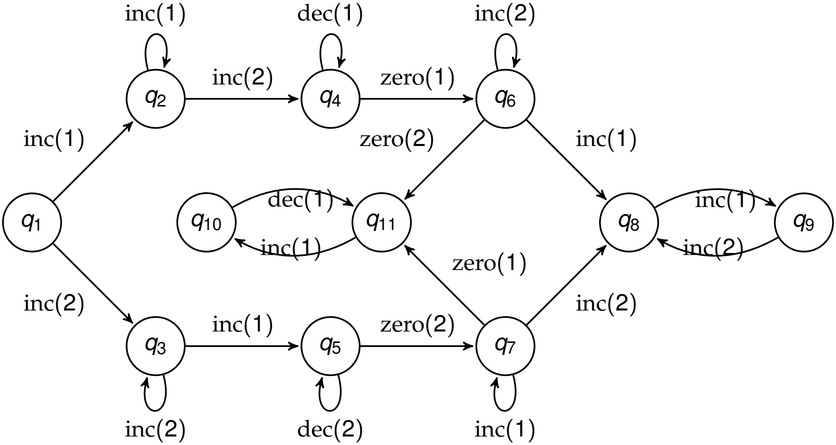
- The reachability problem with bounded number of reversals:

  Input: CM $\mathtt{M}$, $\langle q, \mathbf{x} \rangle$, $\langle q', \mathbf{x}' \rangle$ and $r \geq 0$.
  Question: Is there a run $\langle q, \mathbf{x} \rangle \xrightarrow{*} \langle q', \mathbf{x}' \rangle$ s.t. each counter performs during the run a number of reversals bounded by r?

- The problem is decidable (add tuples in the control states to count the numbers of reversals).

Proof ideas

- Reachability relation of simple loops can be expressed in Presburger arithmetic.
- Runs can be normalized so that:
  - each simple loop is visited at most an exponential number of times,
  - the different simple loops are visited in a structured way.
- Parikh images of context-free languages are effectively semilinear. [Parikh, JACM 66]



$$\phi = (x_1 \geq 2 \wedge x_2 \geq 1 \wedge (x_2 + 1 \geq x_1) \vee (x_2 \geq 2 \wedge x_1 \geq 1 \wedge x_1 + 1 \geq x_2)$$

$$[\![\phi]\!] = \{\mathbf{y} \in \mathbb{N}^2 : \langle q_1, \mathbf{0} \rangle \xrightarrow{*} \langle q_9, \mathbf{y} \rangle\}$$

Complexity of reachability problems

- Reachability problem with bounded number of reversals:

  Input: CM $\mathtt{M}$, $\langle q, \mathbf{x} \rangle$, $\langle q', \mathbf{x}' \rangle$ and $r \geq 0$.

  Question: Is there a run $\langle q, \mathbf{x} \rangle \xrightarrow{*} \langle q', \mathbf{x}' \rangle$ s.t. each counter performs during the run a number of reversals bounded by r?

- The problem is NP-complete, assuming that all the natural numbers are encoded in binary except the number of reversals.

- The problem is NEXPTIME-complete assuming that all the natural numbers are encoded in binary.

  [Gurari & Ibarra, ICALP'81; Howell & Rosier, JCSS 87]

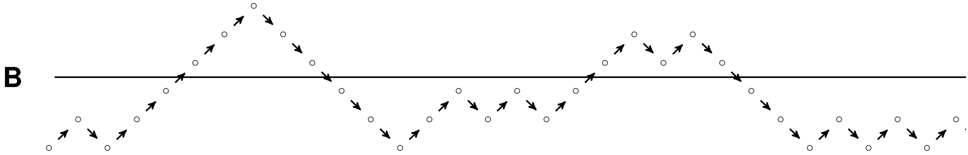
- NEXPTIME-hardness as a consequence of the standard simulation of Turing machines. [Minsky, 67]

Extensions

- Adding a free counter preserves the effective semilinearity of the reachability set. [Ibarra, JACM 78]

- Adding guards of the form $x_i = x_{i'}$ and $x_i \neq x_{i'}$ leads to undecidability of the reachability problem.

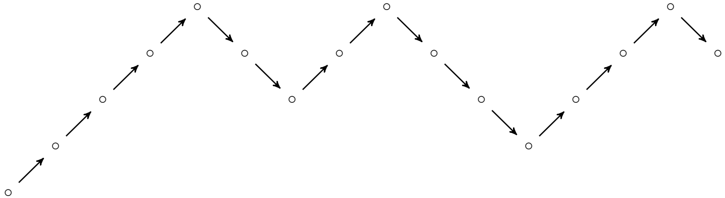
- Reversals are recorded only above a bound **B**:



- This preserves the effective semilinearity of the reachability set. [Finkel & Sangnier, MFCS'08]

Safely enriching the set of guards

- Atomic formulae in guards are of the form $\mathtt{t} \leq k$ or $\mathtt{t} \geq k$ with $k \in \mathbb{Z}$ and $\mathtt{t}$ is of the form $\sum_i a_i \mathrm{x}_i$ with the a_i's in $\mathbb{Z}$.
- $\mathtt{T}$: a finite set of terms including $\{\mathrm{x}_1, \ldots, \mathrm{x}_d\}$.
- A run is r-$\mathtt{T}$-reversal-bounded $\overset{\text{def}}{\Leftrightarrow}$ the number of reversals of each term in $\mathtt{T} \leq r$ times.



Reversal-boundedness leads to semilinearity

- Given a counter machine $\mathtt{M}$, $\mathtt{T}_{\mathtt{M}} \stackrel{\text{def}}{=}$ the set of terms $\mathtt{t}$ occurring in $\mathtt{t} \sim k$ with $\sim \in \{\leq, \geq\}$ + counters in $\{\mathsf{x}_1, \ldots, \mathsf{x}_d\}$.
- $\langle \mathtt{M}, \langle q_0, \mathbf{x}_0 \rangle \rangle$ is reversal-bounded $\stackrel{\text{def}}{\Leftrightarrow}$ there is $r \geq 0$ such that every run from $\langle q_0, \mathbf{x}_0 \rangle$ is r-$\mathtt{T}_{\mathtt{M}}$-reversal-bounded.
- When $\mathtt{T} = \{\mathsf{x}_1, \ldots, \mathsf{x}_d\}$, $\mathtt{T}$-reversal-boundedness is equivalent to reversal-boundedness from [Ibarra, JACM 78].

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- When $\mathtt{T} = \{\mathsf{x}_1, \ldots, \mathsf{x}_d\}$, $\mathtt{T}$-reversal-boundedness is equivalent to reversal-boundedness from [Ibarra, JACM 78].

- Given a counter machine $\mathtt{M}$, $r \geq 0$ and $q, q' \in Q$, one can effectively compute a Presburger formula $\phi_{q,q'}(\overline{\mathsf{x}}, \overline{\mathsf{y}})$ such that for all $\mathfrak{v}$, propositions below are equivalent:
  - $\mathfrak{v} \models \phi_{q,q'}(\overline{\mathsf{x}}, \overline{\mathsf{y}})$,
  - there is an r-$\mathtt{T_C}$-reversal-bounded run from $\langle q, \langle \mathfrak{v}(\mathsf{x}_1), \ldots, \mathfrak{v}(\mathsf{x}_d) \rangle \rangle$ to $\langle q', \langle \mathfrak{v}(\mathsf{y}_1), \ldots, \mathfrak{v}(\mathsf{y}_d) \rangle \rangle$.

[Ibarra, JACM 78; Demri & Bersani, FROCOS'11]

Verifying Temporal Properties

A temporal logic

- Arithmetical terms ($a \in \mathbb{Z}$):

$$\mathtt{t} ::= a\,\mathtt{x} \mid a\,\mathrm{X}\mathtt{x} \mid \mathtt{t} + \mathtt{t}$$

- $\mathrm{X}\mathtt{x}$ is interpreted as the next value of the counter $\mathtt{x}$.

- Formulae:

$$\phi ::= \top \mid q \mid \mathtt{t} \sim k \mid \mathtt{t} \equiv_c k' \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}^{-1}\phi$$

- Linear-time operators X, U and X^{-1}, S.

- Counter values at the previous position can be simulated.

- Models: infinite runs of counter machines.

Reversal-bounded model-checking problem

- $\mathtt{T}_{\phi}$: set of terms of the form $\sum_k (a_k + b_k)\mathrm{x}_k$ when $\mathtt{t} = (\sum_k a_k \mathrm{X}\mathrm{x}_k) + (\sum_k b_k \mathrm{x}_k)$ is a term occurring in ϕ.
- $\mathtt{T}_{\mathtt{M}}$: set of terms $\mathtt{t}$ occurring in $\mathtt{t} \sim k$ with $\sim \in \{\leq, \geq\}$ + counters in $\{\mathrm{x}_1, \ldots, \mathrm{x}_d\}$.
- Problem RBMC:

  Input: a CM $\mathtt{M}$, $\langle q_0, \mathbf{x}_0 \rangle$, a formula ϕ, a bound $r \in \mathbb{N}$ (in binary),

  Question: Is there an infinite run ρ from $\langle q_0, \mathbf{x}_0 \rangle$ such that $\rho, 0 \models \phi$ and ρ is r-$\mathtt{T}$-reversal-bounded with $\mathtt{T} = \mathtt{T}_{\mathtt{C}} \cup \mathtt{T}_{\phi}$?

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- RBMC is NEXPTIME-complete.

  [Howell & Rosier, JCSS 87]

  [Bersani & Demri, FROCOS'11, Hague & Lin, CAV'11]

  (Proof plan: RBMC $\leq$ repeated reachability $\leq$ reachabillty)
- Global model-checking is also possible for RBMC.

The Reversal-Boundedness Detection Problem

The reversal-boundedness detection problem

- The reversal-boundedness detection problem:

  Input: Counter machine $\mathtt{M}$ of dimension d, configuration $\langle \mathtt{M}, \langle q_0, \mathbf{x}_0 \rangle \rangle$ and $i \in [1, d]$.

  Question: Is $\langle \mathtt{M}, \langle q_0, \mathbf{x}_0 \rangle \rangle$ reversal-bounded with respect to the counter x_i?

- Undecidability due to [Ibarra, JACM 78].
- Restriction to VASS is decidable [Finkel & Sangnier, MFCS'08].

Undecidability proof

- Minsky machine $\mathtt{M}$ with halting state q_H (2 counters).
- Either $\mathtt{M}$ has a unique infinite run (and never visits q_H) or $\mathtt{M}$ has a finite run (and halts at q_H).
- Counter machine $\mathtt{M}'$: replace $t = q_i \xrightarrow{\phi} q_j$ by

$$q_i \xrightarrow{\mathrm{inc}(1)} q^{new}_{1,t} \xrightarrow{\mathrm{dec}(1)} q^{new}_{2,t} \xrightarrow{\phi} q_j$$

- We have the following equivalences:
  - $\mathtt{M}$ halts.
  - For $\mathtt{M}'$, q_H is reached from $\langle q_0, \mathbf{0} \rangle$.
  - Unique run of $\mathtt{M}'$ starting by $\langle q_0, \mathbf{0} \rangle$ is finite.
  - $\mathtt{M}'$ is reversal-bounded from $\langle q_0, \mathbf{0} \rangle$.

EXPSPACE-completeness for VASS

- Complexity lower bound is obtained as a slight variant of Lipton's proof for the reachability problem for VASS.
  [Lipton, TR 76]

- EXPSPACE upper bound by reduction into the place-boundedness problem for VASS. [Demri, JCSS 13]

- Place boundedness problem for VASS:

  Input: A VASS $\mathtt{M} = \langle Q, T, C \rangle$ with $\mathrm{card}(C) = d$, an initial configuration $\langle q_0, \mathbf{x}_0 \rangle$ and a counter $\mathsf{x}_j \in C$.

  Question: Is there a bound $B \in \mathbb{N}$ such that $\langle q_0, \mathbf{x}_0 \rangle \xrightarrow{*} \langle q', \mathbf{x}' \rangle$ implies $\mathbf{x}'(j) \leq B$?

- Proof idea: add a new counter that counts the number of reversals for the distinguished counter x_j.

Concluding remarks

- Bounding the number of reversals in counter machines underapproximates its computational behaviors.
- Effective semilinearity holds for (repeated) reachability and even for LTL-like logics (conditions apply).
- Solvers for Presburger arithmetic helpful for decision procedures related to reversal-bounded counter machines.
- VASS witness better computational properties.
- Can the techniques be used for other types of boundedness?

Advances In Modal Logic 2016 (AIML'16)

- 11th Conference on Advances in Modal Logic, Budapest, Hungary.
- Organizer: Andras Maté.
- PC co-chairs: L. Beklemishev & S. Demri.
- Dates
  - Submission **March 10th, 2016**
  - Notification **May 10th, 2016**
  - Conference **August 29th to September 02, 2016**