

# 1 Fast Heuristics

The running times of the local searches started from Random solutions are close to those started from Greedy solutions. The running time of Random itself is negligible and, thus, is also skipped.

Instance	Gap to the best known, %								Time, ms			
	R	G	A(R)	A(G)	F(R)	F(G)	$V_1^{\text{ex}}(\text{R})$	$V_1^{\text{ex}}(\text{G})$	G	A(G)	F(G)	$V_1^{\text{ex}}(\text{G})$
100 × 1000	97.8	0.7	2.8	0.7	<u>0.1</u>	0.2	0.1	0.2	<u>2.1</u>	2.7	3.4	24.5
200 × 1000	103.4	1.8	2.9	0.3	0.3	<u>0.1</u>	0.4	0.2	<u>12.0</u>	14.2	15.7	37.4
400 × 1000	101.1	3.0	1.9	1.6	<u>0.5</u>	0.9	0.6	0.5	<u>7.5</u>	11.9	14.7	54.6
600 × 1000	98.5	4.7	1.3	1.4	<u>0.6</u>	1.0	0.7	0.8	<u>14.2</u>	21.0	31.8	77.3
800 × 1000	100.7	4.1	1.2	0.9	<u>0.5</u>	0.6	0.7	0.8	<u>12.8</u>	19.3	45.6	80.7
1000 × 1000	102.9	4.0	1.2	1.2	1.0	<u>0.5</u>	1.0	1.2	<u>20.2</u>	31.2	79.4	109.9
500 × 5000	99.5	1.1	3.0	1.0	0.2	0.1	0.2	<u>0.1</u>	<u>42.5</u>	71.5	188.3	471.8
1000 × 5000	99.2	2.6	1.7	1.3	0.3	0.3	0.3	<u>0.2</u>	<u>115.5</u>	175.9	421.3	926.4
2000 × 5000	99.2	3.5	1.3	0.9	<u>0.5</u>	0.5	0.6	0.5	<u>180.8</u>	337.3	976.9	2940.1
3000 × 5000	101.3	3.8	1.1	0.9	0.6	<u>0.3</u>	0.7	0.5	<u>289.9</u>	524.2	2596.9	4195.6
4000 × 5000	100.5	4.1	0.9	0.9	0.5	0.6	0.7	<u>0.4</u>	<u>367.7</u>	652.0	1721.3	7538.2
5000 × 5000	99.5	4.4	1.0	1.0	<u>0.8</u>	0.9	0.8	1.0	<u>407.9</u>	774.7	4806.9	1750.1
Average	100.3	3.2	1.7	1.0	<u>0.5</u>	0.5	0.6	0.5	<u>122.8</u>	219.6	908.5	1517.2

Table 1: Random instances.

Instance	Gap to the best known, %								Time, ms			
	R	G	A(R)	A(G)	F(R)	F(G)	$V_1^{\text{ex}}(\text{R})$	$V_1^{\text{ex}}(\text{G})$	G	A(G)	F(G)	$V_1^{\text{ex}}(\text{G})$
100 × 1000	417961.3	1.9	91.6	1.9	<u>0.0</u>	<u>0.0</u>	<u>0.0</u>	<u>0.0</u>	<u>20.1</u>	20.6	20.9	24.0
200 × 1000	499709.4	5.5	100.0	5.5	18.9	2.2	<u>1.3</u>	2.2	<u>13.4</u>	14.5	17.6	67.3
400 × 1000	560477.3	18.0	100.0	13.2	91.3	7.5	<u>3.2</u>	7.5	<u>6.9</u>	10.2	17.4	48.6
600 × 1000	590077.1	22.8	97.4	21.0	92.6	8.3	<u>6.2</u>	8.3	<u>56.5</u>	61.4	76.2	109.9
800 × 1000	647673.5	33.1	99.1	30.4	89.5	11.4	<u>6.7</u>	13.4	<u>13.7</u>	19.8	44.1	91.0
1000 × 1000	633927.8	47.7	97.7	45.5	85.7	27.2	<u>13.7</u>	30.3	<u>15.7</u>	23.7	50.5	137.9
500 × 5000	3239937.6	4.0	100.0	3.6	4.7	0.0	<u>0.0</u>	0.0	<u>44.8</u>	67.8	104.1	248.9
1000 × 5000	3924155.1	15.1	100.0	14.6	76.9	2.9	<u>0.5</u>	5.1	<u>85.8</u>	127.7	262.2	638.3
2000 × 5000	4506581.7	32.6	99.8	30.8	95.6	13.1	<u>12.1</u>	13.1	<u>164.6</u>	241.2	721.6	1666.6
3000 × 5000	4841943.6	49.4	99.4	47.1	96.4	28.5	<u>14.2</u>	28.1	<u>236.0</u>	345.9	950.5	1744.6
4000 × 5000	5155697.4	48.4	99.9	33.1	98.9	32.2	<u>12.1</u>	31.9	<u>322.7</u>	478.0	829.1	1448.2
5000 × 5000	4918821.7	66.3	99.8	65.5	98.9	45.0	<u>19.0</u>	44.6	<u>379.8</u>	584.8	1156.9	3211.4
Average	2494747.0	28.7	98.7	26.0	70.8	14.9	<u>7.4</u>	15.4	<u>113.3</u>	166.3	354.3	786.4

Table 2: Biclique instances.

Instance	Gap to the best known, %								Time, ms			
	R	G	A(R)	A(G)	F(R)	F(G)	$V_1^{\text{ex}}(\text{R})$	$V_1^{\text{ex}}(\text{G})$	G	A(G)	F(G)	$V_1^{\text{ex}}(\text{G})$
100 × 1000	101.2	1.7	5.3	1.7	0.4	<u>0.0</u>	0.4	<u>0.0</u>	<u>1.3</u>	2.2	4.8	8.7
200 × 1000	97.6	1.7	2.0	0.8	0.4	<u>0.2</u>	0.3	0.2	<u>3.1</u>	5.1	8.5	21.0
400 × 1000	98.8	4.1	1.2	0.9	0.3	<u>0.2</u>	0.2	0.3	<u>6.6</u>	12.7	32.6	46.3
600 × 1000	99.9	5.3	1.3	0.9	0.7	0.9	0.6	<u>0.4</u>	<u>15.9</u>	22.8	29.5	72.5
800 × 1000	99.4	6.4	1.4	2.1	0.8	<u>0.5</u>	0.8	0.8	<u>13.2</u>	21.4	64.5	89.3
1000 × 1000	99.8	7.8	1.5	1.3	<u>0.7</u>	0.8	0.9	1.1	<u>18.1</u>	30.8	69.8	92.5
500 × 5000	100.2	2.5	3.2	1.7	<u>0.2</u>	0.2	0.2	0.3	<u>48.7</u>	81.0	176.3	362.4
1000 × 5000	99.1	3.8	2.3	1.9	<u>0.2</u>	0.3	0.2	0.2	<u>79.1</u>	155.8	296.5	931.5
2000 × 5000	100.7	5.6	1.3	1.5	<u>0.5</u>	0.4	<u>0.4</u>	0.7	<u>163.5</u>	320.3	963.2	2423.4
3000 × 5000	99.7	6.2	1.2	1.0	0.6	0.6	0.7	<u>0.4</u>	<u>229.0</u>	493.7	1755.6	3790.7
4000 × 5000	99.9	6.7	1.1	0.8	0.7	0.9	0.8	<u>0.7</u>	<u>349.3</u>	669.4	1889.1	4716.6
5000 × 5000	100.1	7.3	1.1	1.1	0.7	<u>0.7</u>	0.9	0.8	<u>435.2</u>	800.2	3598.0	6573.4
Average	99.7	4.9	1.9	1.3	0.5	<u>0.5</u>	0.5	0.5	<u>113.6</u>	217.9	740.7	1594.0

Table 3: Max Induced Subgraph instances.

Instance	Gap to the best known, %								Time, ms			
	R	G	A(R)	A(G)	F(R)	F(G)	$V_1^{\text{ex}}(\text{R})$	$V_1^{\text{ex}}(\text{G})$	G	A(G)	F(G)	$V_1^{\text{ex}}(\text{G})$
100 × 1000	98.4	8.7	17.2	8.7	4.2	4.2	<u>2.8</u>	4.2	<u>2.6</u>	3.6	5.7	15.2
200 × 1000	94.6	10.5	14.6	10.5	3.6	<u>2.5</u>	4.5	<u>2.5</u>	<u>3.1</u>	5.1	11.8	35.4
400 × 1000	98.4	13.0	12.2	8.8	<u>5.5</u>	5.9	5.5	7.5	<u>7.4</u>	12.0	32.6	45.6
600 × 1000	99.8	14.2	10.6	7.9	5.6	4.9	6.9	<u>4.2</u>	<u>10.6</u>	18.6	43.3	112.0
800 × 1000	99.1	16.0	9.9	10.4	<u>6.7</u>	8.5	7.4	6.9	<u>13.1</u>	22.0	44.4	141.0
1000 × 1000	99.6	16.8	9.3	8.4	<u>7.2</u>	8.3	7.1	<u>6.5</u>	<u>17.3</u>	30.4	60.1	226.9
500 × 5000	100.0	6.5	15.6	6.5	<u>2.5</u>	2.7	2.6	2.7	<u>44.5</u>	69.6	325.0	705.5
1000 × 5000	99.5	9.0	13.4	8.6	3.0	2.9	3.1	<u>2.7</u>	<u>89.3</u>	147.2	453.7	1373.2
2000 × 5000	101.9	11.8	9.0	7.2	4.0	3.2	3.7	<u>2.8</u>	<u>168.4</u>	356.2	1614.2	3227.9
3000 × 5000	99.0	14.0	8.5	7.1	4.6	<u>4.4</u>	5.1	5.4	<u>259.9</u>	578.0	3705.9	6953.6
4000 × 5000	100.9	16.1	7.4	7.2	5.7	<u>4.3</u>	6.1	6.6	<u>358.9</u>	758.4	9079.6	4956.4
5000 × 5000	100.4	15.3	7.4	6.9	<u>6.0</u>	6.6	6.2	6.0	<u>471.7</u>	1013.3	6947.1	14639.6
Average	99.3	12.6	11.3	8.2	4.9	4.9	5.1	<u>4.8</u>	<u>120.6</u>	251.2	1860.3	2702.7

Table 4: MaxCut instances.

Instance	Gap to the best known, %								Time, ms			
	R	G	A(R)	A(G)	F(R)	F(G)	$V_1^{\text{ex}}(\text{R})$	$V_1^{\text{ex}}(\text{G})$	G	A(G)	F(G)	$V_1^{\text{ex}}(\text{G})$
100 × 1000	103.9	5.8	4.5	<u>1.8</u>	4.0	2.6	3.4	<u>1.8</u>	<u>2.2</u>	3.3	3.8	20.3
200 × 1000	101.3	9.8	3.7	3.2	<u>2.5</u>	2.8	2.8	2.8	<u>3.3</u>	5.6	7.5	25.8
400 × 1000	100.1	7.0	2.4	2.1	<u>1.4</u>	1.9	2.1	1.4	<u>5.2</u>	9.9	16.9	26.6
600 × 1000	101.5	6.9	1.7	1.1	<u>1.0</u>	<u>0.8</u>	1.3	0.9	<u>7.8</u>	14.9	25.8	38.3
800 × 1000	101.2	5.8	2.2	2.3	1.7	1.2	1.6	<u>1.1</u>	<u>41.9</u>	49.7	86.2	377.8
1000 × 1000	101.3	5.6	1.6	2.0	<u>0.9</u>	1.7	1.3	1.8	<u>13.2</u>	24.0	29.8	58.1
500 × 5000	99.2	10.5	2.9	4.5	<u>2.5</u>	2.6	2.6	3.0	<u>41.3</u>	73.8	230.3	299.3
1000 × 5000	99.8	9.3	2.5	2.6	<u>1.5</u>	1.7	2.1	2.6	<u>86.3</u>	171.1	409.0	421.3
2000 × 5000	100.5	7.2	1.7	1.7	1.1	<u>0.9</u>	1.3	1.6	<u>184.6</u>	344.3	996.7	1099.3
3000 × 5000	100.1	6.3	1.3	1.2	0.7	<u>0.6</u>	1.0	0.9	<u>292.5</u>	515.2	2641.7	3035.8
4000 × 5000	99.4	5.8	1.2	1.1	0.9	<u>1.0</u>	1.0	<u>0.9</u>	<u>431.5</u>	721.1	1859.3	4737.4
5000 × 5000	99.6	5.4	1.2	1.2	<u>0.8</u>	0.9	1.0	1.0	<u>413.4</u>	745.4	3719.3	5614.7
Average	100.7	7.1	2.2	2.1	1.6	<u>1.6</u>	1.8	1.6	<u>126.9</u>	223.2	835.5	1312.9

Table 5: Matrix Factorization instances.

## 2 Portions-Based and Multi-Start Heuristics

The running times of M(A), M(F), M(V<sub>1</sub><sup>ex</sup>), P<sub>3</sub>, P<sub>4</sub>, P<sub>6</sub> and P<sub>8</sub> are equal to that of V<sub>k</sub>.

### 2.1 Comparison with V<sub>4</sub>

Instance	Gap to the best known, %										Time, sec		
	V <sub>1</sub> <sup>ex</sup>	V <sub>2</sub> <sup>ex</sup>	V <sub>4</sub>	M(A)	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	P <sub>3</sub>	P <sub>4</sub>	P <sub>6</sub>	P <sub>8</sub>	V <sub>1</sub> <sup>ex</sup>	V <sub>2</sub> <sup>ex</sup>	V <sub>4</sub>
100 × 1000	0.18	0.04	0.14	0.86	<u>0.00</u>	<u>0.00</u>	0.10	0.08	0.12	0.13	<u>0.0</u>	0.4	0.1
200 × 1000	0.15	0.10	0.12	0.99	<u>0.00</u>	0.10	0.11	0.10	0.13	0.12	<u>0.0</u>	1.5	0.2
400 × 1000	0.52	0.52	0.52	0.65	0.18	<u>0.12</u>	0.42	0.38	0.39	0.53	<u>0.1</u>	2.5	0.2
600 × 1000	0.79	0.57	0.79	0.46	<u>0.16</u>	0.22	0.72	0.57	0.66	0.73	<u>0.1</u>	18.5	0.4
800 × 1000	0.77	0.62	0.77	0.60	<u>0.31</u>	0.37	0.53	0.53	0.63	0.48	<u>0.1</u>	37.8	0.4
1000 × 1000	1.15	0.98	1.15	0.66	0.52	<u>0.49</u>	0.52	0.63	0.72	0.72	<u>0.1</u>	54.3	0.6
500 × 5000	0.05	<u>0.02</u>	0.05	2.05	0.06	0.07	0.16	0.15	0.12	0.11	<u>0.5</u>	66.0	1.9
1000 × 5000	0.21	0.19	0.21	1.16	<u>0.17</u>	0.21	0.28	0.30	0.27	0.22	<u>0.9</u>	257.2	3.6
2000 × 5000	0.49	0.49	0.49	0.88	<u>0.33</u>	0.38	0.46	0.43	0.45	0.44	<u>2.9</u>	1243.5	8.1
3000 × 5000	0.55	0.55	0.55	0.72	<u>0.42</u>	0.46	0.50	0.48	0.50	0.55	<u>4.2</u>	686.1	13.0
4000 × 5000	0.44	<u>0.40</u>	0.44	0.62	0.44	0.49	0.58	0.55	0.58	0.61	<u>7.5</u>	5490.2	17.4
5000 × 5000	0.97	<u>0.62</u>	0.97	0.75	0.63	0.62	0.75	0.82	0.75	0.75	<u>1.8</u>	29088.8	13.7
Average	0.52	0.43	0.52	0.87	<u>0.27</u>	0.30	0.43	0.42	0.44	0.45	<u>1.5</u>	3078.9	5.0

Table 6: Random instances.

Instance	Gap to the best known, %										Time, sec		
	V <sub>1</sub> <sup>ex</sup>	V <sub>2</sub> <sup>ex</sup>	V <sub>4</sub>	M(A)	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	P <sub>3</sub>	P <sub>4</sub>	P <sub>6</sub>	P <sub>8</sub>	V <sub>1</sub> <sup>ex</sup>	V <sub>2</sub> <sup>ex</sup>	V <sub>4</sub>
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	90.85	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.0</u>	0.2	0.1
200 × 1000	2.21	<u>0.00</u>	2.05	94.76	2.74	0.39	1.51	0.88	1.24	1.27	<u>0.1</u>	9.4	0.6
400 × 1000	7.51	7.41	7.51	97.04	77.12	<u>3.21</u>	7.61	7.51	7.72	7.52	<u>0.0</u>	6.6	0.2
600 × 1000	8.33	<u>1.14</u>	7.83	94.00	88.12	2.75	7.03	7.08	7.43	7.66	<u>0.1</u>	24.2	0.4
800 × 1000	13.45	<u>0.49</u>	12.62	95.89	86.53	6.01	13.53	12.88	13.52	13.47	<u>0.1</u>	51.2	0.6
1000 × 1000	30.35	11.11	28.13	95.85	80.42	<u>10.88</u>	26.83	26.32	26.98	28.16	<u>0.1</u>	85.7	0.8
500 × 5000	0.03	0.03	0.03	99.53	4.72	<u>0.00</u>	0.03	0.03	0.03	0.03	<u>0.2</u>	16.8	1.2
1000 × 5000	5.14	<u>0.20</u>	4.18	99.13	9.92	0.50	2.81	3.65	3.74	3.56	<u>0.6</u>	219.5	3.0
2000 × 5000	13.12	<u>6.23</u>	12.96	98.77	93.78	12.07	13.28	13.73	13.84	13.73	<u>1.7</u>	1684.6	6.7
3000 × 5000	28.11	<u>4.74</u>	27.94	98.36	95.18	9.70	28.09	28.33	28.04	28.37	<u>1.7</u>	5205.6	9.5
4000 × 5000	31.87	12.70	31.66	99.45	98.37	<u>8.92</u>	32.24	32.11	32.27	32.36	<u>1.4</u>	5866.5	11.5
5000 × 5000	44.62	21.47	44.51	99.24	97.95	<u>15.45</u>	44.09	44.42	44.29	44.70	<u>3.2</u>	9012.8	14.4
Average	15.39	<u>5.46</u>	14.95	96.91	61.24	5.82	14.75	14.75	14.93	15.07	<u>0.8</u>	1848.6	4.1

Table 7: Biclique instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_4$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_4$
$100 \times 1000$	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	2.54	<u>0.00</u>	<u>0.00</u>	0.27	0.32	0.15	0.48	<u>0.0</u>	0.2	0.1
$200 \times 1000$	0.23	0.18	0.23	0.75	<u>0.07</u>	0.09	0.25	0.24	0.27	0.22	<u>0.0</u>	1.2	0.1
$400 \times 1000$	0.27	<u>0.06</u>	0.20	0.67	0.08	0.11	0.39	0.38	0.44	0.47	<u>0.0</u>	8.5	0.2
$600 \times 1000$	0.38	<u>0.37</u>	0.38	0.72	0.34	<u>0.30</u>	0.43	0.44	0.51	0.56	<u>0.1</u>	15.4	0.3
$800 \times 1000$	0.79	0.65	0.78	0.79	<u>0.41</u>	<u>0.54</u>	0.71	0.74	0.72	0.72	<u>0.1</u>	32.6	0.5
$1000 \times 1000$	1.06	1.01	1.06	0.82	<u>0.42</u>	0.43	0.69	0.55	0.75	0.81	<u>0.1</u>	42.5	0.5
$500 \times 5000$	0.29	0.18	0.27	2.31	<u>0.04</u>	0.05	0.17	0.19	0.18	0.09	<u>0.4</u>	91.4	1.9
$1000 \times 5000$	0.22	0.21	0.22	1.43	<u>0.13</u>	0.17	0.22	0.20	0.23	0.22	<u>0.9</u>	548.6	3.2
$2000 \times 5000$	0.65	0.54	0.65	0.94	<u>0.27</u>	0.32	0.43	0.43	0.48	0.50	<u>2.4</u>	1825.1	7.2
$3000 \times 5000$	0.42	<u>0.41</u>	0.42	0.84	0.45	0.46	0.59	0.62	0.59	0.66	<u>3.8</u>	2199.0	10.6
$4000 \times 5000$	0.68	<u>0.68</u>	0.68	0.81	<u>0.61</u>	0.66	0.75	0.71	0.67	0.71	<u>4.7</u>	1015.1	14.1
$5000 \times 5000$	0.77	<u>0.54</u>	0.77	0.73	0.56	0.68	0.76	0.76	0.75	0.78	<u>6.6</u>	26134.9	18.1
Average	0.48	0.40	0.47	1.11	<u>0.28</u>	0.32	0.47	0.46	0.48	0.52	<u>1.6</u>	2659.5	4.7

Table 8: Max Induced Subgraph instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_4$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_4$
$100 \times 1000$	4.21	4.06	4.19	11.44	<u>0.98</u>	1.67	4.36	3.82	3.93	4.48	<u>0.0</u>	0.4	0.1
$200 \times 1000$	2.53	2.53	2.53	10.23	<u>1.75</u>	2.46	2.90	2.56	2.94	3.31	<u>0.0</u>	0.6	0.1
$400 \times 1000$	7.47	<u>3.62</u>	7.46	9.10	4.02	4.21	5.55	5.86	5.62	6.15	<u>0.0</u>	16.3	0.2
$600 \times 1000$	4.20	<u>3.20</u>	4.20	7.14	4.27	4.79	4.18	4.44	4.70	4.47	<u>0.1</u>	42.7	0.4
$800 \times 1000$	6.93	6.72	6.93	6.82	<u>5.08</u>	5.32	8.36	8.19	7.49	8.15	<u>0.1</u>	31.5	0.5
$1000 \times 1000$	6.49	6.40	6.49	5.53	<u>4.38</u>	4.47	6.69	6.15	6.91	7.00	<u>0.2</u>	43.9	7.2
$500 \times 5000$	2.67	2.63	2.66	13.27	<u>1.65</u>	1.93	2.97	2.98	3.12	3.01	<u>0.7</u>	50.3	1.9
$1000 \times 5000$	2.72	2.69	2.72	10.80	<u>1.99</u>	2.09	3.14	3.06	2.99	3.14	<u>1.4</u>	290.7	3.9
$2000 \times 5000$	2.76	<u>2.75</u>	2.76	7.51	3.26	3.39	3.25	3.12	2.92	3.37	<u>3.2</u>	671.9	8.2
$3000 \times 5000$	5.42	5.03	5.42	6.50	<u>4.33</u>	4.54	4.80	4.64	4.54	4.64	<u>7.0</u>	2707.7	14.2
$4000 \times 5000$	6.60	6.48	6.60	6.52	5.23	<u>5.21</u>	5.85	5.68	5.66	5.59	<u>5.0</u>	4180.5	14.9
$5000 \times 5000$	6.04	6.04	6.04	5.88	<u>5.02</u>	5.23	6.12	6.16	6.31	5.99	<u>14.6</u>	4784.4	25.8
Average	4.84	4.35	4.83	8.39	<u>3.50</u>	3.78	4.85	4.72	4.76	4.94	<u>2.7</u>	1068.4	6.4

Table 9: MaxCut instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_4$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_4$
100 × 1000	1.78	<u>0.12</u>	0.22	1.84	1.62	1.52	0.16	0.17	0.28	0.63	<u>0.0</u>	0.6	0.2
200 × 1000	2.80	<u>0.32</u>	0.60	1.31	0.88	1.10	0.43	0.48	0.53	1.17	<u>0.0</u>	1.3	0.2
400 × 1000	1.41	<u>0.25</u>	0.46	1.00	0.49	0.75	0.71	0.78	0.93	1.46	<u>0.0</u>	4.7	0.3
600 × 1000	0.85	0.55	0.71	0.69	<u>0.42</u>	0.56	0.54	0.69	0.78	1.13	<u>0.0</u>	19.2	0.3
800 × 1000	1.11	<u>0.91</u>	0.95	1.32	0.97	1.06	1.19	1.23	1.48	1.52	<u>0.4</u>	18.2	0.4
1000 × 1000	1.77	1.47	1.52	0.89	<u>0.48</u>	0.67	1.02	0.86	1.12	1.42	<u>0.1</u>	27.4	0.5
500 × 5000	2.95	0.17	0.25	2.10	1.60	1.95	0.20	<u>0.14</u>	0.22	0.40	<u>0.3</u>	78.4	5.9
1000 × 5000	2.60	<u>0.15</u>	0.54	1.40	0.97	1.18	0.35	0.35	0.42	0.61	<u>0.4</u>	421.3	16.9
2000 × 5000	1.56	<u>0.22</u>	0.84	1.03	0.62	0.77	0.52	0.55	0.63	0.90	<u>1.1</u>	1937.3	19.6
3000 × 5000	0.93	0.64	0.78	0.88	<u>0.61</u>	0.75	0.65	0.71	0.71	0.93	<u>3.0</u>	2296.5	19.2
4000 × 5000	0.86	<u>0.54</u>	0.66	0.74	0.65	0.69	0.68	0.77	0.86	0.88	<u>4.7</u>	5824.3	24.8
5000 × 5000	1.01	0.72	0.96	0.81	<u>0.65</u>	0.75	0.89	0.84	0.94	0.89	<u>5.6</u>	7880.7	25.1
Average	1.64	<u>0.51</u>	0.71	1.17	0.83	0.98	0.61	0.63	0.74	1.00	<u>1.3</u>	1542.5	9.4

Table 10: Matrix Factorization instances.

## 2.2 Comparison with $V_6$

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$
$100 \times 1000$	0.18	0.04	0.08	0.77	<u>0.00</u>	<u>0.00</u>	0.06	0.05	0.09	0.10	<u>0.0</u>	0.4	0.2
$200 \times 1000$	0.15	0.10	0.11	0.80	<u>0.00</u>	0.01	0.11	0.10	0.11	0.15	<u>0.0</u>	1.5	0.5
$400 \times 1000$	0.52	0.52	0.52	0.59	0.09	<u>0.08</u>	0.41	0.40	0.37	0.68	<u>0.1</u>	2.5	0.7
$600 \times 1000$	0.79	0.57	0.74	0.31	<u>0.11</u>	0.13	0.71	0.63	0.71	0.70	<u>0.1</u>	18.5	1.2
$800 \times 1000$	0.77	0.62	0.74	0.46	<u>0.25</u>	0.30	0.54	0.52	0.62	0.54	<u>0.1</u>	37.8	1.4
$1000 \times 1000$	1.15	0.98	1.14	0.48	<u>0.35</u>	0.39	0.53	0.56	0.56	0.59	<u>0.1</u>	54.3	2.1
$500 \times 5000$	0.05	<u>0.02</u>	0.04	1.88	0.05	0.03	0.16	0.15	0.12	0.11	<u>0.5</u>	66.0	4.7
$1000 \times 5000$	0.21	0.19	0.20	1.09	<u>0.15</u>	0.16	0.28	0.30	0.27	0.30	<u>0.9</u>	257.2	10.3
$2000 \times 5000$	0.49	0.49	0.49	0.83	<u>0.30</u>	0.33	0.44	0.44	0.47	0.44	<u>2.9</u>	1 243.5	20.4
$3000 \times 5000$	0.55	0.55	0.55	0.69	<u>0.36</u>	0.42	0.50	0.49	0.55	0.46	<u>4.2</u>	686.1	34.5
$4000 \times 5000$	0.44	0.40	0.44	0.55	<u>0.39</u>	0.45	0.57	0.56	0.58	0.53	<u>7.5</u>	5 490.2	48.5
$5000 \times 5000$	0.97	0.62	0.97	0.68	0.57	<u>0.57</u>	0.73	0.80	0.74	0.72	<u>1.8</u>	29 088.8	42.5
Average	0.52	0.43	0.50	0.76	<u>0.22</u>	0.24	0.42	0.42	0.43	0.44	<u>1.5</u>	3 078.9	13.9

Table 11: Random instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$
$100 \times 1000$	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	87.95	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.0</u>	0.2	0.2
$200 \times 1000$	2.21	<u>0.00</u>	1.49	94.76	2.74	0.13	1.07	0.60	1.08	1.18	<u>0.1</u>	9.4	0.3
$400 \times 1000$	7.51	7.41	7.51	96.81	75.05	<u>3.21</u>	7.53	7.51	7.70	7.52	<u>0.0</u>	6.6	0.6
$600 \times 1000$	8.33	<u>1.14</u>	5.61	92.59	85.64	2.38	6.10	6.03	6.00	7.07	<u>0.1</u>	24.2	1.7
$800 \times 1000$	13.45	<u>0.49</u>	11.46	95.29	85.40	6.01	11.02	11.75	11.76	12.24	<u>0.1</u>	51.2	1.5
$1000 \times 1000$	30.35	11.11	25.64	95.41	78.20	<u>10.88</u>	25.33	24.60	25.54	26.32	<u>0.1</u>	85.7	2.6
$500 \times 5000$	0.03	0.03	0.03	98.58	4.72	<u>0.00</u>	0.03	0.03	0.03	0.03	<u>0.2</u>	16.8	3.6
$1000 \times 5000$	5.14	<u>0.20</u>	2.33	98.56	1.09	0.40	1.37	2.03	1.45	3.04	<u>0.6</u>	219.5	11.6
$2000 \times 5000$	13.12	<u>6.23</u>	12.71	98.47	93.27	12.06	12.82	13.08	13.37	14.06	<u>1.7</u>	1 684.6	20.9
$3000 \times 5000$	28.11	<u>4.74</u>	27.36	98.30	94.81	8.13	27.20	27.53	27.58	27.97	<u>1.7</u>	5 205.6	43.3
$4000 \times 5000$	31.87	12.70	27.09	98.98	98.02	<u>8.92</u>	28.29	27.95	29.83	30.80	<u>1.4</u>	5 866.5	147.3
$5000 \times 5000$	44.62	21.47	43.14	99.12	97.64	<u>15.45</u>	41.85	42.26	42.58	43.89	<u>3.2</u>	9 012.8	121.2
Average	15.39	<u>5.46</u>	13.70	96.24	59.71	5.63	13.55	13.61	13.91	14.51	<u>0.8</u>	1 848.6	29.6

Table 12: Biclique instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	2.17	<u>0.00</u>	<u>0.00</u>	0.06	0.05	0.15	0.31	<u>0.0</u>	0.2	0.2
200 × 1000	<u>0.23</u>	<u>0.18</u>	<u>0.21</u>	0.63	<u>0.03</u>	<u>0.01</u>	0.23	0.21	0.26	0.23	<u>0.0</u>	1.2	0.3
400 × 1000	<u>0.27</u>	<u>0.06</u>	<u>0.16</u>	0.48	<u>0.04</u>	<u>0.08</u>	0.38	0.37	0.44	0.33	<u>0.0</u>	8.5	0.7
600 × 1000	<u>0.38</u>	<u>0.37</u>	<u>0.38</u>	0.58	<u>0.27</u>	<u>0.29</u>	0.40	0.44	0.53	0.47	<u>0.1</u>	15.4	0.9
800 × 1000	<u>0.79</u>	<u>0.65</u>	<u>0.78</u>	0.69	<u>0.35</u>	<u>0.49</u>	0.70	0.78	0.73	0.80	<u>0.1</u>	32.6	1.3
1000 × 1000	<u>1.06</u>	<u>1.01</u>	<u>1.05</u>	0.72	<u>0.34</u>	<u>0.37</u>	0.60	0.52	0.75	0.64	<u>0.1</u>	42.5	1.9
500 × 5000	<u>0.29</u>	<u>0.18</u>	<u>0.23</u>	2.04	<u>0.02</u>	<u>0.04</u>	0.17	0.19	0.16	0.20	<u>0.4</u>	91.4	5.0
1000 × 5000	<u>0.22</u>	<u>0.21</u>	<u>0.22</u>	1.30	<u>0.08</u>	<u>0.12</u>	0.21	0.20	0.23	0.23	<u>0.9</u>	548.6	9.5
2000 × 5000	<u>0.65</u>	<u>0.54</u>	<u>0.65</u>	0.87	<u>0.23</u>	<u>0.26</u>	0.43	0.41	0.43	0.44	<u>2.4</u>	1825.1	20.4
3000 × 5000	<u>0.42</u>	<u>0.41</u>	<u>0.42</u>	0.77	<u>0.36</u>	<u>0.44</u>	0.57	0.62	0.60	0.67	<u>3.8</u>	2199.0	33.0
4000 × 5000	<u>0.68</u>	<u>0.68</u>	<u>0.68</u>	0.75	<u>0.54</u>	<u>0.58</u>	0.73	0.69	0.70	0.73	<u>4.7</u>	1015.1	37.7
5000 × 5000	<u>0.77</u>	<u>0.54</u>	<u>0.77</u>	0.69	<u>0.54</u>	<u>0.62</u>	0.76	0.75	0.75	0.72	<u>6.6</u>	26134.9	47.8
Average	<u>0.48</u>	<u>0.40</u>	<u>0.46</u>	0.97	<u>0.23</u>	<u>0.28</u>	0.44	0.44	0.48	0.48	<u>1.6</u>	2659.5	13.2

Table 13: Max Induced Subgraph instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$
100 × 1000	4.21	4.06	4.18	10.90	0.71	<u>0.49</u>	3.54	3.25	3.50	3.65	<u>0.0</u>	0.4	0.2
200 × 1000	2.53	2.53	2.53	9.18	<u>1.55</u>	1.94	2.78	2.41	2.56	3.39	<u>0.0</u>	0.6	0.4
400 × 1000	7.47	3.62	7.41	7.72	<u>3.25</u>	3.52	5.36	5.62	5.74	5.48	<u>0.0</u>	16.3	0.7
600 × 1000	4.20	<u>3.20</u>	4.19	6.60	3.52	4.24	4.17	4.09	4.41	4.52	<u>0.1</u>	42.7	1.2
800 × 1000	6.93	6.72	6.92	6.07	<u>4.52</u>	4.70	8.34	8.04	7.93	7.37	<u>0.1</u>	31.5	1.5
1000 × 1000	6.49	6.40	6.49	6.01	<u>4.94</u>	5.06	6.77	6.30	6.90	7.03	<u>0.2</u>	43.9	1.9
500 × 5000	2.67	2.63	2.65	12.96	<u>1.38</u>	1.53	2.80	3.11	2.95	3.10	<u>0.7</u>	50.3	5.3
1000 × 5000	2.72	2.69	2.72	10.11	<u>1.67</u>	1.84	3.11	3.09	3.04	3.09	<u>1.4</u>	290.7	10.0
2000 × 5000	2.76	2.75	2.76	7.34	<u>2.70</u>	2.98	3.26	3.05	3.32	3.28	<u>3.2</u>	671.9	18.2
3000 × 5000	5.42	5.03	5.42	6.32	<u>4.13</u>	4.31	4.67	4.71	4.74	4.76	<u>7.0</u>	2707.7	33.9
4000 × 5000	6.60	6.48	6.60	6.26	<u>4.91</u>	5.05	5.63	5.90	5.90	5.79	<u>5.0</u>	4180.5	36.6
5000 × 5000	6.04	6.04	6.04	5.80	<u>4.92</u>	5.03	6.03	6.06	5.95	6.29	<u>14.6</u>	4784.4	56.1
Average	<u>4.84</u>	<u>4.35</u>	<u>4.83</u>	7.94	<u>3.18</u>	<u>3.39</u>	4.71	4.64	4.74	4.81	<u>2.7</u>	1068.4	13.8

Table 14: MaxCut instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_6$
100 × 1000	1.78	0.12	0.12	1.58	1.33	1.51	<u>0.07</u>	0.09	0.14	0.33	<u>0.0</u>	0.6	0.4
200 × 1000	2.80	0.32	0.44	1.12	0.64	0.95	0.28	<u>0.25</u>	0.33	0.89	<u>0.0</u>	1.3	0.6
400 × 1000	1.41	<u>0.25</u>	0.32	0.86	0.40	0.63	0.61	0.66	0.66	0.87	<u>0.0</u>	4.7	1.1
600 × 1000	0.85	0.55	0.66	0.60	<u>0.31</u>	0.38	0.47	0.47	0.56	0.81	<u>0.0</u>	19.2	1.5
800 × 1000	1.11	0.91	0.88	1.18	<u>0.83</u>	0.87	1.08	1.10	1.30	1.40	<u>0.4</u>	18.2	1.9
1000 × 1000	1.77	1.47	1.16	0.79	<u>0.34</u>	0.53	0.99	0.82	0.95	1.06	<u>0.1</u>	27.4	2.7
500 × 5000	2.95	0.17	0.14	2.01	1.56	1.62	0.13	<u>0.11</u>	0.14	0.24	<u>0.3</u>	78.4	16.8
1000 × 5000	2.60	<u>0.15</u>	0.31	1.34	0.88	1.13	0.30	0.29	0.29	0.41	<u>0.4</u>	421.3	46.7
2000 × 5000	1.56	<u>0.22</u>	0.50	0.92	0.58	0.70	0.43	0.47	0.48	0.59	<u>1.1</u>	1937.3	84.4
3000 × 5000	0.93	0.64	0.69	0.82	<u>0.55</u>	0.59	0.57	0.65	0.62	0.77	<u>3.0</u>	2296.5	66.3
4000 × 5000	0.86	<u>0.54</u>	0.65	0.72	0.62	0.64	0.67	0.73	0.78	0.80	<u>4.7</u>	5824.3	59.2
5000 × 5000	1.01	0.72	0.94	0.79	<u>0.60</u>	0.67	0.89	0.82	0.90	0.88	<u>5.6</u>	7880.7	56.1
Average	1.64	<u>0.51</u>	0.57	1.06	0.72	0.85	0.54	0.54	0.59	0.75	<u>1.3</u>	1542.5	28.1

Table 15: Matrix Factorization instances.



### 2.3 Comparison with $V_8$

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$
$100 \times 1000$	0.18	0.04	0.04	0.53	<u>0.00</u>	<u>0.00</u>	0.03	0.03	0.03	0.05	<u>0.0</u>	0.4	0.8
$200 \times 1000$	0.15	0.10	0.11	0.72	0.00	<u>0.00</u>	0.11	0.10	0.11	0.13	<u>0.0</u>	1.5	1.3
$400 \times 1000$	0.52	0.52	0.52	0.46	<u>0.05</u>	0.06	0.41	0.27	0.34	0.41	<u>0.1</u>	2.5	2.6
$600 \times 1000$	0.79	0.57	0.74	0.21	<u>0.07</u>	0.10	0.66	0.62	0.64	0.54	<u>0.1</u>	18.5	3.5
$800 \times 1000$	0.77	0.62	0.72	0.35	<u>0.12</u>	0.23	0.52	0.49	0.54	0.54	<u>0.1</u>	37.8	5.5
$1000 \times 1000$	1.15	0.98	1.13	0.41	0.28	<u>0.27</u>	0.51	0.49	0.56	0.57	<u>0.1</u>	54.3	5.5
$500 \times 5000$	0.05	0.02	0.03	1.73	<u>0.01</u>	0.01	0.15	0.12	0.11	0.09	<u>0.5</u>	66.0	18.8
$1000 \times 5000$	0.21	0.19	0.19	1.02	<u>0.09</u>	0.11	0.28	0.29	0.25	0.27	<u>0.9</u>	257.2	40.4
$2000 \times 5000$	0.49	0.49	0.49	0.76	<u>0.24</u>	0.26	0.42	0.44	0.41	0.43	<u>2.9</u>	1 243.5	69.1
$3000 \times 5000$	0.55	0.55	0.55	0.64	<u>0.31</u>	0.39	0.50	0.48	0.51	0.45	<u>4.2</u>	686.1	92.6
$4000 \times 5000$	0.44	0.40	0.44	0.51	<u>0.35</u>	0.38	0.57	0.56	0.59	0.52	<u>7.5</u>	5 490.2	140.5
$5000 \times 5000$	0.97	0.62	0.97	0.61	<u>0.51</u>	0.51	0.73	0.79	0.74	0.76	<u>1.8</u>	29 088.8	154.0
Average	0.52	0.43	0.50	0.66	<u>0.17</u>	0.19	0.41	0.39	0.40	0.40	<u>1.5</u>	3 078.9	44.6

Table 16: Random instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$
$100 \times 1000$	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	85.76	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.0</u>	0.2	0.6
$200 \times 1000$	2.21	<u>0.00</u>	0.77	94.76	2.74	<u>0.00</u>	0.32	0.29	0.46	0.51	<u>0.1</u>	9.4	1.7
$400 \times 1000$	7.51	7.41	7.49	96.10	72.20	<u>3.21</u>	7.49	7.45	7.59	7.51	<u>0.0</u>	6.6	2.4
$600 \times 1000$	8.33	<u>1.14</u>	4.22	92.23	84.27	2.38	4.01	4.00	4.61	5.23	<u>0.1</u>	24.2	7.6
$800 \times 1000$	13.45	<u>0.49</u>	9.33	94.54	84.31	6.01	7.47	8.39	8.44	9.99	<u>0.1</u>	51.2	9.5
$1000 \times 1000$	30.35	11.11	22.62	94.97	76.31	<u>10.88</u>	22.57	21.25	22.39	23.85	<u>0.1</u>	85.7	14.0
$500 \times 5000$	0.03	0.03	0.03	97.63	4.72	<u>0.00</u>	0.03	0.03	0.03	0.03	<u>0.2</u>	16.8	13.3
$1000 \times 5000$	5.14	<u>0.20</u>	0.71	98.56	1.09	0.34	0.23	0.59	0.36	1.60	<u>0.6</u>	219.5	60.0
$2000 \times 5000$	13.12	<u>6.23</u>	11.88	98.40	92.70	12.02	11.82	11.54	11.70	12.53	<u>1.7</u>	1 684.6	129.9
$3000 \times 5000$	28.11	<u>4.74</u>	25.23	97.95	94.36	8.13	24.67	24.19	24.69	25.72	<u>1.7</u>	5 205.6	273.7
$4000 \times 5000$	31.87	12.70	13.03	98.83	97.99	<u>8.92</u>	12.49	12.60	14.47	22.39	<u>1.4</u>	5 866.5	1 066.6
$5000 \times 5000$	44.62	21.47	24.30	98.88	97.33	<u>15.45</u>	21.82	21.57	22.46	33.66	<u>3.2</u>	9 012.8	1 466.1
Average	15.39	<u>5.46</u>	9.97	95.72	59.00	5.61	9.41	9.33	9.77	11.92	<u>0.8</u>	1 848.6	253.8

Table 17: Biclique instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	1.86	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.01	0.15	<u>0.0</u>	0.2	0.6
200 × 1000	<u>0.23</u>	<u>0.18</u>	<u>0.18</u>	0.45	<u>0.00</u>	<u>0.00</u>	<u>0.21</u>	<u>0.20</u>	0.19	0.21	<u>0.0</u>	1.2	1.5
400 × 1000	<u>0.27</u>	<u>0.06</u>	<u>0.11</u>	0.37	<u>0.02</u>	<u>0.02</u>	<u>0.34</u>	<u>0.34</u>	0.42	0.36	<u>0.0</u>	8.5	2.9
600 × 1000	<u>0.38</u>	<u>0.37</u>	<u>0.37</u>	0.48	<u>0.14</u>	<u>0.18</u>	<u>0.38</u>	<u>0.43</u>	0.51	0.46	<u>0.1</u>	15.4	4.9
800 × 1000	<u>0.79</u>	<u>0.65</u>	<u>0.75</u>	0.53	<u>0.23</u>	<u>0.31</u>	<u>0.69</u>	<u>0.77</u>	0.71	0.64	<u>0.1</u>	32.6	5.2
1000 × 1000	<u>1.06</u>	<u>1.01</u>	<u>1.04</u>	0.52	<u>0.23</u>	<u>0.33</u>	<u>0.58</u>	<u>0.52</u>	0.73	0.80	<u>0.1</u>	42.5	6.4
500 × 5000	<u>0.29</u>	<u>0.18</u>	<u>0.21</u>	1.81	<u>0.01</u>	<u>0.01</u>	<u>0.16</u>	<u>0.17</u>	0.15	0.18	<u>0.4</u>	91.4	18.1
1000 × 5000	<u>0.22</u>	<u>0.21</u>	<u>0.22</u>	1.18	<u>0.05</u>	<u>0.07</u>	<u>0.21</u>	<u>0.20</u>	0.23	0.22	<u>0.9</u>	548.6	35.1
2000 × 5000	<u>0.65</u>	<u>0.54</u>	<u>0.65</u>	0.77	<u>0.20</u>	<u>0.20</u>	<u>0.43</u>	<u>0.42</u>	0.44	0.38	<u>2.4</u>	1825.1	63.7
3000 × 5000	<u>0.42</u>	<u>0.41</u>	<u>0.42</u>	0.73	<u>0.34</u>	<u>0.40</u>	<u>0.57</u>	<u>0.61</u>	0.59	0.65	<u>3.8</u>	2199.0	104.1
4000 × 5000	<u>0.68</u>	<u>0.68</u>	<u>0.68</u>	0.66	<u>0.50</u>	<u>0.52</u>	<u>0.71</u>	<u>0.69</u>	0.68	0.73	<u>4.7</u>	1015.1	136.4
5000 × 5000	<u>0.77</u>	<u>0.54</u>	<u>0.77</u>	0.67	<u>0.47</u>	<u>0.54</u>	<u>0.76</u>	<u>0.75</u>	0.72	0.72	<u>6.6</u>	26134.9	165.7
Average	<u>0.48</u>	<u>0.40</u>	<u>0.45</u>	0.84	<u>0.18</u>	<u>0.22</u>	<u>0.42</u>	<u>0.42</u>	0.45	0.46	<u>1.6</u>	2659.5	45.4

Table 18: Max Induced Subgraph instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$	M(A)	M(F)	M( $V_1^{\text{ex}}$ )	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$
100 × 1000	4.21	4.06	4.18	9.86	<u>0.15</u>	0.18	3.40	2.70	2.84	3.26	<u>0.0</u>	0.4	0.8
200 × 1000	2.53	2.53	2.53	8.48	<u>0.88</u>	0.96	2.23	2.31	2.28	3.09	<u>0.0</u>	0.6	1.3
400 × 1000	7.47	3.62	7.12	6.98	<u>2.67</u>	2.89	5.17	5.50	5.55	5.41	<u>0.0</u>	16.3	2.7
600 × 1000	4.20	3.20	4.12	6.00	<u>3.16</u>	3.35	4.16	4.09	4.28	3.97	<u>0.1</u>	42.7	4.4
800 × 1000	6.93	6.72	6.88	5.52	<u>3.90</u>	3.91	7.89	8.00	7.80	7.26	<u>0.1</u>	31.5	7.3
1000 × 1000	6.49	6.40	6.46	5.36	<u>4.35</u>	4.45	6.69	6.13	6.90	6.97	<u>0.2</u>	43.9	7.9
500 × 5000	2.67	2.63	2.65	12.70	<u>1.04</u>	1.26	2.62	3.03	2.83	2.96	<u>0.7</u>	50.3	17.8
1000 × 5000	2.72	2.69	2.72	9.74	<u>1.40</u>	1.51	3.08	3.04	2.96	3.09	<u>1.4</u>	290.7	35.9
2000 × 5000	2.76	2.75	2.76	6.85	<u>2.35</u>	2.65	3.26	3.07	3.28	3.22	<u>3.2</u>	671.9	68.5
3000 × 5000	5.42	5.03	5.42	6.20	<u>3.63</u>	3.92	4.72	4.74	4.70	5.09	<u>7.0</u>	2707.7	99.1
4000 × 5000	6.60	6.48	6.60	5.99	<u>4.62</u>	4.90	5.63	5.89	5.49	5.76	<u>5.0</u>	4180.5	125.0
5000 × 5000	6.04	6.04	6.04	5.55	<u>4.53</u>	4.82	5.97	6.08	6.13	6.33	<u>14.6</u>	4784.4	191.8
Average	<u>4.84</u>	<u>4.35</u>	<u>4.79</u>	7.43	<u>2.72</u>	<u>2.90</u>	<u>4.57</u>	<u>4.55</u>	4.59	4.70	<u>2.7</u>	1068.4	46.9

Table 19: MaxCut instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_8$
100 × 1000	1.78	0.12	0.10	1.32	0.97	1.04	<u>0.06</u>	0.07	0.07	0.15	<u>0.0</u>	0.6	0.9
200 × 1000	2.80	0.32	0.30	0.92	0.54	0.81	0.23	<u>0.20</u>	0.21	0.29	<u>0.0</u>	1.3	2.2
400 × 1000	1.41	<u>0.25</u>	0.28	0.74	0.31	0.53	0.42	0.42	0.48	0.65	<u>0.0</u>	4.7	3.5
600 × 1000	0.85	0.55	0.63	0.52	<u>0.23</u>	0.31	0.41	0.37	0.50	0.60	<u>0.0</u>	19.2	5.2
800 × 1000	1.11	0.91	0.81	1.04	<u>0.68</u>	0.75	0.95	1.07	1.19	1.15	<u>0.4</u>	18.2	6.9
1000 × 1000	1.77	1.47	1.07	0.58	<u>0.30</u>	0.41	0.88	0.76	0.85	0.92	<u>0.1</u>	27.4	9.5
500 × 5000	2.95	0.17	0.11	1.84	1.50	1.55	0.09	<u>0.09</u>	0.10	0.12	<u>0.3</u>	78.4	38.5
1000 × 5000	2.60	<u>0.15</u>	0.28	1.29	0.84	0.98	0.26	0.22	0.23	0.30	<u>0.4</u>	421.3	103.5
2000 × 5000	1.56	<u>0.22</u>	0.39	0.88	0.55	0.65	0.39	0.38	0.43	0.43	<u>1.1</u>	1937.3	219.1
3000 × 5000	0.93	0.64	0.67	0.74	<u>0.52</u>	0.55	0.54	0.58	0.57	0.66	<u>3.0</u>	2296.5	208.9
4000 × 5000	0.86	<u>0.54</u>	0.63	0.69	0.56	0.55	0.64	0.66	0.74	0.73	<u>4.7</u>	5824.3	151.8
5000 × 5000	1.01	0.72	0.91	0.71	<u>0.52</u>	0.61	0.87	0.79	0.81	0.81	<u>5.6</u>	7880.7	216.1
Average	1.64	0.51	0.51	0.94	0.63	0.73	0.48	<u>0.47</u>	0.52	0.57	<u>1.3</u>	1542.5	80.5

Table 20: Matrix Factorization instances.

## 2.4 Comparison with $V_{10}$

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$
$100 \times 1000$	0.18	0.04	0.04	0.34	<u>0.00</u>	<u>0.00</u>	0.03	0.03	0.03	0.03	<u>0.0</u>	0.4	3.6
$200 \times 1000$	0.15	0.10	0.11	0.56	<u>0.00</u>	<u>0.00</u>	0.11	0.10	0.11	0.12	<u>0.0</u>	1.5	4.6
$400 \times 1000$	0.52	0.52	0.52	0.37	0.02	<u>0.01</u>	0.36	0.24	0.33	0.37	<u>0.1</u>	2.5	8.1
$600 \times 1000$	0.79	0.57	0.70	0.18	<u>0.04</u>	0.05	0.58	0.58	0.63	0.49	<u>0.1</u>	18.5	15.6
$800 \times 1000$	0.77	0.62	0.67	0.28	<u>0.10</u>	0.17	0.51	0.48	0.53	0.51	<u>0.1</u>	37.8	17.9
$1000 \times 1000$	1.15	0.98	1.10	0.33	0.24	<u>0.22</u>	0.49	0.49	0.52	0.57	<u>0.1</u>	54.3	18.6
$500 \times 5000$	0.05	0.02	0.02	1.58	0.00	<u>0.00</u>	0.11	0.11	0.09	0.09	<u>0.5</u>	66.0	74.6
$1000 \times 5000$	0.21	0.19	0.19	0.96	<u>0.05</u>	0.08	0.26	0.29	0.25	0.26	<u>0.9</u>	257.2	134.7
$2000 \times 5000$	0.49	0.49	0.49	0.69	<u>0.22</u>	0.24	0.40	0.42	0.40	0.43	<u>2.9</u>	1 243.5	261.1
$3000 \times 5000$	0.55	0.55	0.55	0.59	<u>0.29</u>	0.33	0.49	0.47	0.51	0.44	<u>4.2</u>	686.1	323.9
$4000 \times 5000$	0.44	0.40	0.44	0.45	<u>0.31</u>	0.35	0.54	0.55	0.56	0.54	<u>7.5</u>	5 490.2	442.0
$5000 \times 5000$	0.97	0.62	0.97	0.54	<u>0.45</u>	0.47	0.71	0.78	0.74	0.77	<u>1.8</u>	29 088.8	590.3
Average	0.52	0.43	0.48	0.57	<u>0.14</u>	0.16	0.38	0.38	0.39	0.38	<u>1.5</u>	3 078.9	157.9

Table 21: Random instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$
$100 \times 1000$	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	84.28	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.0</u>	0.2	2.0
$200 \times 1000$	2.21	<u>0.00</u>	0.23	94.76	2.74	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.07	<u>0.1</u>	9.4	5.7
$400 \times 1000$	7.51	7.41	7.47	94.91	66.19	<u>3.21</u>	7.42	7.39	7.54	7.42	<u>0.0</u>	6.6	9.1
$600 \times 1000$	8.33	<u>1.14</u>	3.69	91.52	83.36	2.38	3.10	2.71	3.23	3.70	<u>0.1</u>	24.2	19.6
$800 \times 1000$	13.45	<u>0.49</u>	6.38	93.95	82.73	5.92	3.26	2.31	3.51	5.47	<u>0.1</u>	51.2	44.9
$1000 \times 1000$	30.35	11.11	18.84	94.42	75.11	<u>10.88</u>	16.21	14.55	18.47	20.28	<u>0.1</u>	85.7	65.7
$500 \times 5000$	0.03	0.03	0.03	97.63	4.72	<u>0.00</u>	0.03	0.03	0.03	0.03	<u>0.2</u>	16.8	58.7
$1000 \times 5000$	5.14	0.20	0.56	98.43	1.09	0.34	<u>0.11</u>	0.29	0.14	0.34	<u>0.6</u>	219.5	152.1
$2000 \times 5000$	13.12	<u>6.23</u>	10.94	98.17	92.38	11.86	9.79	9.54	9.88	11.15	<u>1.7</u>	1 684.6	566.5
$3000 \times 5000$	28.11	<u>4.74</u>	7.91	97.79	93.90	8.13	5.47	5.13	5.95	14.17	<u>1.7</u>	5 205.6	2 917.9
$4000 \times 5000$	31.87	12.70	12.49	98.79	97.96	<u>8.92</u>	11.61	11.60	11.97	16.08	<u>1.4</u>	5 866.5	1 825.5
$5000 \times 5000$	44.62	21.47	21.49	98.82	97.27	<u>15.45</u>	21.09	20.94	21.25	24.21	<u>3.2</u>	9 012.8	2 508.0
Average	15.39	<u>5.46</u>	7.50	95.29	58.12	5.59	6.51	6.21	6.83	8.58	<u>0.8</u>	1 848.6	681.3

Table 22: Biclique instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	1.56	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.01	<u>0.00</u>	<u>0.0</u>	0.2	2.9
200 × 1000	<u>0.23</u>	<u>0.18</u>	<u>0.18</u>	0.41	<u>0.00</u>	<u>0.00</u>	<u>0.21</u>	<u>0.20</u>	0.19	<u>0.19</u>	<u>0.0</u>	1.2	4.8
400 × 1000	<u>0.27</u>	<u>0.06</u>	<u>0.11</u>	0.31	<u>0.01</u>	<u>0.01</u>	<u>0.34</u>	<u>0.33</u>	0.40	<u>0.32</u>	<u>0.0</u>	8.5	9.4
600 × 1000	<u>0.38</u>	<u>0.37</u>	<u>0.37</u>	0.44	<u>0.12</u>	<u>0.13</u>	<u>0.36</u>	<u>0.41</u>	0.49	<u>0.44</u>	<u>0.1</u>	15.4	14.6
800 × 1000	<u>0.79</u>	<u>0.65</u>	<u>0.69</u>	0.40	<u>0.19</u>	<u>0.26</u>	<u>0.67</u>	<u>0.70</u>	0.66	<u>0.63</u>	<u>0.1</u>	32.6	21.1
1000 × 1000	<u>1.06</u>	<u>1.01</u>	<u>1.02</u>	0.42	<u>0.19</u>	<u>0.25</u>	<u>0.58</u>	<u>0.50</u>	0.68	<u>0.70</u>	<u>0.1</u>	42.5	22.1
500 × 5000	<u>0.29</u>	<u>0.18</u>	<u>0.19</u>	1.59	<u>0.00</u>	<u>0.00</u>	<u>0.15</u>	<u>0.15</u>	0.15	<u>0.15</u>	<u>0.4</u>	91.4	71.4
1000 × 5000	<u>0.22</u>	<u>0.21</u>	<u>0.22</u>	1.09	<u>0.04</u>	<u>0.05</u>	<u>0.20</u>	<u>0.19</u>	0.22	<u>0.20</u>	<u>0.9</u>	548.6	107.1
2000 × 5000	<u>0.65</u>	<u>0.54</u>	<u>0.64</u>	0.69	<u>0.16</u>	<u>0.17</u>	<u>0.42</u>	<u>0.41</u>	0.43	<u>0.37</u>	<u>2.4</u>	1825.1	290.4
3000 × 5000	<u>0.42</u>	<u>0.41</u>	<u>0.42</u>	0.68	<u>0.30</u>	<u>0.35</u>	<u>0.57</u>	<u>0.60</u>	0.58	<u>0.62</u>	<u>3.8</u>	2199.0	373.0
4000 × 5000	<u>0.68</u>	<u>0.68</u>	<u>0.68</u>	0.64	<u>0.46</u>	<u>0.46</u>	<u>0.71</u>	<u>0.69</u>	0.68	<u>0.71</u>	<u>4.7</u>	1015.1	489.2
5000 × 5000	<u>0.77</u>	<u>0.54</u>	<u>0.75</u>	0.60	<u>0.42</u>	<u>0.47</u>	<u>0.75</u>	<u>0.75</u>	0.71	<u>0.74</u>	<u>6.6</u>	26134.9	780.3
Average	<u>0.48</u>	<u>0.40</u>	<u>0.44</u>	0.74	<u>0.16</u>	0.18	0.41	0.41	0.43	0.42	<u>1.6</u>	2659.5	182.2

Table 23: Max Induced Subgraph instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$
100 × 1000	4.21	4.06	4.16	9.34	<u>0.01</u>	0.06	3.27	2.52	2.68	2.86	<u>0.0</u>	0.4	2.8
200 × 1000	2.53	2.53	2.53	7.62	<u>0.74</u>	0.83	2.14	2.24	2.22	2.73	<u>0.0</u>	0.6	4.9
400 × 1000	7.47	3.62	7.07	6.49	<u>2.12</u>	2.21	5.12	5.41	5.42	5.25	<u>0.0</u>	16.3	10.8
600 × 1000	4.20	3.20	4.01	5.45	<u>2.89</u>	3.11	4.10	4.06	4.24	3.90	<u>0.1</u>	42.7	15.8
800 × 1000	6.93	6.72	6.87	5.11	<u>3.42</u>	3.61	7.76	7.93	7.77	7.00	<u>0.1</u>	31.5	19.5
1000 × 1000	6.49	6.40	6.43	5.06	<u>3.85</u>	4.25	6.60	6.10	6.58	6.90	<u>0.2</u>	43.9	21.8
500 × 5000	2.67	2.63	2.64	12.20	<u>0.80</u>	0.82	2.46	2.99	2.80	2.77	<u>0.7</u>	50.3	70.3
1000 × 5000	2.72	2.69	2.72	9.41	1.26	<u>1.08</u>	2.98	3.02	2.90	3.11	<u>1.4</u>	290.7	119.6
2000 × 5000	2.76	2.75	2.76	6.47	<u>2.03</u>	2.27	3.18	3.00	3.25	3.09	<u>3.2</u>	671.9	318.5
3000 × 5000	5.42	5.03	5.35	5.81	<u>3.41</u>	3.63	4.69	4.69	4.65	4.74	<u>7.0</u>	2707.7	413.5
4000 × 5000	6.60	6.48	6.56	5.57	<u>4.25</u>	4.50	5.51	5.80	5.39	5.76	<u>5.0</u>	4180.5	629.5
5000 × 5000	6.04	6.04	6.04	5.36	<u>4.21</u>	4.43	5.96	6.07	6.08	6.38	<u>14.6</u>	4784.4	715.9
Average	4.84	4.35	4.76	6.99	<u>2.42</u>	2.57	4.48	4.49	4.50	4.54	<u>2.7</u>	1068.4	195.2

Table 24: MaxCut instances.

Instance	Gap to the best known, %										Time, sec		
	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$	M(A)	M(F)	$M(V_1^{\text{ex}})$	$P_3$	$P_4$	$P_6$	$P_8$	$V_1^{\text{ex}}$	$V_2^{\text{ex}}$	$V_{10}$
100 × 1000	1.78	0.12	0.08	1.09	0.82	0.89	<u>0.06</u>	0.07	0.07	0.14	<u>0.0</u>	0.6	3.1
200 × 1000	2.80	0.32	0.21	0.72	0.49	0.62	0.22	<u>0.18</u>	0.20	0.23	<u>0.0</u>	1.3	7.7
400 × 1000	1.41	<u>0.25</u>	0.26	0.63	0.27	0.37	0.35	0.33	0.42	0.56	<u>0.0</u>	4.7	11.0
600 × 1000	0.85	0.55	0.62	0.44	<u>0.17</u>	0.26	0.36	0.36	0.43	0.38	<u>0.0</u>	19.2	14.0
800 × 1000	1.11	0.91	0.75	0.83	<u>0.53</u>	0.62	0.92	0.96	1.08	0.97	<u>0.4</u>	18.2	28.3
1000 × 1000	1.77	1.47	1.02	0.49	<u>0.27</u>	0.32	0.83	0.75	0.82	0.88	<u>0.1</u>	27.4	30.6
500 × 5000	2.95	0.17	0.08	1.79	1.40	1.51	0.08	<u>0.07</u>	0.08	0.07	<u>0.3</u>	78.4	119.3
1000 × 5000	2.60	<u>0.15</u>	0.21	1.21	0.74	0.91	0.24	0.19	0.20	0.24	<u>0.4</u>	421.3	282.2
2000 × 5000	1.56	<u>0.22</u>	0.38	0.87	0.51	0.60	0.37	0.34	0.39	0.35	<u>1.1</u>	1937.3	567.9
3000 × 5000	0.93	0.64	0.65	0.68	<u>0.46</u>	0.50	0.51	0.55	0.54	0.58	<u>3.0</u>	2296.5	619.9
4000 × 5000	0.86	0.54	0.62	0.62	<u>0.49</u>	0.52	0.63	0.62	0.68	0.67	<u>4.7</u>	5824.3	740.1
5000 × 5000	1.01	0.72	0.91	0.66	<u>0.49</u>	0.55	0.84	0.77	0.80	0.75	<u>5.6</u>	7880.7	646.1
Average	1.64	0.51	0.48	0.84	0.55	0.64	0.45	<u>0.43</u>	0.48	0.49	<u>1.3</u>	1542.5	255.8

Table 25: Matrix Factorization instances.

### 3 Row-Merge Heuristics

The running time of  $M(F)$ ,  $M(V_1^{\text{ex}})$ ,  $R_{\lfloor m/2 \rfloor}^m$ ,  $R_{\lfloor m/3 \rfloor}^m$  and  $R_{\lfloor m/2 \rfloor}^{\text{ls}}$  is fixed to that of  $V_k$ . The algorithm  $M_{100}$  denotes  $M(V_1^{\text{ex}})$  terminated after 100 iterations.

#### 3.1 Comparison with $V_4$

$m \times n$	Gap to the best known, %									Time, sec			
	$V_4$	$M_{100}$	$M(F)$	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_4$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	0.14	<u>0.00</u>	0.00	0.00	<u>0.00</u>	<u>0.00</u>	0.00	0.00	0.07	<u>0.1</u>	0.5	0.5	13.9
200 × 1000	0.12	<u>0.00</u>	0.00	0.10	0.08	0.03	0.02	0.00	0.11	<u>0.2</u>	1.1	0.4	13.8
400 × 1000	0.52	<u>0.06</u>	0.18	0.12	0.15	0.14	0.15	0.13	0.33	<u>0.2</u>	2.2	0.6	13.4
600 × 1000	0.79	<u>0.10</u>	0.16	0.22	0.23	0.22	0.20	0.20	0.53	<u>0.4</u>	3.3	0.6	14.1
800 × 1000	0.77	<u>0.23</u>	0.31	0.37	0.28	0.26	0.26	0.25	0.36	<u>0.4</u>	4.0	0.9	14.7
1000 × 1000	1.15	<u>0.29</u>	0.52	0.49	0.35	0.31	0.54	0.52	0.46	<u>0.6</u>	5.8	1.0	15.9
500 × 5000	0.05	<u>0.01</u>	0.06	0.07	0.03	0.02	0.05	0.06	0.09	<u>1.9</u>	22.5	2.5	71.1
1000 × 5000	0.21	<u>0.09</u>	0.17	0.21	0.12	0.11	0.17	0.18	0.20	3.6	52.9	<u>3.6</u>	71.7
2000 × 5000	0.49	<u>0.25</u>	0.33	0.38	0.28	0.32	0.35	0.33	0.33	8.1	131.6	<u>7.3</u>	73.2
3000 × 5000	0.55	<u>0.35</u>	0.42	0.46	0.33	<u>0.31</u>	0.48	0.46	0.44	13.0	214.3	<u>12.3</u>	79.9
4000 × 5000	0.44	0.35	0.44	0.49	0.33	<u>0.30</u>	0.43	0.46	0.46	17.4	286.9	<u>17.2</u>	94.7
5000 × 5000	0.97	0.49	0.63	0.62	<u>0.33</u>	0.35	0.69	0.64	0.65	<u>13.7</u>	348.5	40.4	111.1
Average	0.52	<u>0.19</u>	0.27	0.30	0.21	0.20	0.28	0.27	0.34	<u>5.0</u>	89.5	7.3	49.0

Table 26: Random instances.

$m \times n$	Gap to the best known, %									Time, sec			
	$V_4$	$M_{100}$	$M(F)$	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_4$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.1</u>	0.5	0.4	10.8
200 × 1000	2.05	<u>0.00</u>	2.74	0.39	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.24	<u>0.6</u>	0.7	<u>0.4</u>	12.1
400 × 1000	7.51	<u>3.21</u>	77.12	3.21	<u>3.21</u>	<u>3.21</u>	0.68	<u>0.21</u>	7.04	<u>0.2</u>	2.0	0.5	12.5
600 × 1000	7.83	2.38	88.12	2.75	2.38	2.38	1.22	<u>1.21</u>	4.07	<u>0.4</u>	3.0	0.6	11.9
800 × 1000	12.62	6.01	86.53	6.01	2.93	2.93	1.90	<u>1.52</u>	7.07	<u>0.6</u>	3.9	0.7	12.2
1000 × 1000	28.13	10.88	80.42	10.88	7.79	7.79	<u>1.49</u>	2.25	22.80	<u>0.8</u>	4.2	1.0	12.2
500 × 5000	0.03	<u>0.00</u>	4.72	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.03	<u>1.2</u>	14.1	2.2	61.1
1000 × 5000	4.18	0.36	9.92	0.50	<u>0.31</u>	<u>0.31</u>	0.36	0.58	0.88	3.0	24.0	<u>2.6</u>	58.2
2000 × 5000	12.96	12.06	93.78	12.07	12.07	12.07	1.89	<u>0.93</u>	11.49	6.7	61.0	<u>5.7</u>	74.3
3000 × 5000	27.94	8.13	95.18	9.70	8.13	8.13	2.14	<u>1.13</u>	20.34	<u>9.5</u>	79.9	10.4	84.4
4000 × 5000	31.66	8.92	98.37	8.92	26.63	26.63	2.91	<u>1.39</u>	16.35	<u>11.5</u>	92.2	15.6	82.2
5000 × 5000	44.51	15.45	97.95	15.45	15.45	15.45	4.93	<u>3.80</u>	27.74	<u>14.4</u>	122.4	36.1	125.5
Average	14.95	5.62	61.24	5.82	6.58	6.58	1.46	<u>1.09</u>	9.84	<u>4.1</u>	34.0	6.3	46.4

Table 27: Biclique instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>4</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>4</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.09	<u>0.1</u>	0.6	0.4	12.0
200 × 1000	0.23	<u>0.00</u>	0.07	0.09	0.08	0.06	0.10	0.10	0.16	<u>0.1</u>	0.8	0.4	12.5
400 × 1000	0.20	<u>0.02</u>	0.08	0.11	0.12	0.11	0.10	0.12	0.26	<u>0.2</u>	2.0	0.5	12.5
600 × 1000	0.38	<u>0.20</u>	0.34	0.30	0.39	0.37	0.32	0.32	0.44	<u>0.3</u>	3.1	0.6	13.1
800 × 1000	0.78	<u>0.31</u>	0.41	0.54	0.51	0.48	0.46	0.43	0.64	<u>0.5</u>	4.4	0.8	14.2
1000 × 1000	1.06	<u>0.33</u>	0.42	0.43	0.36	0.45	0.46	0.46	0.59	<u>0.5</u>	6.4	1.2	15.5
500 × 5000	0.27	<u>0.01</u>	0.04	0.05	0.03	0.04	0.06	0.08	0.09	<u>1.9</u>	22.5	2.5	70.9
1000 × 5000	0.22	<u>0.06</u>	0.13	0.17	0.11	0.13	0.14	0.14	0.20	3.2	53.4	3.0	70.4
2000 × 5000	0.65	<u>0.20</u>	0.27	0.32	0.21	0.21	0.32	0.29	0.37	7.2	141.3	<u>6.1</u>	77.0
3000 × 5000	0.42	0.37	0.45	0.46	<u>0.32</u>	0.34	0.46	0.45	0.52	<u>10.6</u>	223.3	12.1	76.5
4000 × 5000	0.68	0.47	0.61	0.66	<u>0.42</u>	0.44	0.60	0.56	0.65	<u>14.1</u>	318.5	18.3	91.7
5000 × 5000	0.77	0.50	0.56	0.68	<u>0.43</u>	0.43	0.62	0.58	0.64	<u>18.1</u>	378.9	33.9	98.5
Average	0.47	<u>0.21</u>	0.28	0.32	0.25	0.26	0.30	0.29	0.39	<u>4.7</u>	96.3	6.7	47.1

Table 28: Max Induced Subgraph instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>4</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>4</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	4.19	<u>0.19</u>	0.98	1.67	1.02	1.09	1.12	1.30	1.17	<u>0.1</u>	0.7	0.4	13.4
200 × 1000	2.53	<u>0.94</u>	1.75	2.46	1.96	1.82	1.79	2.03	1.40	<u>0.1</u>	1.3	0.4	14.0
400 × 1000	7.46	<u>2.82</u>	4.02	4.21	3.47	3.09	4.01	3.81	3.69	<u>0.2</u>	2.9	0.6	14.7
600 × 1000	4.20	<u>3.31</u>	4.27	4.79	3.48	3.42	4.33	4.64	3.62	<u>0.4</u>	4.9	0.8	15.2
800 × 1000	6.93	3.91	5.08	5.32	<u>3.53</u>	4.00	5.15	4.96	5.12	<u>0.5</u>	6.7	1.0	16.1
1000 × 1000	6.49	4.41	4.38	4.47	4.53	4.38	3.99	4.12	<u>3.81</u>	7.2	8.1	<u>1.3</u>	16.8
500 × 5000	2.66	<u>0.98</u>	1.65	1.93	1.46	1.51	1.56	1.65	1.56	<u>1.9</u>	30.5	2.8	70.3
1000 × 5000	2.72	<u>1.16</u>	1.99	2.09	1.59	1.58	2.27	2.09	1.70	<u>3.9</u>	78.3	4.2	73.9
2000 × 5000	2.76	<u>2.41</u>	3.26	3.39	2.74	2.83	3.03	3.02	2.88	<u>8.2</u>	212.3	14.0	81.1
3000 × 5000	5.42	<u>3.63</u>	4.33	4.54	3.92	4.25	4.02	3.99	3.90	<u>14.2</u>	378.9	20.3	99.5
4000 × 5000	6.60	<u>4.52</u>	5.23	5.21	5.09	5.23	4.94	4.97	5.06	<u>14.9</u>	520.3	30.4	123.3
5000 × 5000	6.04	<u>4.43</u>	5.02	5.23	5.32	5.42	5.33	5.13	5.69	<u>25.8</u>	633.3	45.9	126.3
Average	4.83	<u>2.73</u>	3.50	3.78	3.18	3.22	3.46	3.48	3.30	<u>6.4</u>	156.5	10.2	55.4

Table 29: MaxCut instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>4</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>4</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	<u>0.22</u>	1.46	1.62	1.52	1.02	0.87	1.36	1.69	0.78	<u>0.2</u>	0.4	0.4	13.2
200 × 1000	0.60	0.96	0.88	1.10	1.12	0.96	0.96	0.92	<u>0.54</u>	<u>0.2</u>	0.9	0.5	13.4
400 × 1000	0.46	0.62	0.49	0.75	0.82	0.75	0.55	0.59	<u>0.39</u>	<u>0.3</u>	1.5	0.5	14.0
600 × 1000	0.71	0.34	0.42	0.56	0.54	0.54	0.42	0.51	<u>0.32</u>	<u>0.3</u>	2.4	0.7	14.2
800 × 1000	0.95	0.83	0.97	1.06	0.91	0.98	1.08	1.13	<u>0.83</u>	<u>0.4</u>	3.5	0.9	14.9
1000 × 1000	1.52	0.50	0.48	0.67	0.52	<u>0.41</u>	0.65	0.60	0.58	<u>0.5</u>	4.6	1.2	15.6
500 × 5000	<u>0.25</u>	1.68	1.60	1.95	1.07	1.02	1.49	1.54	0.92	5.9	11.1	<u>2.2</u>	65.9
1000 × 5000	0.54	1.14	0.97	1.18	0.92	0.87	0.94	0.92	<u>0.53</u>	16.9	34.9	<u>3.2</u>	74.6
2000 × 5000	0.84	0.68	0.62	0.77	0.65	0.63	0.68	0.72	<u>0.37</u>	19.6	90.8	<u>6.4</u>	73.4
3000 × 5000	0.78	0.55	0.61	0.75	0.48	0.50	0.59	0.60	<u>0.42</u>	19.2	146.9	<u>12.2</u>	81.1
4000 × 5000	0.66	0.55	0.65	0.69	0.47	<u>0.43</u>	0.59	0.60	0.58	24.8	202.0	<u>19.1</u>	97.7
5000 × 5000	0.96	0.60	0.65	0.75	0.47	<u>0.45</u>	0.70	0.59	0.61	<u>25.1</u>	253.6	31.4	101.4
Average	0.71	0.83	0.83	0.98	0.75	0.70	0.83	0.87	<u>0.57</u>	9.4	62.7	<u>6.6</u>	48.3

Table 30: Matrix Factorization instances.



### 3.2 Comparison with $V_6$

$m \times n$	Gap to the best known, %									Time, sec			
	$V_6$	$M_{100}$	M(F)	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_6$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	0.08	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.00	0.00	0.03	<u>0.2</u>	0.5	0.5	13.9
200 × 1000	0.11	<u>0.00</u>	<u>0.00</u>	<u>0.01</u>	0.08	0.03	0.00	0.00	0.10	0.5	1.1	<u>0.4</u>	13.8
400 × 1000	0.52	<u>0.06</u>	0.09	0.08	0.15	0.14	0.11	0.11	0.23	0.7	2.2	<u>0.6</u>	13.4
600 × 1000	0.74	<u>0.10</u>	0.11	0.13	0.23	0.22	0.11	0.17	0.39	1.2	3.3	<u>0.6</u>	14.1
800 × 1000	0.74	0.23	0.25	0.30	0.28	0.26	0.21	<u>0.20</u>	0.31	1.4	4.0	<u>0.9</u>	14.7
1000 × 1000	1.14	<u>0.29</u>	0.35	0.39	0.35	0.31	0.42	0.34	0.33	2.1	5.8	<u>1.0</u>	15.9
500 × 5000	0.04	<u>0.01</u>	0.05	0.03	0.03	0.02	0.04	0.05	0.07	4.7	22.5	<u>2.5</u>	71.1
1000 × 5000	0.20	<u>0.09</u>	0.15	0.16	0.12	0.11	0.14	0.13	0.18	10.3	52.9	<u>3.6</u>	71.7
2000 × 5000	0.49	<u>0.25</u>	0.30	0.33	0.28	0.32	0.30	0.30	0.31	20.4	131.6	<u>7.3</u>	73.2
3000 × 5000	0.55	0.35	0.36	0.42	0.33	<u>0.31</u>	0.43	0.39	0.40	34.5	214.3	<u>12.3</u>	79.9
4000 × 5000	0.44	0.35	0.39	0.45	0.33	<u>0.30</u>	0.39	0.39	0.43	48.5	286.9	<u>17.2</u>	94.7
5000 × 5000	0.97	0.49	0.57	0.57	<u>0.33</u>	0.35	0.61	0.56	0.63	42.5	348.5	<u>40.4</u>	111.1
Average	0.50	<u>0.19</u>	0.22	0.24	0.21	0.20	0.23	0.22	0.28	13.9	89.5	<u>7.3</u>	49.0

Table 31: Random instances.

$m \times n$	Gap to the best known, %									Time, sec			
	$V_6$	$M_{100}$	M(F)	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_6$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.2</u>	0.5	0.4	10.8
200 × 1000	1.49	<u>0.00</u>	2.74	0.13	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.50	<u>0.3</u>	0.7	0.4	12.1
400 × 1000	7.51	3.21	75.05	3.21	3.21	3.21	0.35	<u>0.03</u>	6.60	0.6	2.0	<u>0.5</u>	12.5
600 × 1000	5.61	2.38	85.64	2.38	2.38	2.38	0.92	<u>0.61</u>	2.48	1.7	3.0	<u>0.6</u>	11.9
800 × 1000	11.46	6.01	85.40	6.01	2.93	2.93	1.70	<u>1.20</u>	4.40	1.5	3.9	<u>0.7</u>	12.2
1000 × 1000	25.64	10.88	78.20	10.88	7.79	7.79	<u>1.10</u>	1.65	20.09	2.6	4.2	<u>1.0</u>	12.2
500 × 5000	0.03	<u>0.00</u>	4.72	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.03	3.6	14.1	<u>2.2</u>	61.1
1000 × 5000	2.33	0.36	1.09	0.40	0.31	0.31	<u>0.23</u>	0.46	0.28	11.6	24.0	<u>2.6</u>	58.2
2000 × 5000	12.71	12.06	93.27	12.06	12.07	12.07	1.14	<u>0.71</u>	10.19	20.9	61.0	<u>5.7</u>	74.3
3000 × 5000	27.36	8.13	94.81	8.13	8.13	8.13	1.58	<u>0.90</u>	10.91	43.3	79.9	<u>10.4</u>	84.4
4000 × 5000	27.09	8.92	98.02	8.92	26.63	26.63	1.85	<u>0.81</u>	6.51	147.3	92.2	<u>15.6</u>	82.2
5000 × 5000	43.14	15.45	97.64	15.45	15.45	15.45	4.34	<u>3.24</u>	10.08	121.2	122.4	<u>36.1</u>	125.5
Average	13.70	5.62	59.71	5.63	6.58	6.58	1.10	<u>0.80</u>	6.01	29.6	34.0	<u>6.3</u>	46.4

Table 32: Biclique instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>6</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>6</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.05	<u>0.2</u>	0.6	0.4	12.0
200 × 1000	0.21	<u>0.00</u>	0.03	0.01	0.08	0.06	0.07	0.07	0.12	<u>0.3</u>	0.8	0.4	12.5
400 × 1000	0.16	<u>0.02</u>	0.04	0.08	0.12	0.11	0.08	0.08	0.12	0.7	2.0	<u>0.5</u>	12.5
600 × 1000	0.38	<u>0.20</u>	0.27	0.29	0.39	0.37	0.25	0.27	0.39	0.9	3.1	<u>0.6</u>	13.1
800 × 1000	0.78	<u>0.31</u>	0.35	0.49	0.51	0.48	0.40	0.37	0.52	1.3	4.4	<u>0.8</u>	14.2
1000 × 1000	1.05	0.33	0.34	0.37	0.36	0.45	0.33	<u>0.31</u>	0.49	1.9	6.4	<u>1.2</u>	15.5
500 × 5000	0.23	<u>0.01</u>	0.02	0.04	0.03	0.04	0.05	<u>0.03</u>	0.08	5.0	22.5	<u>2.5</u>	70.9
1000 × 5000	0.22	<u>0.06</u>	0.08	0.12	0.11	0.13	0.13	0.10	0.17	9.5	53.4	<u>3.0</u>	70.4
2000 × 5000	0.65	<u>0.20</u>	0.23	0.26	0.21	0.21	0.27	0.27	0.31	20.4	141.3	<u>6.1</u>	77.0
3000 × 5000	0.42	0.37	0.36	0.44	<u>0.32</u>	0.34	0.43	0.39	0.47	33.0	223.3	<u>12.1</u>	76.5
4000 × 5000	0.68	0.47	0.54	0.58	<u>0.42</u>	0.44	0.54	0.53	0.61	37.7	318.5	<u>18.3</u>	91.7
5000 × 5000	0.77	0.50	0.54	0.62	<u>0.43</u>	0.43	0.59	0.54	0.59	47.8	378.9	<u>33.9</u>	98.5
Average	0.46	<u>0.21</u>	0.23	0.28	0.25	0.26	0.26	0.25	0.33	13.2	96.3	<u>6.7</u>	47.1

Table 33: Max Induced Subgraph instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>6</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>6</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	4.18	<u>0.19</u>	0.71	0.49	1.02	1.09	0.56	0.71	0.80	<u>0.2</u>	0.7	0.4	13.4
200 × 1000	2.53	<u>0.94</u>	1.55	1.94	1.96	1.82	1.24	1.51	0.97	<u>0.4</u>	1.3	0.4	14.0
400 × 1000	7.41	<u>2.82</u>	3.25	3.52	3.47	3.09	3.58	3.23	2.85	0.7	2.9	<u>0.6</u>	14.7
600 × 1000	4.19	<u>3.31</u>	3.52	4.24	3.48	3.42	4.00	4.22	3.34	1.2	4.9	<u>0.8</u>	15.2
800 × 1000	6.92	3.91	4.52	4.70	<u>3.53</u>	4.00	4.56	4.29	3.95	1.5	6.7	<u>1.0</u>	16.1
1000 × 1000	6.49	4.41	4.94	5.06	4.53	<u>4.38</u>	4.63	4.60	4.70	1.9	8.1	<u>1.3</u>	16.8
500 × 5000	2.65	<u>0.98</u>	1.38	1.53	1.46	1.51	1.44	1.43	1.14	5.3	30.5	<u>2.8</u>	70.3
1000 × 5000	2.72	<u>1.16</u>	1.67	1.84	1.59	1.58	1.83	1.79	1.35	10.0	78.3	<u>4.2</u>	73.9
2000 × 5000	2.76	<u>2.41</u>	2.70	2.98	2.74	2.83	2.81	2.56	2.63	18.2	212.3	<u>14.0</u>	81.1
3000 × 5000	5.42	<u>3.63</u>	4.13	4.31	3.92	4.25	3.92	3.88	3.79	33.9	378.9	<u>20.3</u>	99.5
4000 × 5000	6.60	<u>4.52</u>	4.91	5.05	5.09	5.23	4.72	4.79	4.96	36.6	520.3	<u>30.4</u>	123.3
5000 × 5000	6.04	<u>4.43</u>	4.92	5.03	5.32	5.42	5.02	4.87	5.60	56.1	633.3	<u>45.9</u>	126.3
Average	4.83	<u>2.73</u>	3.18	3.39	3.18	3.22	3.19	3.16	3.01	13.8	156.5	<u>10.2</u>	55.4

Table 34: MaxCut instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>6</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>6</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	<u>0.12</u>	1.46	1.33	1.51	1.02	0.87	1.28	1.56	0.48	<u>0.4</u>	0.4	0.4	13.2
200 × 1000	0.44	0.96	0.64	0.95	1.12	0.96	0.79	0.69	<u>0.39</u>	0.6	0.9	<u>0.5</u>	13.4
400 × 1000	0.32	0.62	0.40	0.63	0.82	0.75	0.44	0.48	<u>0.31</u>	1.1	1.5	<u>0.5</u>	14.0
600 × 1000	0.66	0.34	0.31	0.38	0.54	0.54	0.32	0.32	<u>0.18</u>	1.5	2.4	<u>0.7</u>	14.2
800 × 1000	0.88	0.83	0.83	0.87	0.91	0.98	0.88	0.80	<u>0.70</u>	1.9	3.5	<u>0.9</u>	14.9
1000 × 1000	1.16	0.50	<u>0.34</u>	0.53	0.52	0.41	0.42	0.42	0.45	2.7	4.6	<u>1.2</u>	15.6
500 × 5000	<u>0.14</u>	1.68	1.56	1.62	1.07	1.02	1.40	1.44	0.76	16.8	11.1	<u>2.2</u>	65.9
1000 × 5000	<u>0.31</u>	1.14	0.88	1.13	0.92	0.87	0.85	0.84	0.44	46.7	34.9	<u>3.2</u>	74.6
2000 × 5000	0.50	0.68	0.58	0.70	0.65	0.63	0.62	0.62	<u>0.30</u>	84.4	90.8	<u>6.4</u>	73.4
3000 × 5000	0.69	0.55	0.55	0.59	0.48	0.50	0.54	0.57	<u>0.38</u>	66.3	146.9	<u>12.2</u>	81.1
4000 × 5000	0.65	0.55	0.62	0.64	0.47	<u>0.43</u>	0.55	0.58	0.52	59.2	202.0	<u>19.1</u>	97.7
5000 × 5000	0.94	0.60	0.60	0.67	0.47	<u>0.45</u>	0.65	0.57	0.58	56.1	253.6	<u>31.4</u>	101.4
Average	0.57	0.83	0.72	0.85	0.75	0.70	0.73	0.74	<u>0.46</u>	28.1	62.7	<u>6.6</u>	48.3

Table 35: Matrix Factorization instances.

### 3.3 Comparison with $V_8$

$m \times n$	Gap to the best known, %									Time, sec			
	$V_8$	$M_{100}$	M(F)	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_8$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	0.04	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.02</u>	0.8	0.5	<u>0.5</u>	13.9
200 × 1000	0.11	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.08	0.03	<u>0.00</u>	<u>0.00</u>	0.08	1.3	1.1	<u>0.4</u>	13.8
400 × 1000	0.52	0.06	<u>0.05</u>	0.06	0.15	0.14	0.08	0.07	0.12	2.6	2.2	<u>0.6</u>	13.4
600 × 1000	0.74	0.10	0.07	0.10	0.23	0.22	<u>0.07</u>	0.10	0.21	3.5	3.3	<u>0.6</u>	14.1
800 × 1000	0.72	0.23	0.12	0.23	0.28	0.26	0.15	<u>0.10</u>	0.25	5.5	4.0	<u>0.9</u>	14.7
1000 × 1000	1.13	0.29	0.28	<u>0.27</u>	0.35	0.31	0.37	0.33	0.32	5.5	5.8	<u>1.0</u>	15.9
500 × 5000	0.03	0.01	<u>0.01</u>	0.01	0.03	0.02	0.02	0.02	0.06	18.8	22.5	<u>2.5</u>	71.1
1000 × 5000	0.19	0.09	<u>0.09</u>	0.11	0.12	0.11	0.09	0.11	0.16	40.4	52.9	<u>3.6</u>	71.7
2000 × 5000	0.49	0.25	<u>0.24</u>	0.26	0.28	0.32	0.25	0.26	0.27	69.1	131.6	<u>7.3</u>	73.2
3000 × 5000	0.55	0.35	<u>0.31</u>	0.39	0.33	0.31	0.37	0.38	0.35	92.6	214.3	<u>12.3</u>	79.9
4000 × 5000	0.44	0.35	0.35	0.38	0.33	<u>0.30</u>	0.33	0.34	0.41	140.5	286.9	<u>17.2</u>	94.7
5000 × 5000	0.97	0.49	0.51	0.51	<u>0.33</u>	0.35	0.51	0.49	0.57	154.0	348.5	<u>40.4</u>	111.1
Average	0.50	0.19	<u>0.17</u>	0.19	0.21	0.20	0.19	0.18	0.24	44.6	89.5	<u>7.3</u>	49.0

Table 36: Random instances.

$m \times n$	Gap to the best known, %									Time, sec			
	$V_8$	$M_{100}$	M(F)	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_8$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.6	0.5	<u>0.4</u>	10.8
200 × 1000	0.77	<u>0.00</u>	2.74	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.04	1.7	0.7	<u>0.4</u>	12.1
400 × 1000	7.49	3.21	72.20	3.21	3.21	3.21	0.06	<u>0.00</u>	5.91	2.4	2.0	<u>0.5</u>	12.5
600 × 1000	4.22	2.38	84.27	2.38	2.38	2.38	0.68	<u>0.38</u>	1.73	7.6	3.0	<u>0.6</u>	11.9
800 × 1000	9.33	6.01	84.31	6.01	2.93	2.93	0.97	<u>0.62</u>	2.03	9.5	3.9	<u>0.7</u>	12.2
1000 × 1000	22.62	10.88	76.31	10.88	7.79	7.79	<u>0.61</u>	1.15	16.37	14.0	4.2	<u>1.0</u>	12.2
500 × 5000	0.03	<u>0.00</u>	4.72	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.02	13.3	14.1	<u>2.2</u>	61.1
1000 × 5000	0.71	0.36	1.09	0.34	0.31	0.31	0.13	0.33	<u>0.05</u>	60.0	24.0	<u>2.6</u>	58.2
2000 × 5000	11.88	12.06	92.70	12.02	12.07	12.07	0.63	<u>0.31</u>	6.53	129.9	61.0	<u>5.7</u>	74.3
3000 × 5000	25.23	8.13	94.36	8.13	8.13	8.13	1.18	<u>0.57</u>	3.22	273.7	79.9	<u>10.4</u>	84.4
4000 × 5000	13.03	8.92	97.99	8.92	26.63	26.63	1.48	<u>0.41</u>	2.01	1066.6	92.2	<u>15.6</u>	82.2
5000 × 5000	24.30	15.45	97.33	15.45	15.45	15.45	3.36	2.64	<u>1.80</u>	1466.1	122.4	<u>36.1</u>	125.5
Average	9.97	5.62	59.00	5.61	6.58	6.58	0.76	<u>0.53</u>	3.31	253.8	34.0	<u>6.3</u>	46.4

Table 37: Biclique instances.

$m \times n$	Gap to the best known, %									Time, sec			
	$V_8$	$M_{100}$	M(F)	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_8$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.04	0.6	0.6	<u>0.4</u>	12.0
200 × 1000	0.18	<u>0.00</u>	0.00	<u>0.00</u>	0.08	0.06	0.02	0.03	0.05	1.5	0.8	<u>0.4</u>	12.5
400 × 1000	0.11	0.02	0.02	<u>0.02</u>	0.12	0.11	0.04	0.03	0.05	2.9	2.0	<u>0.5</u>	12.5
600 × 1000	0.37	0.20	<u>0.14</u>	0.18	0.39	0.37	0.17	0.15	0.33	4.9	3.1	<u>0.6</u>	13.1
800 × 1000	0.75	0.31	<u>0.23</u>	0.31	0.51	0.48	0.30	0.26	0.46	5.2	4.4	<u>0.8</u>	14.2
1000 × 1000	1.04	0.33	<u>0.23</u>	0.33	0.36	0.45	0.25	0.24	0.45	6.4	6.4	<u>1.2</u>	15.5
500 × 5000	0.21	0.01	0.01	0.01	0.03	0.04	<u>0.00</u>	0.01	0.05	18.1	22.5	<u>2.5</u>	70.9
1000 × 5000	0.22	0.06	<u>0.05</u>	0.07	0.11	0.13	0.07	0.06	0.11	35.1	53.4	<u>3.0</u>	70.4
2000 × 5000	0.65	<u>0.20</u>	0.20	0.20	0.21	0.21	0.21	0.20	0.23	63.7	141.3	<u>6.1</u>	77.0
3000 × 5000	0.42	0.37	0.34	0.40	<u>0.32</u>	0.34	0.36	0.37	0.43	104.1	223.3	<u>12.1</u>	76.5
4000 × 5000	0.68	0.47	0.50	0.52	<u>0.42</u>	0.44	0.51	0.50	0.55	136.4	318.5	<u>18.3</u>	91.7
5000 × 5000	0.77	0.50	0.47	0.54	<u>0.43</u>	0.43	0.51	0.49	0.53	165.7	378.9	<u>33.9</u>	98.5
Average	0.45	0.21	<u>0.18</u>	0.22	0.25	0.26	0.20	0.20	0.27	45.4	96.3	<u>6.7</u>	47.1

Table 38: Max Induced Subgraph instances.

$m \times n$	Gap to the best known, %									Time, sec			
	$V_8$	$M_{100}$	M(F)	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_8$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	4.18	0.19	<u>0.15</u>	0.18	1.02	1.09	0.22	0.16	0.36	0.8	0.7	<u>0.4</u>	13.4
200 × 1000	2.53	0.94	0.88	0.96	1.96	1.82	1.16	1.09	<u>0.45</u>	1.3	1.3	<u>0.4</u>	14.0
400 × 1000	7.12	2.82	2.67	2.89	3.47	3.09	2.65	2.59	<u>2.31</u>	2.7	2.9	<u>0.6</u>	14.7
600 × 1000	4.12	3.31	3.16	3.35	3.48	3.42	3.38	3.61	<u>2.79</u>	4.4	4.9	<u>0.8</u>	15.2
800 × 1000	6.88	3.91	3.90	3.91	3.53	4.00	3.98	3.51	<u>3.01</u>	7.3	6.7	<u>1.0</u>	16.1
1000 × 1000	6.46	4.41	4.35	4.45	4.53	4.38	3.86	4.12	<u>3.63</u>	7.9	8.1	<u>1.3</u>	16.8
500 × 5000	2.65	0.98	1.04	1.26	1.46	1.51	1.18	1.21	<u>0.76</u>	17.8	30.5	<u>2.8</u>	70.3
1000 × 5000	2.72	1.16	1.40	1.51	1.59	1.58	1.29	1.49	<u>0.90</u>	35.9	78.3	<u>4.2</u>	73.9
2000 × 5000	2.76	2.41	2.35	2.65	2.74	2.83	2.30	2.35	<u>2.01</u>	68.5	212.3	<u>14.0</u>	81.1
3000 × 5000	5.42	3.63	3.63	3.92	3.92	4.25	3.71	3.66	<u>3.54</u>	99.1	378.9	<u>20.3</u>	99.5
4000 × 5000	6.60	4.52	4.62	4.90	5.09	5.23	<u>4.47</u>	4.47	4.68	125.0	520.3	<u>30.4</u>	123.3
5000 × 5000	6.04	<u>4.43</u>	4.53	4.82	5.32	5.42	4.59	4.50	5.26	191.8	633.3	<u>45.9</u>	126.3
Average	4.79	2.73	2.72	2.90	3.18	3.22	2.73	2.73	<u>2.47</u>	46.9	156.5	<u>10.2</u>	55.4

Table 39: MaxCut instances.

$m \times n$	Gap to the best known, %									Time, sec			
	$V_8$	$M_{100}$	M(F)	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_8$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	<u>0.10</u>	1.46	0.97	1.04	1.02	0.87	1.17	1.34	0.27	0.9	<u>0.4</u>	0.4	13.2
200 × 1000	0.30	0.96	0.54	0.81	1.12	0.96	0.58	0.49	<u>0.13</u>	2.2	0.9	<u>0.5</u>	13.4
400 × 1000	0.28	0.62	0.31	0.53	0.82	0.75	0.31	0.36	<u>0.24</u>	3.5	1.5	<u>0.5</u>	14.0
600 × 1000	0.63	0.34	0.23	0.31	0.54	0.54	0.24	0.23	<u>0.16</u>	5.2	2.4	<u>0.7</u>	14.2
800 × 1000	0.81	0.83	0.68	0.75	0.91	0.98	0.73	0.70	<u>0.62</u>	6.9	3.5	<u>0.9</u>	14.9
1000 × 1000	1.07	0.50	0.30	0.41	0.52	0.41	<u>0.26</u>	0.31	0.41	9.5	4.6	<u>1.2</u>	15.6
500 × 5000	<u>0.11</u>	1.68	1.50	1.55	1.07	1.02	1.27	1.36	0.68	38.5	11.1	<u>2.2</u>	65.9
1000 × 5000	<u>0.28</u>	1.14	0.84	0.98	0.92	0.87	0.79	0.79	0.31	103.5	34.9	<u>3.2</u>	74.6
2000 × 5000	0.39	0.68	0.55	0.65	0.65	0.63	0.54	0.55	<u>0.28</u>	219.1	90.8	<u>6.4</u>	73.4
3000 × 5000	0.67	0.55	0.52	0.55	0.48	0.50	0.51	0.47	<u>0.34</u>	208.9	146.9	<u>12.2</u>	81.1
4000 × 5000	0.63	0.55	0.56	0.55	0.47	<u>0.43</u>	0.52	0.52	0.49	151.8	202.0	<u>19.1</u>	97.7
5000 × 5000	0.91	0.60	0.52	0.61	0.47	<u>0.45</u>	0.57	0.54	0.55	216.1	253.6	<u>31.4</u>	101.4
Average	0.51	0.83	0.63	0.73	0.75	0.70	0.62	0.64	<u>0.37</u>	80.5	62.7	<u>6.6</u>	48.3

Table 40: Matrix Factorization instances.

### 3.4 Comparison with $V_{10}$

$m \times n$	Gap to the best known, %									Time, sec			
	$V_{10}$	$M_{100}$	$M(F)$	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_{10}$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	0.04	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	3.6	0.5	<u>0.5</u>	13.9
200 × 1000	0.11	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.08	0.03	<u>0.00</u>	<u>0.00</u>	0.07	4.6	1.1	<u>0.4</u>	13.8
400 × 1000	0.52	0.06	0.02	<u>0.01</u>	0.15	0.14	0.06	0.06	0.10	8.1	2.2	<u>0.6</u>	13.4
600 × 1000	0.70	0.10	0.04	0.05	0.23	0.22	<u>0.04</u>	0.04	0.15	15.6	3.3	<u>0.6</u>	14.1
800 × 1000	0.67	0.23	0.10	0.17	0.28	0.26	0.12	<u>0.06</u>	0.18	17.9	4.0	<u>0.9</u>	14.7
1000 × 1000	1.10	0.29	0.24	<u>0.22</u>	0.35	0.31	0.26	0.25	0.25	18.6	5.8	<u>1.0</u>	15.9
500 × 5000	0.02	0.01	0.00	<u>0.00</u>	0.03	0.02	0.01	0.00	0.05	74.6	22.5	<u>2.5</u>	71.1
1000 × 5000	0.19	0.09	0.05	0.08	0.12	0.11	<u>0.05</u>	0.06	0.15	134.7	52.9	<u>3.6</u>	71.7
2000 × 5000	0.49	0.25	0.22	0.24	0.28	0.32	<u>0.21</u>	0.23	0.23	261.1	131.6	<u>7.3</u>	73.2
3000 × 5000	0.55	0.35	<u>0.29</u>	0.33	0.33	0.31	0.30	0.31	0.33	323.9	214.3	<u>12.3</u>	79.9
4000 × 5000	0.44	0.35	0.31	0.35	0.33	0.30	<u>0.28</u>	0.31	0.37	442.0	286.9	<u>17.2</u>	94.7
5000 × 5000	0.97	0.49	0.45	0.47	<u>0.33</u>	0.35	0.43	0.44	0.54	590.3	348.5	<u>40.4</u>	111.1
Average	0.48	0.19	<u>0.14</u>	0.16	0.21	0.20	0.15	0.15	0.20	157.9	89.5	<u>7.3</u>	49.0

Table 41: Random instances.

$m \times n$	Gap to the best known, %									Time, sec			
	$V_{10}$	$M_{100}$	$M(F)$	$M(V_1^{\text{ex}})$	$R_{15}$	$R_{20}$	$R_{\lfloor m/2 \rfloor}^m$	$R_{\lfloor m/3 \rfloor}^m$	$R_{\lfloor m/2 \rfloor}^{\text{ls}}$	$V_{10}$	$M_{100}$	$R_{15}$	$R_{20}$
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	2.0	0.5	<u>0.4</u>	10.8
200 × 1000	0.23	<u>0.00</u>	2.74	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	5.7	0.7	<u>0.4</u>	12.1
400 × 1000	7.47	3.21	66.19	3.21	3.21	3.21	<u>0.00</u>	<u>0.00</u>	5.37	9.1	2.0	<u>0.5</u>	12.5
600 × 1000	3.69	2.38	83.36	2.38	2.38	2.38	0.39	<u>0.24</u>	1.36	19.6	3.0	<u>0.6</u>	11.9
800 × 1000	6.38	6.01	82.73	5.92	2.93	2.93	0.54	<u>0.42</u>	0.67	44.9	3.9	<u>0.7</u>	12.2
1000 × 1000	18.84	10.88	75.11	10.88	7.79	7.79	<u>0.29</u>	0.56	4.01	65.7	4.2	<u>1.0</u>	12.2
500 × 5000	0.03	<u>0.00</u>	4.72	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	0.01	58.7	14.1	<u>2.2</u>	61.1
1000 × 5000	0.56	0.36	1.09	0.34	0.31	0.31	0.10	0.27	<u>0.02</u>	152.1	24.0	<u>2.6</u>	58.2
2000 × 5000	10.94	12.06	92.38	11.86	12.07	12.07	0.47	<u>0.19</u>	1.31	566.5	61.0	<u>5.7</u>	74.3
3000 × 5000	7.91	8.13	93.90	8.13	8.13	8.13	0.79	<u>0.27</u>	0.66	2917.9	79.9	<u>10.4</u>	84.4
4000 × 5000	12.49	8.92	97.96	8.92	26.63	26.63	1.33	<u>0.34</u>	0.75	1825.5	92.2	<u>15.6</u>	82.2
5000 × 5000	21.49	15.45	97.27	15.45	15.45	15.45	3.28	2.52	<u>0.36</u>	2508.0	122.4	<u>36.1</u>	125.5
Average	7.50	5.62	58.12	5.59	6.58	6.58	0.60	<u>0.40</u>	1.21	681.3	34.0	<u>6.3</u>	46.4

Table 42: Biclique instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>10</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>10</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	2.9	0.6	<u>0.4</u>	12.0
200 × 1000	0.18	<u>0.00</u>	0.00	<u>0.00</u>	0.08	0.06	<u>0.00</u>	0.01	0.04	4.8	0.8	<u>0.4</u>	12.5
400 × 1000	0.11	0.02	0.01	<u>0.01</u>	0.12	0.11	0.02	0.02	0.04	9.4	2.0	<u>0.5</u>	12.5
600 × 1000	0.37	0.20	0.12	0.13	0.39	0.37	<u>0.08</u>	0.11	0.27	14.6	3.1	<u>0.6</u>	13.1
800 × 1000	0.69	0.31	0.19	0.26	0.51	0.48	0.20	<u>0.18</u>	0.34	21.1	4.4	<u>0.8</u>	14.2
1000 × 1000	1.02	0.33	0.19	0.25	0.36	0.45	<u>0.17</u>	0.20	0.40	22.1	6.4	<u>1.2</u>	15.5
500 × 5000	0.19	0.01	0.00	<u>0.00</u>	0.03	0.04	0.00	<u>0.00</u>	0.04	71.4	22.5	<u>2.5</u>	70.9
1000 × 5000	0.22	0.06	0.04	0.05	0.11	0.13	0.04	<u>0.03</u>	0.09	107.1	53.4	<u>3.0</u>	70.4
2000 × 5000	0.64	0.20	0.16	0.17	0.21	0.21	0.16	<u>0.16</u>	0.22	290.4	141.3	<u>6.1</u>	77.0
3000 × 5000	0.42	0.37	0.30	0.35	0.32	0.34	<u>0.30</u>	0.31	0.39	373.0	223.3	<u>12.1</u>	76.5
4000 × 5000	0.68	0.47	0.46	0.46	<u>0.42</u>	0.44	0.45	0.44	0.51	489.2	318.5	<u>18.3</u>	91.7
5000 × 5000	0.75	0.50	<u>0.42</u>	0.47	0.43	0.43	0.45	0.44	0.49	780.3	378.9	<u>33.9</u>	98.5
Average	0.44	0.21	0.16	0.18	0.25	0.26	<u>0.16</u>	0.16	0.24	182.2	96.3	<u>6.7</u>	47.1

Table 43: Max Induced Subgraph instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>10</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>10</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	4.16	0.19	0.01	0.06	1.02	1.09	0.05	<u>0.00</u>	0.06	2.8	0.7	<u>0.4</u>	13.4
200 × 1000	2.53	0.94	0.74	0.83	1.96	1.82	0.75	0.68	<u>0.20</u>	4.9	1.3	<u>0.4</u>	14.0
400 × 1000	7.07	2.82	2.12	2.21	3.47	3.09	2.27	2.09	<u>1.55</u>	10.8	2.9	<u>0.6</u>	14.7
600 × 1000	4.01	3.31	2.89	3.11	3.48	3.42	2.87	3.01	<u>2.42</u>	15.8	4.9	<u>0.8</u>	15.2
800 × 1000	6.87	3.91	3.42	3.61	3.53	4.00	3.56	3.36	<u>2.57</u>	19.5	6.7	<u>1.0</u>	16.1
1000 × 1000	6.43	4.41	3.85	4.25	4.53	4.38	3.46	3.65	<u>3.19</u>	21.8	8.1	<u>1.3</u>	16.8
500 × 5000	2.64	0.98	0.80	0.82	1.46	1.51	0.84	0.90	<u>0.35</u>	70.3	30.5	<u>2.8</u>	70.3
1000 × 5000	2.72	1.16	1.26	1.08	1.59	1.58	1.07	1.11	<u>0.37</u>	119.6	78.3	<u>4.2</u>	73.9
2000 × 5000	2.76	2.41	2.03	2.27	2.74	2.83	1.89	1.94	<u>1.53</u>	318.5	212.3	<u>14.0</u>	81.1
3000 × 5000	5.35	3.63	3.41	3.63	3.92	4.25	<u>3.10</u>	3.48	3.24	413.5	378.9	<u>20.3</u>	99.5
4000 × 5000	6.56	4.52	4.25	4.50	5.09	5.23	4.20	<u>4.02</u>	4.37	629.5	520.3	<u>30.4</u>	123.3
5000 × 5000	6.04	4.43	<u>4.21</u>	4.43	5.32	5.42	4.42	4.38	5.02	715.9	633.3	<u>45.9</u>	126.3
Average	4.76	2.73	2.42	2.57	3.18	3.22	2.37	2.38	<u>2.07</u>	195.2	156.5	<u>10.2</u>	55.4

Table 44: MaxCut instances.

$m \times n$	Gap to the best known, %									Time, sec			
	V <sub>10</sub>	M <sub>100</sub>	M(F)	M(V <sub>1</sub> <sup>ex</sup> )	R <sub>15</sub>	R <sub>20</sub>	R <sub>[m/2]</sub> <sup>m</sup>	R <sub>[m/3]</sub> <sup>m</sup>	R <sub>[m/2]</sub> <sup>ls</sup>	V <sub>10</sub>	M <sub>100</sub>	R <sub>15</sub>	R <sub>20</sub>
100 × 1000	<u>0.08</u>	1.46	0.82	0.89	1.02	0.87	0.83	1.12	0.12	3.1	<u>0.4</u>	0.4	13.2
200 × 1000	0.21	0.96	0.49	0.62	1.12	0.96	0.47	0.39	<u>0.09</u>	7.7	0.9	<u>0.5</u>	13.4
400 × 1000	0.26	0.62	0.27	0.37	0.82	0.75	0.28	0.26	<u>0.23</u>	11.0	1.5	<u>0.5</u>	14.0
600 × 1000	0.62	0.34	0.17	0.26	0.54	0.54	0.20	0.18	<u>0.15</u>	14.0	2.4	<u>0.7</u>	14.2
800 × 1000	0.75	0.83	0.53	0.62	0.91	0.98	<u>0.50</u>	0.54	0.56	28.3	3.5	<u>0.9</u>	14.9
1000 × 1000	1.02	0.50	0.27	0.32	0.52	0.41	<u>0.20</u>	0.28	0.38	30.6	4.6	<u>1.2</u>	15.6
500 × 5000	<u>0.08</u>	1.68	1.40	1.51	1.07	1.02	1.21	1.25	0.50	119.3	11.1	<u>2.2</u>	65.9
1000 × 5000	<u>0.21</u>	1.14	0.74	0.91	0.92	0.87	0.75	0.73	0.24	282.2	34.9	<u>3.2</u>	74.6
2000 × 5000	0.38	0.68	0.51	0.60	0.65	0.63	0.51	0.50	<u>0.26</u>	567.9	90.8	<u>6.4</u>	73.4
3000 × 5000	0.65	0.55	0.46	0.50	0.48	0.50	0.47	0.43	<u>0.33</u>	619.9	146.9	<u>12.2</u>	81.1
4000 × 5000	0.62	0.55	0.49	0.52	0.47	<u>0.43</u>	0.47	0.47	0.44	740.1	202.0	<u>19.1</u>	97.7
5000 × 5000	0.91	0.60	0.49	0.55	0.47	<u>0.45</u>	0.53	0.51	0.53	646.1	253.6	<u>31.4</u>	101.4
Average	0.48	0.83	0.55	0.64	0.75	0.70	0.54	0.55	<u>0.32</u>	255.8	62.7	<u>6.6</u>	48.3

Table 45: Matrix Factorization instances.

## 4 Comparison to QP Solver

Evaluation of the QP Based approach in comparison to M(F).

	Random			Max Induced			MaxCut			Matrix Fact		
	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP
100 × 1000	<u>0.00</u>	0.00	100.00	<u>0.00</u>	<u>0.00</u>	100.00	<u>0.98</u>	1.30	100.00	1.62	1.69	<u>0.66</u>
200 × 1000	<u>0.00</u>	0.00	100.00	<u>0.07</u>	0.10	100.00	<u>1.75</u>	2.03	100.00	<u>0.88</u>	0.92	70.16
400 × 1000	0.18	<u>0.13</u>	100.00	<u>0.08</u>	0.12	100.00	4.02	<u>3.81</u>	100.00	<u>0.49</u>	0.59	100.00
600 × 1000	<u>0.16</u>	0.20	100.00	0.34	<u>0.32</u>	100.00	<u>4.27</u>	4.64	100.00	<u>0.42</u>	0.51	100.00
800 × 1000	0.31	<u>0.25</u>	100.00	<u>0.41</u>	0.43	100.00	5.08	<u>4.96</u>	90.03	<u>0.97</u>	1.13	100.00
1000 × 1000	0.52	<u>0.52</u>	100.00	<u>0.42</u>	0.46	100.00	4.38	4.12	<u>0.57</u>	<u>0.48</u>	0.60	100.00
500 × 5000	0.06	<u>0.06</u>	100.00	<u>0.04</u>	0.08	1.62	1.65	<u>1.65</u>	39.38	1.60	1.54	<u>0.87</u>
1000 × 5000	<u>0.17</u>	0.18	100.00	<u>0.13</u>	0.14	70.18	<u>1.99</u>	2.09	9.06	0.97	0.92	<u>0.39</u>
2000 × 5000	<u>0.33</u>	<u>0.33</u>	100.00	<u>0.27</u>	0.29	30.12	3.26	<u>3.02</u>	3.93	0.62	0.72	<u>0.11</u>
3000 × 5000	<u>0.42</u>	0.46	100.00	0.45	<u>0.45</u>	60.07	4.33	<u>3.99</u>	12.11	0.61	0.60	<u>0.10</u>
4000 × 5000	<u>0.44</u>	0.46	100.00	0.61	0.56	<u>0.21</u>	5.23	4.97	<u>2.13</u>	0.65	0.60	<u>0.12</u>
5000 × 5000	<u>0.63</u>	0.64	100.00	0.56	0.58	<u>0.15</u>	5.02	5.13	<u>2.16</u>	0.65	<u>0.59</u>	10.08
	0.27	<u>0.27</u>	100.00	<u>0.28</u>	0.29	63.53	3.50	<u>3.48</u>	46.62	<u>0.83</u>	0.87	40.21

Table 46: The running time of the solvers is equal to that of  $V_4$

	Random			Max Induced			MaxCut			Matrix Fact		
	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP
100 × 1000	<u>0.00</u>	0.00	0.61	<u>0.00</u>	<u>0.00</u>	1.14	<u>0.71</u>	0.71	9.02	1.33	1.56	<u>0.24</u>
200 × 1000	<u>0.00</u>	0.00	0.23	<u>0.03</u>	0.07	0.12	1.55	<u>1.51</u>	4.08	0.64	0.69	<u>0.04</u>
400 × 1000	<u>0.09</u>	0.11	0.27	<u>0.04</u>	0.08	0.11	3.25	<u>3.23</u>	<u>2.33</u>	0.40	0.48	<u>0.14</u>
600 × 1000	<u>0.11</u>	0.17	0.14	0.27	0.27	<u>0.21</u>	3.52	4.22	<u>1.24</u>	0.31	0.32	<u>0.12</u>
800 × 1000	0.25	0.20	<u>0.08</u>	0.35	0.37	<u>0.11</u>	4.52	4.29	<u>1.81</u>	0.83	0.80	<u>0.12</u>
1000 × 1000	0.35	0.34	<u>0.16</u>	0.34	0.31	<u>0.07</u>	4.94	4.60	<u>0.72</u>	0.34	0.42	<u>0.11</u>
500 × 5000	<u>0.05</u>	0.05	1.41	<u>0.02</u>	0.03	1.60	<u>1.38</u>	1.43	11.74	1.56	1.44	<u>0.60</u>
1000 × 5000	0.15	<u>0.13</u>	0.46	<u>0.08</u>	0.10	0.60	<u>1.67</u>	1.79	7.73	0.88	0.84	<u>0.14</u>
2000 × 5000	0.30	0.30	<u>0.15</u>	0.23	0.27	<u>0.14</u>	2.70	<u>2.56</u>	2.75	0.58	0.62	<u>0.04</u>
3000 × 5000	0.36	0.39	<u>0.07</u>	0.36	0.39	<u>0.12</u>	4.13	3.88	<u>2.02</u>	0.55	0.57	<u>0.07</u>
4000 × 5000	0.39	0.39	<u>0.05</u>	0.54	0.53	<u>0.16</u>	4.91	4.79	<u>1.95</u>	0.62	0.58	<u>0.08</u>
5000 × 5000	0.57	0.56	<u>0.08</u>	0.54	0.54	<u>0.14</u>	4.92	4.87	<u>1.81</u>	0.60	0.57	<u>0.06</u>
	<u>0.22</u>	0.22	0.31	<u>0.23</u>	0.25	0.38	3.18	<u>3.16</u>	3.93	0.72	0.74	<u>0.15</u>

Table 47: The running time of the solvers is equal to that of  $V_6$

	Random			Max Induced			MaxCut			Matrix Fact		
	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP
100 × 1000	<u>0.00</u>	<u>0.00</u>	0.20	<u>0.00</u>	<u>0.00</u>	0.38	<u>0.15</u>	0.16	10.49	0.97	1.34	<u>0.33</u>
200 × 1000	<u>0.00</u>	<u>0.00</u>	0.09	<u>0.00</u>	0.03	0.12	<u>0.88</u>	1.09	3.65	0.54	0.49	<u>0.10</u>
400 × 1000	<u>0.05</u>	0.07	0.14	<u>0.02</u>	0.03	0.04	2.67	2.59	<u>1.20</u>	0.31	0.36	<u>0.02</u>
600 × 1000	<u>0.07</u>	0.10	0.10	<u>0.14</u>	0.15	<u>0.05</u>	3.16	3.61	<u>0.79</u>	0.23	0.23	<u>0.03</u>
800 × 1000	<u>0.12</u>	0.10	<u>0.04</u>	0.23	0.26	<u>0.02</u>	3.90	3.51	<u>0.53</u>	0.68	0.70	<u>0.02</u>
1000 × 1000	0.28	0.33	<u>0.09</u>	0.23	0.24	<u>0.04</u>	4.35	4.12	<u>0.34</u>	0.30	0.31	<u>0.04</u>
500 × 5000	<u>0.01</u>	0.02	0.37	<u>0.01</u>	0.01	0.35	<u>1.04</u>	1.21	8.69	1.50	1.36	<u>0.26</u>
1000 × 5000	<u>0.09</u>	0.11	0.18	<u>0.05</u>	0.06	0.18	<u>1.40</u>	1.49	4.89	0.84	0.79	<u>0.09</u>
2000 × 5000	0.24	0.26	<u>0.07</u>	0.20	0.20	<u>0.05</u>	2.35	2.35	<u>1.60</u>	0.55	0.55	<u>0.04</u>
3000 × 5000	0.31	0.38	<u>0.03</u>	0.34	0.37	<u>0.07</u>	3.63	3.66	<u>1.38</u>	0.52	0.47	<u>0.04</u>
4000 × 5000	0.35	0.34	<u>0.02</u>	0.50	0.50	<u>0.09</u>	4.62	4.47	<u>1.27</u>	0.56	0.52	<u>0.04</u>
5000 × 5000	0.51	0.49	<u>0.02</u>	0.47	0.49	<u>0.07</u>	4.53	4.50	<u>1.00</u>	0.52	0.54	<u>0.03</u>
	0.17	0.18	<u>0.11</u>	0.18	0.20	<u>0.12</u>	<u>2.72</u>	2.73	2.99	0.63	0.64	<u>0.09</u>

Table 48: The running time of the solvers is equal to that of  $V_8$

	Random			Max Induced			MaxCut			Matrix Fact		
	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP	M(F)	$R_{\lfloor m/3 \rfloor}^m$	QP
100 × 1000	<u>0.00</u>	<u>0.00</u>	0.02	<u>0.00</u>	<u>0.00</u>	0.07	0.01	<u>0.00</u>	5.34	0.82	1.12	<u>0.05</u>
200 × 1000	0.00	<u>0.00</u>	0.01	<u>0.00</u>	0.01	0.04	0.74	<u>0.68</u>	2.06	0.49	0.39	<u>0.01</u>
400 × 1000	<u>0.02</u>	0.06	0.07	0.01	0.02	<u>0.00</u>	2.12	2.09	<u>0.49</u>	0.27	0.26	<u>0.00</u>
600 × 1000	0.04	0.04	<u>0.00</u>	0.12	0.11	<u>0.01</u>	2.89	3.01	<u>0.49</u>	0.17	0.18	<u>0.02</u>
800 × 1000	0.10	0.06	<u>0.01</u>	0.19	0.18	<u>0.01</u>	3.42	3.36	<u>0.34</u>	0.53	0.54	<u>0.00</u>
1000 × 1000	0.24	0.25	<u>0.03</u>	0.19	0.20	<u>0.03</u>	3.85	3.65	<u>0.19</u>	0.27	0.28	<u>0.02</u>
500 × 5000	<u>0.00</u>	0.00	0.18	0.00	<u>0.00</u>	0.20	<u>0.80</u>	0.90	5.32	1.40	1.25	<u>0.12</u>
1000 × 5000	<u>0.05</u>	0.06	0.10	0.04	<u>0.03</u>	0.09	1.26	<u>1.11</u>	2.72	0.74	0.73	<u>0.05</u>
2000 × 5000	0.22	0.23	<u>0.05</u>	0.16	0.16	<u>0.02</u>	2.03	1.94	<u>0.42</u>	0.51	0.50	<u>0.03</u>
3000 × 5000	0.29	0.31	<u>0.02</u>	0.30	0.31	<u>0.03</u>	3.41	3.48	<u>0.38</u>	0.46	0.43	<u>0.03</u>
4000 × 5000	0.31	0.31	<u>0.01</u>	0.46	0.44	<u>0.05</u>	4.25	4.02	<u>0.39</u>	0.49	0.47	<u>0.02</u>
5000 × 5000	0.45	0.44	<u>0.01</u>	0.42	0.44	<u>0.03</u>	4.21	4.38	<u>0.41</u>	0.49	0.51	<u>0.02</u>
	0.14	0.15	<u>0.04</u>	0.16	0.16	<u>0.05</u>	2.42	2.38	<u>1.55</u>	0.55	0.55	<u>0.03</u>

Table 49: The running time of the solvers is equal to that of  $V_{10}$