COLOPHON

The attached paper has no connection with my current research in Computer Science and Digital Documents. It is another in a series of experiments to see how long it takes for me to re-build electronic versions of my published early papers as properly re-typeset ‘PDF Normal’ rather than just as a bitmap scan.

This particular paper was co-authored with my PhD supervisor, Andrew Robertson, and it appeared in the International Journal of Mass Spectrometry and Ion Physics (Elsevier) in 1968. This, of course, was well before the era of desktop publishing and all I had was a reprint of the original paper. I decided against scanned bitmap pages and/or the use of an OCR package, reasoning that the time taken to rebuild the paper to my liking, with something like *Acrobat Capture*, would probably exceed the time taken to re-key the whole thing.

So, the paper was totally re-keyed. UNIX *troff* was used to set up the correct typeface (Times) and to get the line and page breaks as accurate as possible. Specialist material within the paper was re-set with the *refer*, *grap*, *pic*, *eqn*, and *psfig* pre-processors for *troff*.

Of course, another candidate for this exercise would have been LaTeX but although I use this quite a lot for journal and conference papers I am not very skilled in the tweaks needed to get exact page layout. Hence the decision to use the *troff* suite.

The first two diagrams were recreated from scratch using the *grap* pre-processor. The remaining three diagrams were scanned in from a reprint of the paper. They were then cleaned up in Adobe Photoshop and vectorized using Adobe Streamline. The lettering on these 3 diagrams was re-set in Adobe Illustrator, using Helvetica and Helvetica-Oblique, before exporting each diagram to version 3.0 of Illustrator’s Encapsulated PostScript (but with no TIFF preview). This Encapsulated PostScript was then incorporated into the paper using *psfig*.

This form of “rescue” of an early paper can produce a very pleasing result but the time taken to do it (12 hours for this paper) makes it prohibitively expensive for any publisher to undertake as a general procedure, especially given that one needs to be totally familiar with something like *troff*/*eqn*, or LaTeX before one can even begin.

The International Journal of Mass Spectrometry and Ion Physics is now very simply the “International Journal of Mass Spectrometry”. The earlier incarnation of this journal has its home page at:
`http://www.sciencedirect.com/science/journal/00207381`
PDF versions of the papers in that journal are available (to subscribers) from volume 1 onwards though the early PDFs are, of course, just bitmap scans of pages. Indeed, a PDF bitmap version of this very paper is available via the above Web page (but, again, one needs to be a subscriber to the journal).

CALCULATION OF ELECTRIC FIELD STRENGTHS AT A SHARP EDGE

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Sharp edges were first used for field ionisation mass spectrometry by Beckey[1]. Although Cross and Robertson[2] found that etched metal foils were more effective than razor blades for field ionisation, blades are very convenient for determination of field ionisation mass spectra, as reported by Robertson and Viney [3,4]. The electric field at the vertex of a sharp edge can be calculated by the method of the conformal transformation[5]. Here we give some equations for the field deduced with the assumption that the edge surface can be approximated by a hyperbola. We also compare two hyperbolae with radii of curvature at the vertex of 500 Å and 1000Å with the profile of a commercial carbon-steel razor blade.

CALCULATIONS OF FIELD STRENGTH AT THE VERTEX OF A SHARP EDGE

Sharp edge above a flat plate

If $z = x+iy$ and $w = u+iv$ ($i = \sqrt{-1}$), the transformation

$$w = k \cosh z \tag{1}$$

where k is a constant, will transform a series of lines parallel to the x-axis, and in the z-plane, to a series of confocal hyperbolae in the w-plane. The line $y = \frac{1}{2}\pi$ in the z-plane is transformed into the line $u = 0$ in the w-plane; the line $y = 0$ is transformed to $v = 0$, but u only has values greater than a certain minimum. A physical picture of the transformation is to regard it as transforming an infinite parallel-plate condenser, with plates at $y = 0$ and $\frac{1}{2}\pi$, into two perpendicular plates, separated by a gap, in the w-plane. A straight line in the z-plane, parallel to the x-axis and very close to it, transforms to a hyperbola in the w-plane which corresponds to an idealised edge; at $v = 0$ it has a very small radius of curvature (see Fig. 1). The hyperbola in fact approximates quite well to the profile of the razor blades used for field ionisation by Robertson and Viney[3,4].

We require to find the electric field at the vertex of the idealised edge. From eqn. (1)

$$u + iv = k \cosh x \cosh iy + k \sinh x \sinh iy \tag{2}$$

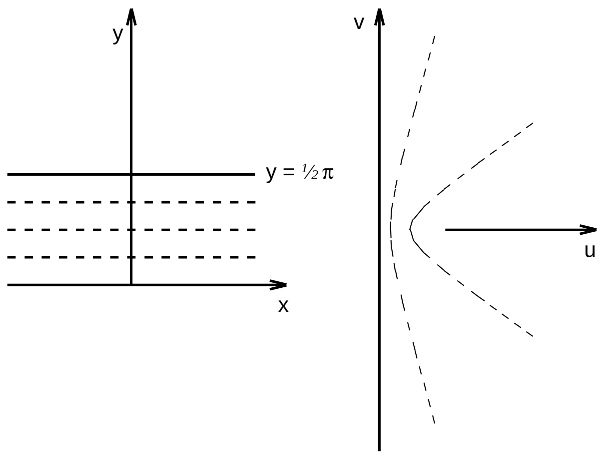

Fig. 1. Transformation of a parallel-plate condenser in the *z*-plane into two perpendicular plates separated by a gap. Equipotentials between the condenser plates are transformed into hyperbolae. Equipotentials are shown as dashed lines.

But $\cosh iy = \cos y$ and $\sinh iy = i \sin y$, hence by separating real and imaginary parts

$$u = k \cosh x \cos y \tag{3}$$

$$v = k \sinh x \sin y \tag{4}$$

When the curves intersect the *u*-axis, $v = 0$ and since in general $y \neq 0$ this implies

$$k \sinh x = 0 \tag{5}$$

hence $x = 0$. Thus eqn. (3) gives

$$u = k \cos y \tag{6}$$

at the intersection of the hyperbola with the *u*-axis. Hence the distance *l* from the fixed plate to this point of intersection is

$$l = k \cos y \tag{7}$$

To determine the equation of the hyperbolae in the *w*-plane we must eliminate *x* from eqns. (3) and (4). Putting

$$u/k \cos y = \cosh x \tag{8}$$

$$v/k \sin y = \sinh x \tag{9}$$

and squaring and subtracting we obtain

$$u^2/k^2 \cos^2 y - v^2/k^2 \sin^2 y = 1 \tag{10}$$

Thus the lines y = constant transform into hyperbolae (of which only the positive branches are considered in this discussion). The hyperbolae have eccentricities, e, of $\sec\, y$ and asymptotes given by[5].

$$v = \pm u \tan y \tag{11}$$

The radius of curvature r at any point on the hyperbolae is given by

$$r = [1 + (\mathrm{d}u/\mathrm{d}v)^2]^{\frac{3}{2}} / (\mathrm{d}^2u/\mathrm{d}v^2) \tag{12}$$

Differentiation of eqn. (10) gives

$$\mathrm{d}u/\mathrm{d}v = (v/u)\cot^2 y \tag{13}$$

$$\mathrm{d}^2u/\mathrm{d}v^2 = (u^2 \cot^2 y - v^2 \cot^4 y)/u^3 \tag{14}$$

Clearly from eqn. (13) $\mathrm{d}u/\mathrm{d}v$ is zero at the vertex where $v = 0$, thus eqn. (12) reduces to

$$r_{\mathrm{v}} = (\mathrm{d}^2u/\mathrm{d}v^2)^{-1}_{v=0} \tag{15}$$

where r_{v} is the value of r at the vertex. Thus from eqn. (14)

$$r_{\mathrm{v}} = u \tan^2 y \tag{16}$$

and from eqns. (6) and (7)

$$r_{\mathrm{v}} = l \tan^2 y \tag{17}$$

In general the field at the vertex of the hyperbola will have two components, in the u and v directions, $\partial V/\partial u$ and $\partial V/\partial v$, where V is the potential difference between the the edge and the flat plate, and corresponds to the potential difference between the two plates of the parallel-plate condenser. We can write

$$\partial V/\partial u = (\partial V/\partial y)(\partial y/\partial u) + (\partial V/\partial x)(\partial x/\partial u)$$

$$\partial V/\partial v = (\partial V/\partial y)(\partial y/\partial v) + (\partial V/\partial x)(\partial x/\partial v)$$

However $\partial V/\partial x = 0$ for the parallel-plate condenser, so

$$(\partial V/\partial u)_v = (\mathrm{d}V/\mathrm{d}y)\,(\partial y/\partial u)_v \tag{18}$$

$$(\partial V/\partial v)_u = (\mathrm{d}V/\mathrm{d}y)\,(\partial y/\partial v)_u \tag{19}$$

Differentiating eqn. (10) with respect to u and v in turn gives

$$\left(\frac{\partial y}{\partial u}\right)_v = -\frac{u \cos y \sin^3 y}{u^2 \sin^4 y + v^2 \cos^4 y} \tag{20}$$

$$\left(\frac{\partial y}{\partial v}\right)_u = \frac{v \sin y \cos^3 y}{u^2 \sin^4 y + v^2 \cos^4 y} \tag{21}$$

Since $v = 0$ and $u = k \cos y$ at the vertex

$$(\mathrm{d}y/\mathrm{d}u)_{v=0} = -1/k \sin y \tag{22}$$

$$(\mathrm{d}y/\mathrm{d}v)_{v=0} = 0 \tag{22}$$

Now in eqn. (18), $\mathrm{d}V/\mathrm{d}y$ is the uniform field in the parallel-plate condenser. The negative sign of eqns. (20) and (22) shows that the field in the edge and plate arrangement is in the opposite sense to that in the condenser. Evidently from Fig. 1, if the potential increases for decrease of y from $\frac{1}{2}\pi$ to zero, then the potential in the w-plane increases as u increases from zero to l. Let F_c be the uniform field in the parallel-plate condenser, and F_v be the field at the vertex of the edge, when we obtain from eqns. (18) and (7)

$$F_\mathrm{v} = -F_\mathrm{c}/l \tan y \tag{24}$$

The lower plate of the parallel-plate condenser must transform to the required edge profile, so this plate is not at $y = 0$ but at $y = y'$ (where $y' \ll \frac{1}{2}\pi$), hence

$$F_\mathrm{c} = -V/(\tfrac{1}{2}\pi - y') \tag{25}$$

but $y' \sim 0$, hence

$$F_\mathrm{c} \sim -2V/\pi \tag{26}$$

and from eqn. (24)

$$F_\mathrm{v} = 2V/\pi l \tan y \tag{27}$$

From eqn. (17)

$$\tan y = (r_\mathrm{v}/l)^{\frac{1}{2}} \tag{28}$$

Thus

$$F_\mathrm{v} \sim 2V/\pi(r_\mathrm{v} l)^{\frac{1}{2}} \tag{29}$$

The elegant deduction of Gilliland and Viney[5], which allows for a slit in the flat plate, reduces to eqn. (29) when the slit width becomes zero. It is of interest that the result in eqn. (29) can also be found in a simple way. This relation is valid if the values of $y = y'$ which must be taken to give the correct edge profile is almost zero. From eqn. (17) the condition is satisfied for all the blades and edges used in our work.

The above derivation takes no account of end effects, or of the presence of microtips and whiskers on the edge which lead to a field enhancement factor β. Robertson and Viney [4] estimated β to be about 7 for razor blades.

Edge suspended above a rod

Another transformation gives the macroscopic field at the vertex of an edge above a circular rod. This system may be useful in a mass spectrometer, and is similar to the two-wire source of Williams[6]. The sources previously described[3, 4] have a blade (or foil) above a slit, and at high fields the blade or foil tends to be pulled to one side or other of the slit because of slight imperfections in alignment. However the edge-rod source would be self-centering.

A suitable transformation is

$$\ln c = k' \cosh z \tag{30}$$

where $c = a + \mathrm{i}b$, $z = x + \mathrm{i}y$ and k' is a constant. This transforms the parallel-plate condenser of Fig. 2 into the edge and rod configuration shown.

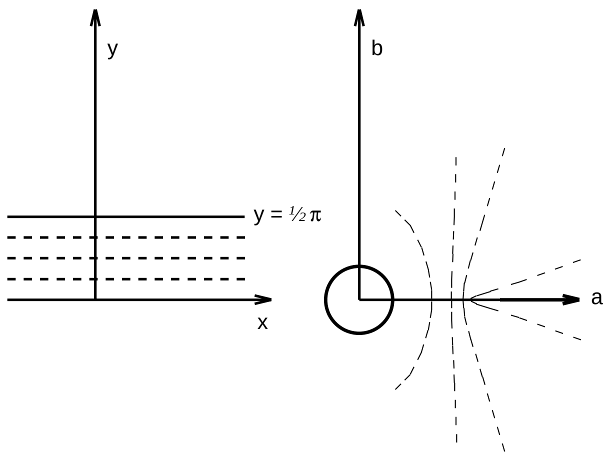

Fig. 2. Transformation of a parallel-plate condenser in the *z*-plane into a rod and plate arrangement in the *c*-plane. Equipotentials are shown as dashed lines.

The complex number c can be expressed in the form

$$c = R \exp(\mathrm{i}\theta)$$

whence

$$\ln c = \ln R + \mathrm{i}\theta \tag{31}$$

where

$$R = (a^2 + b^2)^{\frac{1}{2}} \tag{32}$$

$$\theta = \tan^{-1}(b/a) \tag{33}$$

Also from eqn. (30)

$$\ln R = k' \cosh x \cos y \tag{34}$$

$$\theta = k' \sinh x \sin y \tag{35}$$

From eqns. (32) and (33)

$$b = R \sin\theta \tag{36}$$

$$a = R \cos\theta \tag{37}$$

From eqns. (34)–(37) we find for the real and imaginary parts of c

$$a = \exp(k' \cosh x \cos y) \cos (k' \sinh x \sin y) \tag{38}$$

$$b = \exp(k' \cosh x \cos y) \sin (k' \sinh x \sin y) \tag{39}$$

A program was written for the University of London "Atlas" computer to evaluate and plot out values of a and b for various straight lines y = constant in the z-plane. The results, shown in Fig. 3 show that the transformation is the desired one.

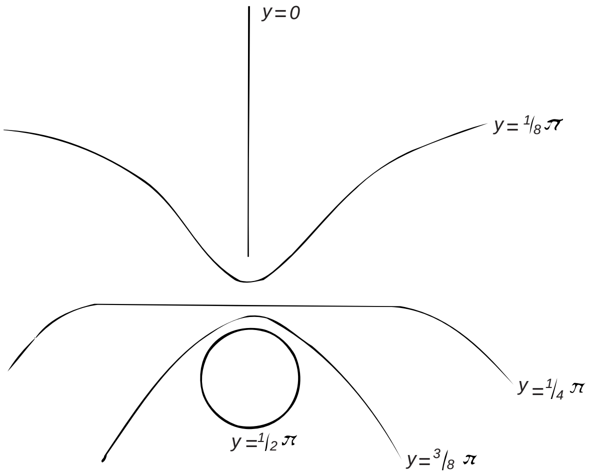

Fig. 3. Equipotentials in the c-plane between a plate and a rod, calculated with a computer, which gave the points shown. The equipotentials are shown as continuous lines.

When $y = \frac{1}{2}\pi$, eqn. (35) gives

$$\theta = k' \sinh x \tag{40}$$

and eqn. (34) gives

$$\ln R = 0 \tag{41}$$

hence $R = 1$, and thus the line $y = \frac{1}{2}\pi$ transforms to a circle of unit radius. Similarly eqns.(32)–(37) show that the line $y = 0$ transforms to the a-axis ($b = 0$), with $a \geq \exp k'$. Thus the distance l of the edge from the surface of the rod is given by

$$l = \exp k' - 1 \tag{42}$$

since the circle has unit radius.

To find the radius of curvature of the edge, it is represented, as before, by a curve lying very close to the straight line corresponding to $y = 0$. The radius of curvature r at any point is

$$r = [1+(\mathrm{d}a/\mathrm{d}b)^2]^{\frac{3}{2}} \,/\, (\mathrm{d}^2a/\mathrm{d}b^2) \tag{43}$$

Proceeding as before (eqns. (13)–(15)) we have

$$\frac{\mathrm{d}a}{\mathrm{d}b} = \frac{2a\cos^2 y\cdot\tan^{-1}(b/a) - b\sin^2 y\cdot\ln(b^2+a^2)}{a\sin^2 y\cdot\ln(b^2+a^2) + 2b\cos^2 y\cdot\tan^{-1}(b/a)} \tag{44}$$

From this equation $\mathrm{d}a/\mathrm{d}b = 0$ for $b = 0$, as is required. Also

$$\left(\frac{\mathrm{d}^2a}{\mathrm{d}b^2}\right)_{b=0} = \frac{\cos^2 y - \sin^2 y\cdot\ln a}{a\sin^2 y\cdot\ln a} \tag{45}$$

Thus eqn. (43) reduces to

$$r_\mathrm{v} = \frac{a\sin^2 y\cdot\ln a}{\cos^2 y - \sin^2 y\cdot\ln a} \tag{46}$$

The components in the a and b directions of the field F_v at the vertex are, since $(\partial V/\partial x) = 0$ as before,

$$\partial V/\partial a = (\mathrm{d}V/\mathrm{d}y)(\partial y/\partial a) \tag{47}$$

$$\partial V/\partial b = (\mathrm{d}V/\mathrm{d}y)(\partial y/\partial b) \tag{48}$$

Now x can be eliminated from eqns. (32)–(35) to give

$$\frac{[\ln(a^2+b^2)]^2}{4k'^2\cos^2 y} - \frac{[\tan^{-1}(b/a)]^2}{k'^2\sin^2 y} = 1 \tag{49}$$

and when $b = 0$,

$$\ln a = k'\cos y \tag{50}$$

Differentiating eqn. (49) with respect to a and b in turn gives

$$\left(\frac{\partial y}{\partial a}\right)_b = -\frac{\cos y\,\sin y\,[a\sin^2 y\cdot\ln(a^2+b^2) + 2b\cos^2 y\,\tan^{-1}(b/a)]}{(a^2+b^2)\{\frac{1}{2}[\ln(a^2+b^2)]^2\sin^4 y + 2\,[\tan^{-1}(b/a)]^2\cos^4 y\}}$$

$$\left(\frac{\partial y}{\partial b}\right)_a = -\frac{\cos y\,\sin y\,[2a\cos^2 y\cdot\tan^{-1}(b/a) - b\sin^2 y\cdot\ln(a^2+b^2)]}{(a^2+b^2)\{\frac{1}{2}[\ln(a^2+b^2)]^2\sin^4 y + 2\,[\tan^{-1}(b/a)]^2\cos^4 y\}}$$

These two equations with eqn. (50) simplify to

$$(\mathrm{d}y/\mathrm{d}a)_{b=0} = -1/ak'\sin y \tag{51}$$

$$(\mathrm{d}y/\mathrm{d}b)_{a=0} = 0 \tag{52}$$

Letting $F_\mathrm{c} = \mathrm{d}V/\mathrm{d}y$ (the uniform field in the parallel-plate condenser) we find

$$F_\mathrm{v} = -F_\mathrm{c}/ak'\sin y \tag{53}$$

Here again the two fields in Fig. 2 are in opposite senses. Rearranging eqn. (46)

and substituting for $\sin y$ in eqn. (53) yields

$$F_v = \frac{-F_c}{ak' \, [r_v / \{a \ln a + r_v(1+\ln a)\}]^{\frac{1}{2}}} \tag{54}$$

In eqn. (54) if $a = 1$ the field is found at the surface of the rod; to obtain the field at the edge the value of a must exceed 1. Then if $r_v \ll a$ (which is so in the experimental arrangements), the term $r_v(1+\ln a)$ in eqn. (48) is negligible compared with $a \ln a$ and the equation becomes

$$F_v = -F_c / ak' \, (r_v / a \ln a)^{\frac{1}{2}} \tag{55}$$

However, for $y \sim 0$ (which transforms to the edge profile)

$$a = 1 + l \tag{56}$$

and so from eqn. (50)

$$k' = \ln(1+l) \tag{57}$$

and substituting these values and $F_c = -2V/\pi$ in eqn. (55) gives

$$F_v \sim 2V/\pi[r_v(1+l) \ln (1+l)]^{\frac{1}{2}} \tag{58}$$

However, this field is for the case where the rod has unit radius. For the general case we regard Fig. 2 as a scale representation of the true physical arrangement, but in which the unit of measurement is the radius R of the rod. In this representation all true dimensions are multiplied by R^{-1}. For the general case, therefore, one multiplies all true dimensions in the rod-plate arrangement, and the field in the parallel plate condenser, by R^{-1}, and substituting into eqn. (58) gives

$$F_v \sim 2V/\pi[r_v R(1+R^{-1}) \ln (1+(l/R)]^{\frac{1}{2}} \tag{59}$$

Also $r_v \ll l$ and $R_v \ll R$. Clearly as l/R becomes small compared with unity, $(1+l/R) \to 1$ and $\ln(1+l/R) \to l/R$, thus F_v reduces to

$$F_v \sim 2V/\pi(r_v l)^{\frac{1}{2}} \tag{60}$$

which is identical with eqn. (29).

Thus when the edge is very near the rod, the rod behaves as a flat plane.

Calculation of field strengths for conditions used

From eqn. (29) with $r_V = 1000$ Å (an approximate value for razor blades [4]) and with $l = 0.025$ cm and $V = 8000$V, $F_v = 1.02 \times 10^7$ V cm^{-1}. If all distances are kept constant and the plate is replaced by a rod of 0.05 cm radius, F_v from eqn. (59) is 0.925×10^7 V cm^{-1}. The field at the blade edge is then only 0.9 of that with the plate. Neither of these fields is large enough for field ionisation of most molecules without some field intensification by surface roughness.

Field at the cathode surface

A knowledge of this field is important in considering the possibility of field emission of electrons from the rod or the plate, and possible growth of whiskers on these electrodes in the presence of certain gases and electrostatic fields [7,8].

The maximum field F_m at the cathode evidently occurs directly underneath the edge, and its value can be calculated for both arrangements considered. For the edge and flat plate, from eqn. (22) at $x = 0$ and $y = \frac{1}{2}\pi$,

$$dy/du = -k^{-1} = -l^{-1}$$

Hence from eqns. (18) and (26)

$$F_m = 2V/\pi l \tag{61}$$

For the edge and rod, from eqn (51) at $x = 0$ and $y = \frac{1}{2}\pi$,

$$dy/da = -1/k'a$$

Hence for a rod of unit radius $[a = 1, k' = \ln(1+l)]$ we have from eqn. (47)

$$F_m = 2V/[\pi R \ln(1 + l/R)] \tag{62}$$

As might be expected eqns. (63) and (64) become the same for $l/R \ll 1$.

With the values of voltage and dimensions already given, eqn. (61) gives $F_m = 2.04 \times 10^5$ V cm^{-1} and eqn. (63) gives $F_m = 2.51 \times 10^5$ V cm^{-1}. From Duell's work [7] these fields seem too small to initiate whisker growth, except perhaps on pre-existing projections. Preliminary experiments with the edge-rod system showed that electron emission from the rod (0.05 cm radius tungsten) was in fact important, and extreme care had to be taken in cleaning and polishing the rod.

APPLICATION OF EQUATIONS TO RAZOR BLADES

Electron-micrography shows that the radius of curvature of a razor blade edge is not a well-defined quantity. Perhaps a better description is given by the average thickness at various distances back from the edge. One type of carbon-steel blade is ground with three facets (Fig. 4). Fig. 5 shows a magnified profile of the final facet, and, for comparison, the profile of an idealised hyperbolic edge calcu-

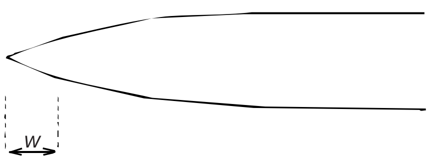

Fig. 4. Shape of a normal carbon-steel razor blade. The length W varies from 10 to 20 µm. The angle at the tip of the final facet is approximately 10°.

lated as follows. For the curve representing the edge profile, $y \to 0$ and $k \to l$, hence eqn. (10) becomes

$$(u^2/l^2) - (v^2/l^2 y^2) = 1 \tag{64}$$

Since the half-thickness $t_{½}$ of the blade at any distance d from the vertex (d being measured along the axis of symmetry of the hyperbola) is $t_{½} = v$, and since $d = u - l$, eqn. (64) gives

$$(d + l)^2 = l^2 + (t_{½}^2 / y^2) \tag{65}$$

From eqn. (17) as $y \to 0$, $y = (r_v / l)^{½}$, and since $l \gg d$, eqn. (65) gives

$$d = t_{½} / 2r_v \tag{66}$$

The blade profile was calculated from eqn. (66) taking r_v as 1000 Å, and as 500 Å.

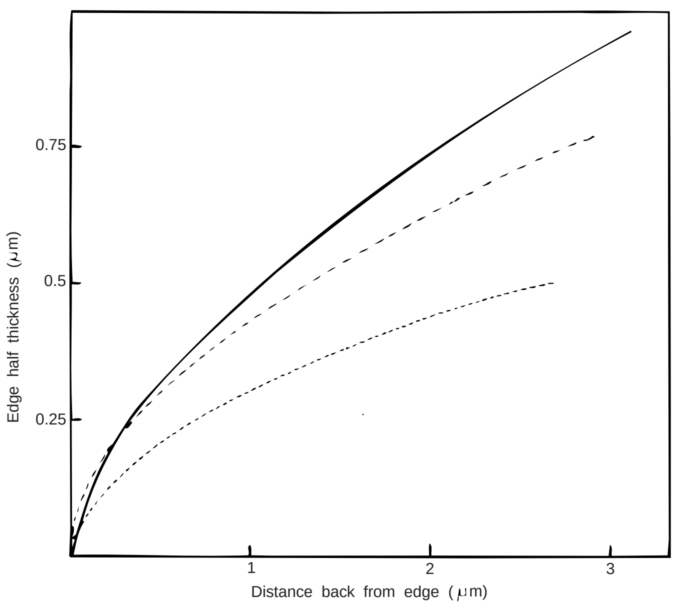

Fig. 5. Blade half thickness against distance from the edge (continuous line). Two idealised hyperbolic edges are also shown (dashed lines), with radii at the vertex of 1000 and 500 Å.

The correspondence of the hyperbola and the actual edge near the vertex is reasonably close for $r_v = 1000$ Å.

Electron micrographs show razor blade edges to be fairly smooth; any undulations are on a relatively large scale. This accounts for the low field intensification factor found for blades [4].

We thank the Institute of Petroleum for support to D.F.B, and Messrs. Gillette Industries for razor blades and information on them. We thank Dr. C. M. Cross for checking the equations.

SUMMARY

The sharp edges of a typical carbon-steel razor blade may be approximated by a hyperbola. The electric field strength at the vertex of the edge can then be calculated by the method of the conformal transformation. Equations are deduced for a blade above a plate and a blade above a rod. The results are considered in relation to the use of blades in field ionisation mass spectrometry.

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