Consensus Action Games

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Abstract

We present Consensus Action Games (CAGs), a novel approach to modelling consensus action in multi-agent systems inspired by quorum sensing and other forms of decision making found in biological systems. In a consensus action game, each agent's degree of commitment to the joint actions in which it may participate is expressed as a quorum function, and an agent is willing to participate in a joint action if and only if a quorum consensus can be achieved by all the agents participating in the action.We study the computational complexity of several decision problems associated with CAGs and give tractable algorithms for problems such as determining whether an action is a consensus action. We briefly compare CAGs to related work such as Qualitative Coalitional Games.

1 Introduction

There are many reasons why agents may wish to, or indeed have to, cooperate, for example, where resources are constrained or otherwise in contention, where agents have differing abilities, or where they possess differing information. Even self interested agents may be motivated towards cooperative behaviour where this is consistent with individual rationality, for example, where cooperation increases their individual utility. While there has been considerable research in AI into joint actions and the collective execution of a shared plan [Levesque *et al.*, 1990; Grosz and Kraus, 1993; Tambe, 1997], the main focus of this work has been to examine how teams of autonomous agents may collectively achieve some goal. Relatively little attention has been paid to the selection of the joint actions that agents may perform.

However, the problem of collective action selection has been extensively studied in the fields of behavioural ecology and theoretical biology. In this paper, we propose a game-theoretic model which is an abstraction of several mechanisms for collective action selection occurring in nature.

Many natural systems, including bacteria [Jacob *et al.*, 2004], ants [Pratt *et al.*, 2005], bees [Seeley and Visscher, 2004], and fish [Ward *et al.*, 2008] exhibit a behaviour known as quorum sensing. Through a process termed the *quorum response*, the probability that an individual will select a particular action is increasing in the proportion of individuals already having made that choice. This relationship is typically non-linear such that the probability that an action is selected by an agent increases sharply once the number of agents that have already selected that action passes some threshold [Sumpter and Pratt, 2008]. The macroscopic behaviour of such a self-organising system resembles one in which individuals converge upon consensus with respect to a joint action. The prevalence of quorum decision making in nature suggests that this is an efficient, effective and stable mechanism through which group activities can be coordinated. Theoretical models support this view, predicting that where the quorum threshold is adaptive, decisions can not only be optimal [List, 2004] but also provide a trade-off between speed and accuracy [Pratt and Sumpter, 2006]. Conradt and Roper [2005] term this type of group decision *combined decisions*.

Another mechanism for collective action selection is found in spatially cohesive groups of (often social) animals, where decisions must be made regarding, e.g., movement direction, travel destination and activity timing. For example, a group of primates may need to decide whether to forage or go to a water source. To minimise the risk of predation, it is critical that whatever action is chosen is a consensus action, i.e., is performed jointly by all the agents, but each agent will typically have differing preferences for each joint action and for which other agents participate in the action (e.g., mutual grooming with a high status individual). The mechanisms by which such consensus actions are selected are poorly understood. However there is evidence from field observations to suggest that the more individuals indicate they are in favour of a particular action (for example, by making tentative moves in a particular direction), the more likely are the other animals to 'agree' to the action (see, for example [Stueckle and Zinner, 2008]). Conradt and Roper [2005] term this type of group decision *consensus decisions*.

In this paper we present *Consensus Action Games* (CAGs), a novel approach to modelling collective action selection in multi-agent systems inspired by mechanisms for reaching combined and consensus decisions in natural systems. In a consensus action game, each agent's degree of commitment to the joint actions in which it may participate is expressed as a quorum function, and its decision whether to support a joint action is mediated by the quorum thresholds of the other agents that may participate in the action. Consensus is reached, where possible, through a series of individual commitments. We study the computational complexity of several decision problems associated with CAGs and give tractable algorithms for problems such as determining whether an action is a consensus action. We briefly compare CAGs to related work such as Qualitative Coalitional Games.

Although the immediate motivation for consensus action games are the phenomena underlying combined and consensus decisions in biological systems, we believe the model has wider application, for example, modelling trend adoption in people, and consensus action in multiagent systems. As such, it extends current work in coalition formation in multiagent systems, in considering not only which coalition an agent should join, but which action the agent performs as part of that coalition.

The remainder of this paper is organised as follows. In section 2 we introduce Consensus Action Games (CAGs), and in section 3 we consider the complexity of decision problems associated with CAGs. We discuss related research in section 4, and in section 5, we conclude and suggest directions for future work.

2 Consensus Action Games

A consensus action game (CAG) is a tuple $\Gamma = \langle G, A, J, q\rangle$ where:

G is a finite set of agents, $\{1, \ldots, n\}, n \geq 2$

A is a finite, non empty set of possible actions $\{1, \ldots, m\}$

J is a set of joint actions; each joint action is a set of pairs (i, a), where $i \in G$ and $a \in A$, specifying the action performed by each agent participating in the joint action. We write $J_i = \{j \in J \mid (i, a) \in j\}$ to indicate the set of joint actions in which agent i may participate, and $J_{G'} = \{j \in J \mid \{i \mid (i, a) \in j\} = G'\}$ for the set of all joint actions that can be performed by the set of agents $G' \subseteq G$.[1]

q is a quorum function which takes an agent $i \in G$ and an action j in J_i and returns a number in the interval [0,1], formally $q : \{(i, j) \mid i \in G, j \in J_i\} \rightarrow [0, 1]$. We will sometimes write $q_i(j)$ for $q(i, j)$. For an agent $i \in G' \subseteq G$ and joint action $j \in J_{G'}$, the quorum function $q_i(j)$ gives the minimum proportion of agents in G' which must support j in order that i will support j. Where $q_i(j) = 0$ agent i shows unconditional support for j, where $0 < q_i(j) < 1$ the agent shows conditional support for j; where $q_i(j) = 1$ the agent does not support j.

We say there is a *quorum consensus* about a joint action j if and only if all agents participating in j support j. Let $G' \subseteq G$, $j \in J_{G'}$, and $Q \subseteq G'$. Consider a function $Support_j : Q \mapsto Q \cup \{i \in G' \mid q_i(j) \times |G'| \leq |Q|\}$. Then the joint action j is a *quorum consensus action* if and only if G' is the least fixed point of $Support_j$. We will refer to each invocation of $Support_j$ as a *round*.

[1]Note that the set of joint actions is not simply the Cartesian product of all possible individual actions.

2.1 Example

Consider a group of six agents which have actions sing (s), play (p) and have a party (h). There are three joint actions:

$j_1 = \{(6, s), (2, p)\}$ with $q(6, j_1) = 0$ and $q(2, j_1) = 1/4$
$j_2 = \{(6, s), (3, p)\}$ with $q(6, j_2) = 0$ and $q(3, j_2) = 3/4$
$j_3 = \{(1, h), (2, h), \ldots, (6, h)\}$ with $q(i, j_3) = (i - 1)/6$

Intuitively, agent 6 is keen to sing, and agent 2 will consent to participate in the joint action j_1 where 6 sings and 2 plays accompaniment, because 2 requires at least a quarter of the agents involved in the action to support it before it declares its support, and agent 6 (half of the agents) supports it. Hence j_1 is a quorum consensus action. Action j_2 is not a quorum consensus action (agent 6 has unconditional support for it, but taking this into account only half of the agents support the action, and agent 3 requires three quarters). Finally, action 3 is a quorum consensus action: agent 1 has unconditional support for it, agent 2 supports it provided 1/6 of the agents do (which agent 1 does), agent 3 supports it if 2 out of 6 agents do (which 1 and 2 do), and so on. Observe that if we had $q(6, j_3) = 1$ rather than 5/6, then j_3 would not be a quorum consensus action.

The first two actions illustrate joint actions which are 'joint activities' (actions which require several participants to be performed) while the third action can be seen as somewhat similar to the quorum sensing in bacteria (all agents do the same thing, and the larger the number of agents that support the action, the larger the number of agents who are willing to participate in the action).

3 Computational Complexity of CAGs

Our characterisation of the computational complexity of consensus action games focuses on three natural decision problems associated with the selection of joint actions.

Consensus Action (CA): is an action a consensus action?

Group Consensus (GC): does a particular group of agents have a consensus action?

No Consensus (NC): is it the case that no group of agents has a consensus action?

We begin by considering the size of the input to the decision problems, namely the size of the representation of a CAG. Given a set of agents of size n and a set of actions of size m, in the worst case (when every set of agents can jointly execute any possible combination of actions) the set of joint actions J has cardinality $O(m^n)$, i.e., exponential in the number of agents. However, for any particular CAG $|J|$ may be significantly smaller than m^n.

We assume a concise representation of the input in which each joint action j is encoded as a set of triples rather than pairs: each triple consists of an agent, an action and the value of the quorum function for the agent and joint action. Thus q is encoded in J. We also assume the function $agents : J \rightarrow \mathfrak{P}(G)$ returns $G' \subseteq G$, the set of agents that may participate in action j, which runs in at most $O(n)$. Finally, we assume that J is implemented as a random access data structure and that we can determine the size (number of elements) in J in $O(log|J|)$.

The first three decision problems consider the complexity of determining CA, GC and NC for quorum consensus actions.

QUORUM CONSENSUS ACTION(QCA)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$ and a joint action $j \in J$, is j a quorum consensus action?

Algorithm: The algorithm must verify that $agents(j)$ is the least fixed point of $Support_j$.

Time Complexity: $O(n)$.

Algorithm 1 Is j a quorum consensus action.

```
function QCA(j, Γ)
    array support[|j| + 1] ← {0, ..., 0}
    for all (i, a, q) ∈ j do
        k ← ⌈q × |j|⌉
        support[k] ← support[k] + 1
    s ← support[0]
    for k from 1 to |j| do
        if k ≤ s then
            s ← s + support[k]
        else
            return false
    return true
```

Note that we can also obtain an $O(n \times log(n))$ algorithm, which runs in constant space by sorting j.

QUORUM GROUP CONSENSUS (QGC)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$ and a subset of agents $G' \subseteq G$, is there a quorum consensus action for G'?

Algorithm: The algorithm must verify that $\exists j \in J_{G'}$ such that G' is the least fixed point of $Support_j$.

Time Complexity: $O(n \times |J|)$

A non-deterministic algorithm first guesses an index of an action j in J (this can be done in $O(log(|J|)) \leq O(n)$ by the assumption that we can get the size of J in $O(log(|J|))$, and then checks that $agents(j) = G'$ and that j is a consensus action. This can be done in time linear in n using Algorithm 1. This gives us a non-deterministic linear time algorithm for a random access machine.[2] Hence, the problem is in NP(n) for RAM.

QUORUM NO CONSENSUS (QNC)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$, is it the case that no subset $G' \subseteq G$ has a quorum consensus action?

Algorithm: The algorithm must verify that $\neg\exists j \in J$ such that G' is the least fixed point of $Support_j$.

Time Complexity: $O(n \times |J|)$

Since the problem of the existence of a quorum consensus action is in NP(n) for RAM (guess an action in J and verify it is a quorum consensus action), its complement QNC is in co-NP(n) for RAM.

[2] As Immerman [1998] has observed, such machines correspond more closely to real computers than do multi-tape Turing machines.

3.1 The Core of Consensus Action Games

The core is a key solution concept in game theory that aggregates stable outcomes which are both individually and collectively rational. In CAGs, agents are willing to participate in any joint action where the degree of support for the action exceeds the agent's quorum threshold. However, a rational agent will disregard joint actions in which not all agents are willing to participate as these are unlikely to be performed. Thus the only joint actions in which an agent would actually participate are consensus actions. Consensus actions are therefore individually rational and, in one sense, stable. Collectively rational outcomes are, traditionally, those where no subset of agents can find improvement through unilateral defection. We consider the complexity of two complimentary solution concepts for the core of CAGs.

G'-Minimal Consensus

Our first solution concept takes a similar approach to the qualitative model of the core introduced in [Wooldridge and Dunne, 2004]. We define the G'-minimal core of CAGs as containing only G'-minimal consensus actions. A *G'-minimal consensus action* is a quorum consensus action for which no subset $G'' \subset G'$ of agents have a quorum consensus action. The G'-minimal core aggregates quorum consensus actions which are collectively rational in the sense that they are immune to unilateral defection by some agents $G'' \subset G'$.

Below we consider the complexity of determining CA, GC and NC under the solution concept of the G'-minimal core.

G'-MINIMAL CONSENSUS ACTION (GMCA)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$ and a joint action $j \in J$ by the agents $G' \subseteq G$, is j a G'-minimal consensus action for G'?

Algorithm: The algorithm must verify that G' is the least fixed point of $Support_j$ and that $\forall G'' \subset G', \neg\exists k \in J_{G''}$ such that G'' is the least fixed point of $Support_k$.

Time Complexity: $O(n \times |J|)$.

A non-deterministic algorithm to solve the complement of this problem (decide whether an action is *not* a G'-minimal consensus action) first checks whether j is a quorum consensus action (and returns true if it is not); if j is a quorum consensus action, it will guess an index of an action $k \in J$ and check that $agents(k) \subset G'$ and k is a quorum consensus action. So the problem of deciding whether an action is *not* a minimal quorum consensus action is in NP(n) on RAM. Hence deciding whether an action is a G'-minimal consensus action is in co-NP(n) for RAM.

G'-MINIMAL GROUP CONSENSUS (GMGC)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$ and a subset of agents $G' \subseteq G$, is there a minimal quorum consensus action for G'?

Algorithm: The algorithm must verify that $\exists j \in J_{G'}$ such that G' is the least fixed point of $Support_j$ and that $\forall G'' \subset G', \neg\exists k \in J_{G''}$ such that G'' is the least fixed point of $Support_k$.

Time Complexity: $O(n \times |J|)$

A nondeterministic algorithm first calls an NP(n) oracle to check that G' has a quorum consensus action; if G' does have a quorum consensus action, it then calls an NP(n) oracle to check whether any $G'' \subset G'$ has a quorum consensus action. Hence the problem is in $D^p(n)$ (on RAM).[3]

G'-MINIMAL NO CONSENSUS (GMNC)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$, is it the case that no subset $G' \subseteq G$ has a G'-minimal consensus action?

Algorithm: The algorithm must verify that $\neg\exists j \in J$ by agents $G' \subseteq G$ s.t. G' is the least fixed point of $Support_j$ and that $\forall G'' \subset G', \neg\exists k \in J_{G''}$ such that G'' is the least fixed point of $Support_k$.

Time Complexity: $O(n \times |J|)$

Note that if any subgroup of agents has a quorum consensus action then either that joint action, or some joint action by a subset of those agents will be minimal.

A non-deterministic polynomial time algorithm on RAM for solving the *complement* of this problem (to accept CAGs with non-empty G'-minimal core) would guess an action in J and verify that it is a quorum consensus action. Hence the problem of deciding whether the G'-minimal core is empty is co-NP(n).

$q-$Minimal Consensus

Our second solution concept focuses on the difficulty of reaching consensus. We define the *q-minimal core* of a CAG as containing only those joint actions for which the number of rounds required for quorum consensus is minimal. Specifically, a quorum consensus action j by the agents G' is a *q-minimal consensus action* if there is no other quorum consensus action for G' where the number of rounds required to reach consensus is less than the number of rounds required to reach consensus regarding j.

Below we consider the complexity of determining CA, GC and NC under the solution concept of the q-minimal core. We begin by defining a function $rounds$ that computes the number of rounds before the least fixed point of $Support_j$ is encountered.

Algorithm 2 Number of consensus rounds for j.

```
function rounds(j)
    Q ← 0
    r ← 0
    i_1 ← 0
    i_2 ← −1
    sort(j) by ascending q_i(j)
    for all (i, a, q) ∈ j do
        if q × |j| ≤ Q then
            Q ← Q + 1
            i_1 ← ⌊(q × |j|)⌋
            if (i_1 > i_2) then
                r ← r + 1
                i_2 ← i_1
    return r
```

Algorithm 2 has time complexity of $O(n \times log(n))$.

[3] The Difference class is the class of problems which are in the difference of two NP classes of problems [Papadimitriou, 1994]. Wooldridge and Dunne [2004] have shown that similar decision problems for Qualitative Coalitional Games (such as minimal successful coalition) are D^p-complete.

We can now consider the following decision problems for the q-minimal-core of CAGs.

QUORUM MINIMAL CONSENSUS ACTION (QMCA)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$, and a joint action $j \in J$, is j a q-minimal consensus action?

Algorithm: The algorithm must verify that $G' = agents(j)$ is the least fixed point of $Support_j$, and that no other quorum consensus action for G' reaches the least fixed point of $Support_j$ in fewer rounds than required for j.

Time Complexity: $O(n \times log(n) \times |J|)$

Algorithm 3 Is j a q-minimal consensus action.

```
function QMCA(j, Γ)
    G' ← agents(j)
    r ← 0
    if QCA(j, Γ) then
        r ← rounds(j)
    else
        return false
    for all k ∈ J do
        if agents(k) = G' ∧ QCA(k, Γ) then
            if rounds(k) < r then
                return false
    return true
```

A non-deterministic algorithm for deciding that j is *not* a quorum minimal consensus action will first check whether it is a consensus action (and return yes if it is not) and if it is, compute $rounds(j)$ and guess an action $k \in J$ and verify that $agents(k) = agents(j)$ and $rounds(k) < rounds(j)$. The problem of deciding that j is *not* a quorum minimal consensus action is therefore in NP(n) on RAM. Hence deciding whether j is a quorum minimal consensus action is in co-NP(n) on RAM.

QUORUM MINIMAL GROUP CONSENSUS (QMGC)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$ and a subset of agents $G' \subseteq G$, is there a q-minimal consensus action for G'?

Algorithm: Observe that if G' has a quorum consensus action then G' has a q-minimal consensus action; therefore this problem is equivalent to QCG.

QUORUM MINIMAL NO CONSENSUS (QMNC)

Given a CAG $\Gamma = \langle G, A, J, q\rangle$, is it the case that no subset $G' \subseteq G$ has a q-minimal consensus action?

Algorithm: Observe that if any G' has a quorum consensus action then at least one G' has a q-minimal consensus action. Therefore this problem is equivalent to QNC.

A summary of our results is given in table 1.

	QC	G'-minimal	q-minimal
CA	$P(n)$	$co\text{-}NP(n)$	$co\text{-}NP(n)$
GC	$NP(n)$	$D^p(n)$	$NP(n)$
NC	$co\text{-}NP(n)$	$co\text{-}NP(n)$	$co\text{-}NP(n)$

Table 1: Summary of Results (upper bounds). QC – Quorum Consensus, CA – Action Consensus, GC – Group Consensus, NC – No Consensus. Note that we assume random access to indices in J, so the complexity classes are for (N)RAM.

4 Related Work

CAGs have some similarities to Qualitative Coalitional Games (QCGs) [Wooldridge and Dunne, 2004]). It is therefore interesting to compare CAGs and QCGs, especially with respect to the size of representation and the complexity of similar decision problems.

A QCG Γ may be represented as an $(n+3)$ tuple $\Gamma = \langle \mathsf{G}, \mathsf{Ag}, \mathsf{G}_1 \ldots \mathsf{G}_n, \mathsf{V} \rangle$ where $\mathsf{G_i} \subseteq \mathsf{G}$ represents each agent's $\mathsf{i} \in \mathsf{Ag}$ set of goals and $\mathsf{V} : 2^{\mathsf{Ag}} \to 2^{2^{\mathsf{G}}}$ is the characteristic function of the game mapping each possible coalition of agents to the sets of goals that coalition can achieve. In QCGs:

- A set of goals $\mathsf{G}' \subseteq \mathsf{G}$ is *feasible* for a coalition $\mathsf{C} \subseteq \mathsf{Ag}$ if $\mathsf{G}' \in \mathsf{V}(\mathsf{C})$.
- A set of goals $\mathsf{G}' \subseteq \mathsf{G}$ *satisfies* an agent $\mathsf{i} \in \mathsf{C} \subseteq \mathsf{Ag}$ if $\mathsf{G}' \cap \mathsf{G_i} \neq \emptyset$.
- A coalition $\mathsf{C} \subseteq \mathsf{Ag}$ is *successful* if there exists some set of goals $\mathsf{G}' \subseteq \mathsf{G}$ such that G' is feasible for C and G' satisfies at least all agents $\mathsf{i} \in \mathsf{C}$. A coalition C is *selfishly successful* if G' is feasible for C and satisfies only the agents in $\mathsf{i} \in \mathsf{C}$.
- A coalition $\mathsf{C} \subseteq \mathsf{Ag}$ is in the *core* if it is both (selfishly) successful and minimal, i.e., there is no strict subset $\mathsf{C}' \subset \mathsf{C}$ which is successful.

To compare QCGs and CAGs, we can identify agents' goals with quorum consensus actions that they would participate in. CAGs thus correspond to a particular kind of QCGs, namely those where the characteristic function consists of singleton sets (since the agents can perform only one joint action at a time).

The worst case size of the game representation for QCGs is the characteristic function where each coalition can enforce any subset of goals. There are 2^n coalitions and 2^m subsets of goals, so the worst case size of V is $O(2^{n+m})$. This is different from CAGs where the worst case size of J is only exponential in n but not in m.

Complexity results for QCGs in [Wooldridge and Dunne, 2004] are given as a function of the size of representation, where the characterisitic function is replaced by a propositional formula Ψ (which as noted may be exponential in the number of agents and goals, but generally will be more concise than a naive representation of V). The successful coalition problem is NP in the size of the representation. It corresponds to our QGC problem which is also in NP, however it is NP in the number of agents (assuming random access). Alternatively, QGC can be characterised as linear in the size of representation since it involves a single iteration over J, doing a linear (in n) amount of work.

Relationships between CAGs and other game theoretic models can also be identified. A central premise in CAGs is that agents' choices are conditioned by the number of other agents also making some choice. Anonymous Games [Daskalakis and Papadimitriou, 2007] consider situations where the utility of participation in some coalition is independent of the identities of the agents concerned; in such situations other factors, including the size of the coalition become determinants of an agent's choice. In general, however, CAGs are non-anonymous therefore, for example, an agent could refuse ($q_i(j) = 1$) to participate in any joint action in which some other, specific, agent participates. In Imitation Games [McLennan and Tourky, 2010] two players take the roles of leader and follower; through the payoff structure the follower is motivated to act in consensus with the leader. McLennan and Tourky [2010] find that the complexity of several decision problems concerning Nash equilibria in such is games is no less than for the general two-player case.

CAGs are also related to work on conditional preference. In a CAG the agents must choose between potentially exponentially many joint actions. For an individual agent each joint action encodes: an action for that agent, the subset of agents with which it acts and the actions performed by those agents. Agents in CAGs must therefore make decisions over multiple domains.

It is not our intention that the quorum function be interpreted as a comparator or scale of preference over joint actions; however certain basic correspondences between the quorum function and preferences do exist. For example it is reasonable to identify those joint actions for which $q_i(j) = 0$ as being the 'most preferred' joint actions of agent i. Where $q_i(j) > 0$ support for a joint action becomes conditional, and an agent will only support j if the proportion of other agents supporting j exceeds $q_i(j)$.

Boutilier *et al* [1999] have proposed conditional preference, or CP-nets, as a natural and compact representation suitable for capturing conditional preferences over combinatorial domains. Succinctness is a useful property, as explicit representation of preference over exponentially many outcomes is often impractical. Preferences in CP-nets are formed under the assumption of ceteris paribus (all else being equal) and can be described as having the form: given x, $y > z$. This gives rise to preference structures which are potentially non-linear and may be incomplete.

There is considerable work in the social choice literature on preference aggregation. Much of this work has focused on the problem of aggregating the preferences of a large number of decision makers when making decisions over a single, relatively small, domain. Comparatively little attention has been given to collective decisions where the reverse is true, as is the case for CAGs. A notable exception is [Lang, 2007] where the potential of structure within CP-nets to reduce the computational overhead associated with combinatorial problems is explored. However Lang [2007] has shown that the complexity of all positional scoring voting rules, including Borda and even simple majority, cannot be reduced using CP-nets.

5 Discussion and Future Work

We have introduced consensus action games, in which agents' willingness to participate in joint actions is mediated by a biologically inspired quorum function. We have analysed the complexity of several natural decision problems associated with individual and collective rationality in CAGs and shown that tractable algorithms exist (at worst polynomial in the size of the input). We conjecture that the upper bounds are tight (that the lower bounds for the problems in table 1 are the same).

We have chosen to study consensus action selection in a context where individual decisions are conditioned through a quorum response as opposed to the more common setting where decisions are guided by preference. It seems likely that collective decisions in natural systems are not taken on a purely preferential basis; inherent difficulties associated with the representation, elicitation and aggregation of preferences in combinatorial domains are well known. Our results suggest that quorum behaviours may make comparatively lower cognitive demands on a decision maker. This may explain why even the simplest organisms are able to effectively coordinate group-level activities through the quorum mechanism.

A robust decision making procedure should reliably produce beneficial outcomes for the decision makers under diverse conditions. Our present model considers agents acting under a single set of constraints, those joint actions given in J. A natural extension to this work would be to consider iterated consensus action selection, where decisions regarding joint actions are taken repeatedly in differing states of the world. An iterated version of CAGs would allow us to investigate the performance of quorum consensus decisions over time. For example, it would be interesting to examine the implications of this decision process for both individual and social welfare. Of equal interest are the questions of how an agent's quorum function is implemented, the strategies that agents may employ in selecting quorum thresholds and the effects of these on individual and group well being.

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