

Toward Better Build Volume Packing In Additive Manufacturing: Classification Of Existing Problems And Benchmarks

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Abstract

In many cases, the efficient operation of Additive Manufacturing (AM) technology relies on build volumes being packed effectively. Packing algorithms have been developed in response to this requirement. The configuration of AM build volumes is particularly challenging due to the multitude of irregular geometries encountered and the potential benefits of nesting parts. Currently proposed approaches to address this packing problem are routinely evaluated on data sets featuring shapes that are not representative of targeted manufacturing products. This study provides a useful classification of AM build volume packing problems and an overview of existing benchmarks for the analysis of such problems. Additionally, this paper discusses characteristics of future, more realistic, benchmarks with the intention of promoting research toward effective and efficient AM build volume packing being integrated into AM production planning methodologies.

Introduction

Additive Manufacturing (AM), also known as 3D Printing, is an umbrella term that refers to a set of technologies related to the production of three-dimensional shapes by successively adding horizontal layers of geometry. AM finds its use in industry for the manufacture of end-use components with the aim of building up components in a single process step and for rapid prototyping purposes. The benefits of its adoption include the production of complex irregular shapes that can be customized according to user preference and an ability to deposit multiple components concurrently within a build operation. Additional benefits arise through the possibility of precisely estimating build time, raw material utilization, energy consumption and, consequently, total cost [1, 2]. Considering these characteristics, packing algorithms that are able to handle irregular shapes have been used in the AM workflow in order to determine suitable packing configurations for efficient technology

operation [2]. The effective utilization of the available build volume capacity promotes AM’s usefulness as an industrial process by simultaneously reducing the required amount of time, process energy consumption and cost, as well as the number of builds.

However, one recurring challenge faced is the absence of a widely used benchmark with representative part geometries and properties adequately simulating real-world scenarios. In fact, it can be observed that the implemented methods are often tested against data sets which are composed of simple shapes which are very different to typical geometries required in industrial applications of AM.

Among the spectrum of available AM technology, this paper focuses on Laser Sintering (LS) which carries the advantage of being able to build configurations of geometries that have been arranged freely in the platform’s internal build volume without the need for support structures [3, 4]. For details on the operating principle of LS see, for example, Gibson et al. [5]. Concentrating on LS, this paper discusses existing benchmarks covering major classes of problems and configurations. This provides a context for the evaluation of key characteristics proposed for a future standard benchmark enabling the assessment of the strengths and weaknesses of individual build volume packing approaches for the implementation of AM in the real world.

The following section of this paper summarizes the most accepted classification for cutting and packing problems involving irregular shapes and the main variations. It also explains how these are applicable to AM technology and proposes a notation for the treatment of such problems. The subsequent section reviews existing benchmarks and build volume packing algorithms in the literature, grouping them by the type of solution adopted. Building on this context, the following section proposes characteristics for a new and generally applicable benchmark for AM-related build volume packing problems. Conclusions are drawn in a final section.

Classification of packing problems

A suitable starting point for this paper is to outline an accepted categorization of packing problems. Wäscher et al. [6] present a typology for cutting and packing problems, extending the typology by Dyckhoff [7]. This research applies this typology to the 3D packing problems encountered in the AM process variant LS.

In AM, information on part geometry is exchanged via 3D CAD data describing as part’s *shape*, normally using the STL file format. This paper will use the term *item* to describe an individual instance of such a 3D shape. A *profile* consists of the characteristics of a build chamber (i.e. the available build volume) and one item of each shape included in the problem. The term *demand* describes the number of instances of each shape requested for manufacturing. Thus, a build volume packing problem requires the packing of the demanded items into the available build space, optimizing certain criteria while respecting relevant constraints.

A simple form of this model assumes an overall level of demand exceeding the manufacturing capacity of a single build operation. Applied to a single machine, this implies that not all required parts can be inserted into a single build volume. Thus, the production of all items is not possible within the build volume and a subset of items needs to be selected. In a simple specification, the selection could be determined to maximize the total volume of all parts contained in the build (*Output Maximization*). In a more advanced model, the overall demand must be handled

in a of series builds with a fixed Z-height. In this case, the optimal packing will minimize the number of builds. In a further specification treating Z-height as variable, the objective may be to minimize the overall required Z-height of the packed builds. Similarly, the problem can be set up to minimize the height of a single infinitely tall hypothetical build volume (i.e. with an open dimension) accommodating all requested shapes [8, 9, 10].

A build volume packing problem that treats Z-height as variable is extremely relevant to LS. This is the case as reducing the Z-height of the build reduces the number of required layers, consequently reducing both time [11], process energy consumption [2, 12] and cost [13], thereby contributing to the objective of *Input Minimization*. It should be noted that other variants of AM problems exist, which may require the optimization of a different objective, or multiple conflicting objectives simultaneously. This could be, for example, the minimization of surface roughness, build platform contact area or support structure volume [8, 9, 10].

The 3D packing problems encountered in AM can be captured as a four-tuple using the notation proposed by Dyckhoff [7]: $\alpha/\beta/\gamma/\delta$. Applied to AM, the first element α denotes the dimensionality of the packing problem, β represents the optimization criterion, γ denotes the characteristics of the AM build volume and δ represents the assortment of demand of parts. For the AM technology variant in the focus of this paper, LS, the packing problem is fully three dimensional, hence α is specified as 3. In terms of the optimization criterion (β) both Input minimization (In) and Output maximization (Ou) are possible. As outlined above, the build volumes to be filled may be treated in alternative ways (γ). It is proposed that these specifications correspond to one of the following:

- one build volume with fixed dimensions (Of),
- one build volume with open Z-height (Oo),
- multiple identical build volumes (I),
- strongly heterogeneous build volumes (S), or
- weakly heterogeneous build volumes (W)

In terms of the geometry of parts requested (δ) a useful distinction can be made between identical (I), strongly heterogeneous (S) and weakly heterogeneous (W) geometries. It is noteworthy that Wäscher et al. [6] uses the term *regular* to identify convex shapes such as rectangles, spheres or cylinders. Of course, more *irregular* shapes are of interest in the context of AM.

To maintain the generality of this notation, it is important denote the presence of non-convexity, irregularity and geometric complexity in shapes and build volumes. This is done by inserting ' as a suffix in the notation. For example, $3/\text{In}'/\text{S}'/\text{I}'$ could denote an AM packing problem which requires placement of many identical items with irregular shapes in the profile into irregular build volumes of different shapes with the goal of minimizing the total volume of support structures, a variation of Input minimization denoted by In' .

It is clear that using the proposed typology requires judgment to select the appropriate classification. For example, the definition of *weakly* and *strongly heterogeneous* by Wäscher et al. [6] does not quantify characteristics that could be applied to AM build volumes. Instead, linguistic terms such as 'few' or 'many' are used for each demand or build volume entity in a given problem.

A basic type of packing problem found in AM is the *bin packing* problem, which can be denoted as $3/\text{In}/\text{I}/*$ using '*' as a wildcard symbol that can take any value. In this specification, the entire

heterogeneous set of shapes must be packed/placed into a minimum number of build volumes with fixed dimensions [2, 14, 15, 16]. This corresponds to the minimization of inputs used to create a given number of parts. Another common problem dealt with in the AM literature is the *open dimensional* problem, denoted as 3/In/Oo/*. In this model, all required part geometries must be organized in such a way that the Z-height of the container is minimized [10, 17]. Effectively, the objective in such an open dimensional packing problems is to reduce the overall number of layers. An additional way of viewing the packing problem encountered in AM is as a *knapsack* problem, denoted as 3/Ou/Of/*. Here, a subset of shapes must be selected from the overall demand and placed into a single build volume with fixed dimensions to optimize a given objective [2].

Existing 3D packing benchmarks and solution methods

In the field of algorithm design, benchmark data sets are defined to reflect standard problems for the application of algorithms in order to test their efficacy. Thus, a *benchmark* forms a dataset consisting of one or more profiles, as defined above. This section presents a survey of the AM literature for existing 3D packing problems formulated in this manner, along with the proposed methods for solving those problems. Figure 1 presents a diverse range of 3D packing benchmarks identified in this research, also stating the number n of shapes included in these benchmarks. To aid reader orientation throughout this paper, Figure 1 applies labels to each benchmark. As shown, these shapes range from basic polygonal shapes that do not reflect actual products to real part geometries.

Many packing problems, especially those operating in three dimensions, are computationally complex to solve. In algorithm design such problems are defined as being NP-hard [18]. Therefore, most existing methodologies do not aim to identify a globally optimal solution but to establish an optimized, ‘good enough’, solution in an acceptable amount of time. Such methodologies are also described as heuristic solution methods. To limit this survey to work which is relevant to AM, only packing algorithms handling three-dimensional irregular shapes have been included in this paper, perhaps with the exception of Stoyan et al. [17]. This decision was made as the problems faced in real AM processes concentrate heavily on non-convex geometries.

As part of packing algorithms, placement heuristics such as bottom-left-right placement [15], aim to find the best position for 2D or 3D shapes by defining a rule which will determine where the next piece will be placed within the container, given the packing of the already inserted items. Baumers et al. [2] adopted a combined bottom-right-left and center-of-mass placement algorithm in order to demonstrate that an improved utilization of available machine capacity positively impacts process efficiency. This result highlights the importance of developing and utilizing efficient packing algorithms rather than manually packing build volumes. The dataset used in this study is shown in Figure 1(c).

Other placement heuristic approaches have also been used, such as no-fit-polygon [19, 20] and geometrical translation implementation [21], which uses a guided local search to minimize the overlap between envelope shapes. The latter approach was able to cope with non-convex polygons and interior holes and was applied to S2005 [17] and by Ikonen et al. [16].

One of the most common approaches to the problem of determining the order of insertion of shapes into build volumes is to use a Genetic Algorithm (GA) [8, 22, 23]. In these methods, information is

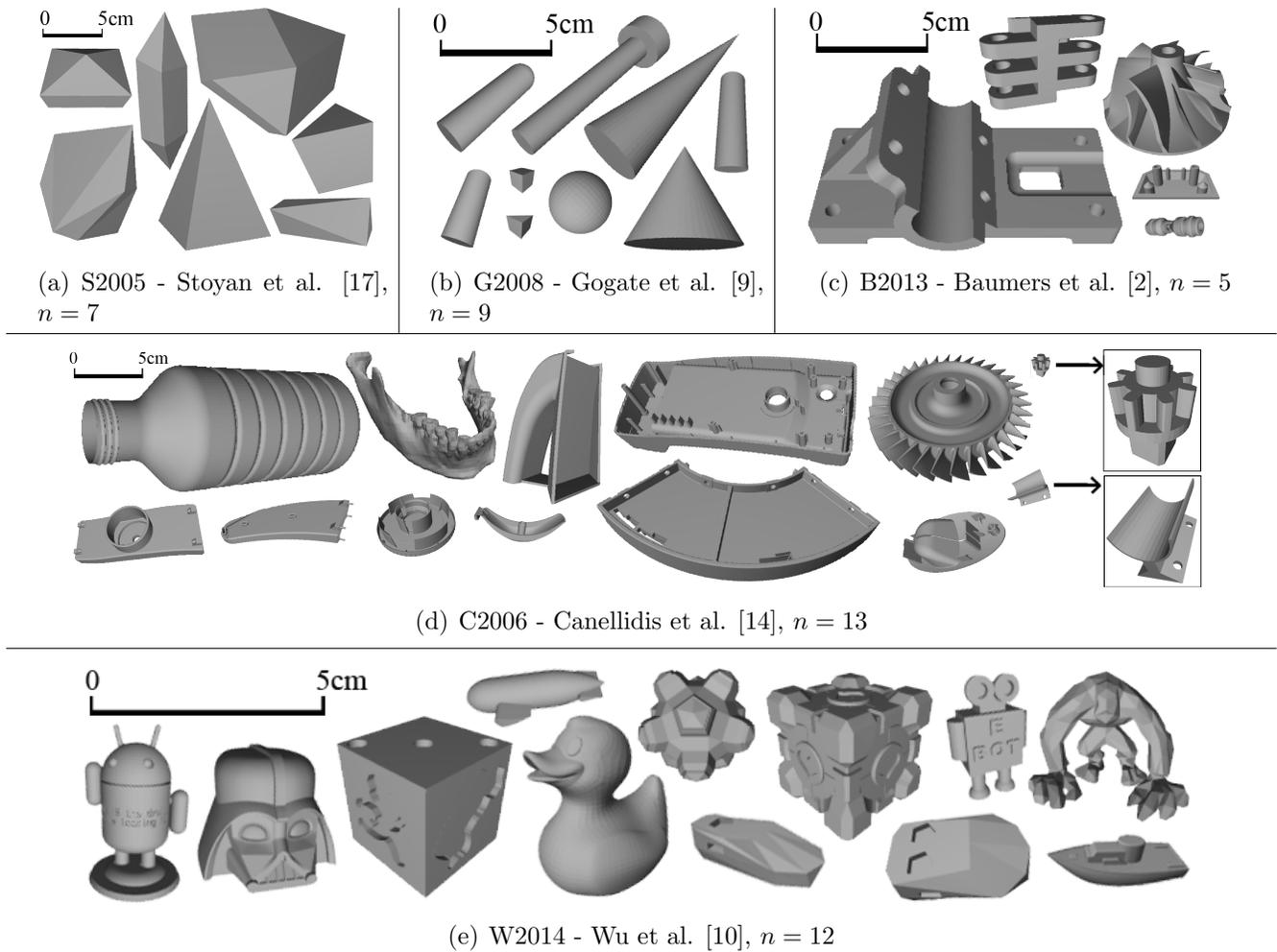


Figure 1: Existing benchmarks for 3D AM build volume packing.

placed in a ‘chromosome’ constituting a vector of values (often binary values) in which the position conveys meaning and the values determine what to do, then operations are applied to modify or combine these chromosomes. However, the efficiency of such approaches is highly dependent on a human operator’s skill in determining the attachment points for each part, the parameters which were used for the GA and the weights which were assigned to each of the various objectives in the fitness function.

One of the first papers to use this method is Ikonen et al. [16], which considers the $3/In'/Of'/\{I,S'\}$ problem. In their implementation, the GA aims to minimize the volume which is not currently contained in the build chamber and the chromosome contains information about the order of placement and orientation (with a limited number of orientations) for each part. Similarly, to solve the $3/Ou'/Of/S'$ problem, Canellidis et al. [14] apply their own dataset. Interestingly, this approach determines the appropriate surface quality, support volume, build time (total height of parts) and projection area in an initial manual phase. Only after this operator intervention, a GA coupled with a customized placement algorithm is employed to determine the packing configuration.

Gogate and Pande [9] approach the multi-objective problem of minimizing part build height, stair case effect, volume and area-of-contact of support structures in a variant of the $3/In'/Oo/S'$ problem, using a GA to determine the packing sequence and the orientation and rotation of each

part. The parts are rotated on a plane in 45° increments (8 possible rotations for each part shown in the Figure 1(b)) rather than allowing free rotation. A bottom-left placement algorithm (based on [24]) is then used to pack the items using a voxel representation. In this case, the quality of the packing result is dependent upon finding good relative weights for the various objectives in the fitness function.

Wu et al. [10] propose an additional packing approach using a GA to solve 3/In'/Oo/S', aiming to minimize the total Z-height of the build volume, the surface roughness (using a function for measuring the average surface roughness) and the support volume. This approach combines a GA with a modified bottom-left-front heuristic, but this method utilizes a Pareto optimal frontier for considering objective trade-offs rather than weighting the objectives.

Due to the complex characteristics of the irregular packing problem and the multiplicity of its objectives, only few methods attempt to find exact/optimal solutions to the problems via mathematical modeling. Those which do so consider only small problems with limited numbers of parts, potentially making these methods infeasible in real-world cases. A mathematical model of this type was developed by Stoyan et al. [17] to find good placement patterns for a set of convex shapes, packing between 7 and 25 items of seven different types shown in the benchmark S2005 (Figure 1(a)). Identifiable as a 3/In/Oo/S' problem, this approach aims to minimize build volume Z-height. In addition to limiting the quantity of items, this approach requires that complex part geometries are simplified to approximated convex shapes. A similar model, associated with a local search method, is reapplied by Stoyan et al. [25] to the same problem. The model is validated against five configurations with up to 80 parts and is reported to effectively pack up to 250 shapes.

It is noteworthy that in some cases individual benchmarks have been reapplied in additional studies of AM-related 3D packing problems. To provide an overview, Table 1 summarizes the use of the benchmarks identified in this paper across the literature. It further categorizes the used methodology as outlined in this section, and classifies the packing problem using the proposed standard notation. Benchmarks which the authors were not able to obtain enter Table 1 using the label 'Other'.

Table 1: Summary of studies of AM-related 3D packing problems in AM

Method	Reference	Problem	S2005	C2006	G2008	B2013	W2014	Other
Placement Heuristic	Dickison and Knopf (1998) [15]	3/Ou'/Of'/S'						x
	Egeblad et al. (2009) [21]	3/In/Oo/S'	x					x
	Baumers et al. (2013) [2]	3/Ou/Of/*				x		
Genetic Algorithm + Placement Heuristic	Ikonen et al.[16]	3/In'/Of'/{I,S'}						x
	Hur et al. [8]	3/In'/Oo'/S'						x
	Canellidis et al. [14]	3/Ou'/Of/S'		x				
	Gogate and Pande [9]	3/In'/Oo/S'			x			
	Wu et al. [10]	3/In'/Oo/S'			x		x	
Mathematical Models	Stoyan et al. (2004) [25]	3/In/Oo/S'						x
	Stoyan et al. (2005) [17]	3/In/Oo/S'	x					

As is apparent from Table 1, none of the studies in the literature have applied more than two benchmarks. The authors assume that a reason for this is that existing benchmarks are rarely

designed to cover multiple categories of packing problems. A further impediment may be that obtaining multiple datasets to test against can be time consuming.

Key characteristics for a new standard benchmark

For a generally applicable benchmark for AM-related 3D packing problems, it is important to incorporate part shapes with features that realistically reflect AM product geometry. As shown above in Figure 1, this realism is lacking in several benchmarks encountered in the literature. In addition to irregularity of shape structures, AM products often exhibit a high degree of geometric complexity. In fact, the ability to deposit such complexity is seen as one of the prime reasons for adopting AM [26].

To construct an initial comparison between shapes, it is useful to initially assess the shapes contained in the benchmarks with respect to the number of vertices, edges, facets (f), area of surface (a) and volume (v). Additionally, a basic metric intended to reflect shape complexity, as proposed by Valentan et al. [27], can be applied. This simple metric k is specified as follows:

$$k = \frac{f \times a}{v} \quad (1)$$

Figure 2 summarizes the mean values and standard deviations of the geometric characteristics present in the benchmarks found in the literature. It is important to note in this context that these data are obtained from polygonal geometric information, hence they are not consistent with respect to the approximation of curvature. Please also note the logarithmic scale of the Y axis.

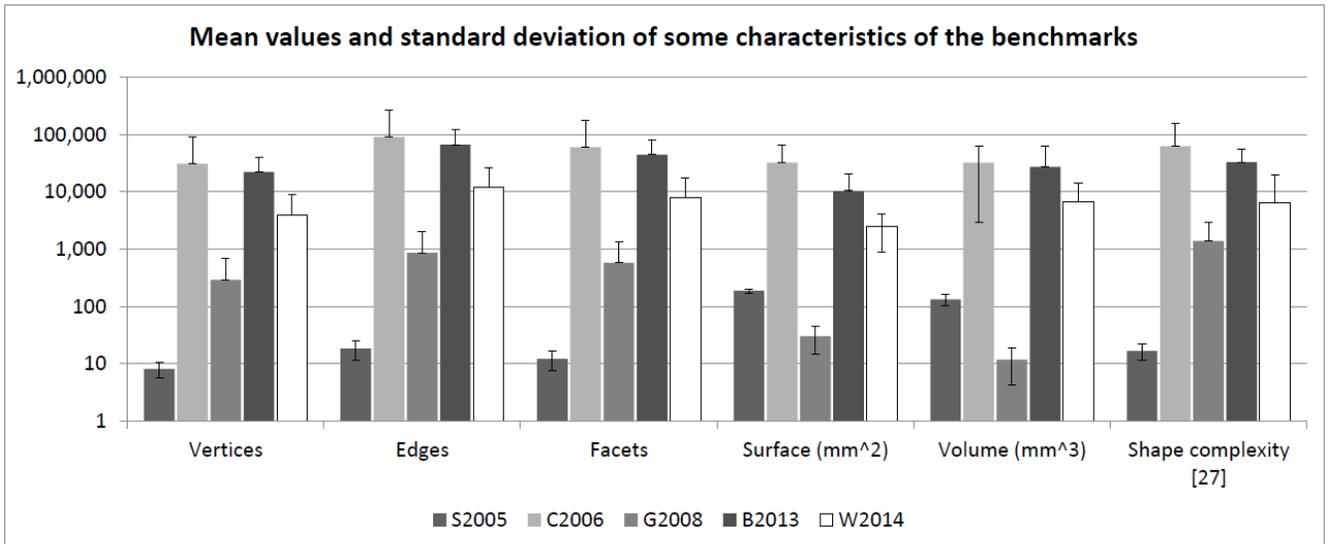


Figure 2: Comparison of selected 3D packing benchmarks used in AM

The high degree of variance in the geometric characteristics of the benchmarks assessed in Figure 2 suggests initially that different levels of realism are reflected. It must be emphasized that to be useful for application in real world build volume configuration, 3D packing approaches must be able to handle geometries that are typically produced via AM. Additionally, the high degree of dispersion

in terms of the above metrics suggests that a consistent methodology should be considered for the conversion of continuous curvature into polygonal shapes.

A further aspect that a new benchmark should address and which limits the usefulness of the typology in Wäscher et al. [6] is the possibility of different quantity requirements for each of the shapes. This aspect is contained in the above definition of demand. In a typical AM process, especially when considering LS, individual builds routinely contain large numbers of identical items to maximize process efficiency [28]. This indicates that studies focusing on the insertion of single units of each geometry into the available build space [9, 10, 14] may not be reflective of real world technology usage. However, this aspect is dealt with in other studies [2, 17] which model variation in demanded quantity, as illustrated in Table 2.

Table 2: Characteristics of demand per shape in Stoyan et al. [17] and Baumers et al. [2]

Profile	Stoyan et al. [17]			Baumers et al. [2]							
	A	B	C	A	B	C	D	E	F	G	H
Mean quantity	1.00	1.71	3.57	∞	5.00	3.00	1.00	∞	11.50	72.50	1.00
Std. dev.	0.00	0.88	1.29	0.00	0.00	0.00	0.00	0.00	1.50	27.50	0.00

To aid the formulation of a new benchmark, this paper proposes to identify variation as ‘low’ or ‘high’ for a given demand. It is proposed that *low variation* occurs when the mean is greater than or equal to the standard deviation of the quantity of items per shape, and *high variation* occurs otherwise.

Conclusion

This paper has identified existing benchmarks developed to study AM-related 3D build volume packing problems. By applying a standard notation to these benchmarks, a classification system for such problems has been proposed. Based on the characteristics of the shapes contained in these benchmarks and the demand level governing how many of the individual shapes enter the build volume packing problem, this paper has recommended characteristics for a future benchmark for AM build volume packing. This will ensure that the future benchmark will be reflective of realistic AM build volume packing tasks and is able to cover the main categories of packing problems encountered in AM. Of course, the goal of applying AM build volume packing methodologies is to increase the efficiency of AM processes by maximizing build volume capacity utilization.

Two avenues for further research are proposed by the authors: outside of AM, advanced packing algorithms are applied by the component layout industry [29, 30]. One example for such an algorithm is the implementation of a Simulated Annealing metaheuristic by Cagan et al. [31]. One particularity of this method is the use of octree-models, an alternative data-structure to the voxel that hierarchically splits the three-dimensional space, allowing very fast detection of overlaps between parts. As such methodologies have been applied in AM design methodologies [32], they should also be considered for AM build volume packing. Additionally, it will be necessary to relate existing and future benchmarks to the characteristics of various AM technology variants. For example, many AM platforms require that the shapes inserted in to build volumes are connected via support structures to a build platform, which may result in 2D build volume packing problems, rather than the 3D problems discussed in this research.

Further information

As an additional output of the research leading to this paper, new benchmarks will be made available via the corresponding authors' website (see <http://www.cs.nott.ac.uk/~lja/benchmark.html>), together with other supporting material. Where possible, the existing benchmarks outlined in this paper will also be made available, subject to author approval.

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