

# Interval type-2 hesitant fuzzy Entropy-based WASPAS approach for aircraft type selection

Muhammet Deveci<sup>a</sup>, Sultan Ceren Öner<sup>b</sup>, Muharrem Enis Ciftci<sup>c</sup>, Ender Özcan<sup>d</sup>, Dragan Pamucar<sup>\*e</sup>

<sup>a</sup>*Department of Industrial Engineering, Turkish Naval Academy, National Defence University, 34940 Tuzla, Istanbul, Turkey.*

<sup>b</sup>*Department of Industrial Engineering, Faculty of Management, Istanbul Technical University, 34367 Mada, Istanbul, Turkey*

<sup>c</sup>*Department of Industrial Engineering, Faculty of Mechanical Engineering, Yildiz Technical University, 34349 Yildiz, Istanbul, Turkey*

<sup>d</sup>*Computational Optimisation and Learning (COL) Lab, School of Computer Science, University of Nottingham, NG8 1BB, Nottingham, UK.*

<sup>e</sup>*Department of Logistics, Military Academy, University of Defence in Belgrade, 11000 Belgrade, Serbia*

---

## Abstract

Choosing the most appropriate aircraft type for a given route is one of the crucial issues that the decision makers at airline companies have to address under uncertainty based on various commercial, marketing and operational criteria. A novel multi-criteria decision making approach integrating Entropy-based Weighted Aggregated Sum Product Assessment (WASPAS) method and interval type-2 hesitant fuzzy sets (IT2HFS) is introduced for tackling this problem and tested using a particular case study obtained from a full service carrier in Turkey. This study contributes to representing and handling degrees of uncertainty in the decision making process of aircraft type selection based on the IT2HFS. The results showed that Airbus 32C is the suitable alternative for a given route in between Kuwait and Istanbul airports. The experts evaluated the results and confirmed that the proposed approach is the most suitable one when compared to four other IT2HFS based approaches.

*Keywords:* Multi-criteria decision making, airline, hybrid fuzzy method.

---

## 1. Introduction

The global airline industry has grown rapidly in recent years and this trend will continue with technological advances in the area. Cargo transportation

---

\*Corresponding author

*Email addresses:* muhammetdeveci@gmail.com (Muhammet Deveci), csalkin@itu.edu.tr (Sultan Ceren Öner), muharremenis@gmail.com (Muharrem Enis Ciftci), ender.ozcan@nottingham.ac.uk (Ender Özcan), dpamucar@gmail.com (Dragan Pamucar\*)

plays a critical role in international trade, and passenger transportation is crucial for tourism. According to the International Air Transport Association, goods valued at \$5.7 trillion were transported by air and tourists using air transport spent \$620 billion in 2015 [1]. The airline industry in Turkey has been growing considerably due to deregulation of the Turkish aviation industry, with the strong economic growth and tourist appeal since 2003 [2, 3]. Currently, a giant third airport is being built on the northern European side of Istanbul. By 29 October 2018, Istanbul's third airport has been opened.

The seat capacity of Turkish Airlines grew by 349% from 2003 to 2014. Middle Eastern and African destinations play a very important role in the growth of the Turkish aviation market. The common history and similar cultural background have an important effect on that growth [4]. The number of destinations in the Middle East served by Turkish Airlines increased from 10 to 30, necessitating an increase in the airline transport capacity and then handling of aircraft type selection. Aircraft type selection from current fleet for a given route is a difficult task requiring consideration of a number of criteria, including customer expectation, operating costs, revenue potential, competition, utilization of aircraft and capacity allocation. Therefore, airlines carefully choose their aircrafts for the routes in order to maximize profit and hit their business targets.

For the frequent flyers from not only the Middle Eastern countries but also other countries, the most important factor turns out to be the comfort during a flight. Aircraft manufacturers work with psychologists, anthropologists and frequent flyers in the cabin design phase of an aircraft. The majority of the airlines provide their customers the aircraft types along with the timetables via their reservation systems based on websites and mobile applications. According to [5], it has been found that "business travelers prefer itineraries with standard jets relative to propeller or regional jets, and interestingly, also to wide-body jets. The interaction model shows that the preference for standard jets is particularly pronounced for frequent travelers". Therefore, aircraft type, especially the wide body aircrafts play a major role along with pricing policy in the competition between airlines that fly on the same routes.

This is one of the first studies that hybridises an interval type-2 hesitant fuzzy system with a multi-criteria decision making method for solving a real-world aircraft type selection problem for the Istanbul-Kuwait route of a Turkish airline company. This particular route is the focus of this study, since there is potential for high revenue for the company but high customer satisfaction as well considering that Kuwait has a very high gross domestic product (GDP) per capita. We propose entropy-based WASPAS (Weighted aggregated sum product assessment) [6] to be embedded into an interval type-2 hesitant fuzzy system for capturing the uncertainty associated with the problem when the decision makers evaluate different aircraft types for the route. IT2HFSs are a special kind of hesitant fuzzy sets that presents both primary and secondary memberships with intervals. WASPAS integrated IT2HFS is provided as a multi-criteria decision making method applied, to aircraft type selection as a case study in our paper. The final rankings of the proposed approach are compared to the performance of the interval type-2 hesitant fuzzy embedding other multi-criteria methods,

including MABAC [7], ARAS [8], TODIM [9] and COPRAS [10]. The results indicate the feasibility of the proposed approach, which has been validated and confirmed by experts from the airline company, handling conflicting criteria and decision making under uncertainty for aircraft type selection for a given route.

The key reasons for applying entropy based WASPAS embedded IT2HFS approach to the problem are listed as follows:

- Interval type-2 fuzzy sets (IT2FS) enable more degrees of freedom in decision making process when modelling the uncertainty compared with T1FNs [11].
- IT2HFS captures the vague information better than the hesitant fuzzy sets enabling inclusion of additional information (data) in the decision making process using primary and secondary memberships with intervals [12, 13, 14]. Those intervals indicate explanations of linguistic terms by considering optimistic and pessimistic perspective.
- The criteria weights that are crucial for the final decision cannot be directly determined as using only weighted sum or weighted product of selected criteria. Thus, weighted aggregation of both additive and multiplicative methods strikes a balance between them and enables more realistic criteria weights when WASPAS is integrated into IT2HFSs.
- Additionally, WASPAS aggregates weighted sum model and weighted product model which enables evaluating alternatives according to three aspects: The first aspect is related to Weighted Sum Model (WSM), was applied for evaluating a number of alternatives considering a number of decision criteria. The second aspect is extracted from Weighted Product Model (WPM) to evaluate alternatives in terms of multiplicative exponential generalized criterion. Finally, as the third aspect, weighted aggregation of additive and multiplicative methods allow gathering more realistic criteria weights when WASPAS is integrated to the information under interval type-2 hesitant fuzzy sets [15].
- Using the concept of the global ideal alternative for qualitative criteria such as *Customer Expectation and Competition*, has some significant outcomes: (1) all of alternatives can be compared with each other, (2) all of alternatives compared with the optimal alternative according to each criterion. The optimal solution can be defined based on standards and regulations or considering passengers' needs, which in general cannot exist in all cases. Thus, using of the best alternative among chosen or using value defined by laws or regulations, like for *Capacity* is a necessity while adapting WASPAS. For attributes, like *Price or Competition*, it is difficult to define the global ideal solution, due to the lowest value, like minimum price or maximum capacity, is favored. Suggesting best alternative among several alternatives is conducted by a normalization process, which assists the evaluation of objective pairwise comparisons [16].

- To find the proper way to derive the weight vector and aggregate the decision makers preferences, entropy-based WASPAS method is proposed.

The remainder of this paper is organized as follows: Section 2 presents an overview of the relevant work and previous studies. The criteria and alternatives for aircraft type selection on route are defined in Section 3. Section 4 give the basic steps of the interval-type type-2 fuzzy and the proposed approach of integrated interval type-2 hesitant fuzzy sets and entropy-based WASPAS. Section 5 presents a case study from Turkey. In addition, sensitivity analysis is performed in this section and results are compared for validation. Finally, Section 6 includes a general evaluation of the study with concluding remarks and potential improvements.

## 2. Related Work on Fuzzy Systems and Airline Problems

Decision-making is one of the essential processes for selecting the most appropriate alternative among diversified options and underpins many applications that could be encountered in different fields. Since 1965 [17] type-1 fuzzy sets (T1FS) for capturing vagueness in information, has been successfully applied to various multi-criteria decision-making (MCDM) problems[18]. Fuzzy MCDM is widely adopted in the evaluation of airline industry problems, as seen in airline new route selection [3], performance measurement of airlines [19, 20], efficiency assessment of airlines [21, 22], supplier selection in the airline retail industry [23], efficient improvement strategy [24], strategic management decision in the airline industry [25], aircraft selection [26, 27], airline fleet planning [28], selection of the strategic alliance partner [29, 30, 31], and evaluating the strategic design parameters of airports [32]. In none of these studies, the issue of aircraft type selection has been tackled on the route between two points.

In order to manage vague information more efficiently, researchers have extended fuzzy sets to different forms [13], like general type-2 fuzzy set (GT2FS) [33], interval type-2 fuzzy set [11] and interval-valued fuzzy set (IVFS) [34, 35, 36]. There have been also several extensions of FSs that are very successfully apart from them: hesitant fuzzy set (HFS) [37] and intuitionistic fuzzy sets (IFSs) [38].

Hesitant fuzzy MCDM approach has recently been utilized quite often. Some of these studies include a theorem for hesitant fuzzy sets [39], hesitant fuzzy based ELECTRE [40, 41, 42], dual hesitant fuzzy elements based VIKOR [43], linguistic hesitant fuzzy set based VIKOR [44], hesitant fuzzy linguistic entropy and cross-entropy methods [45], double hierarchy hesitant fuzzy linguistic term set [46], interval type-2 hesitant fuzzy set [13], new hesitant fuzzy MCDM [47], hesitant pythagorean fuzzy sets [48], cosine-distance-based hesitant fuzzy linguistic (HFL)-TOPSIS method and the cosine-distance-based HFL-VIKOR [49], the correlation measures and correlation coefficients of the HFL [50], hesitant fuzzy VIKOR [51], hesitant fuzzy sets based SWARAWASPAS [52], and hesitant fuzzy sets based TODIM method [53],

In addition, various MCDM methods have been employed in different applications. For example, Xu and Gong [54] proposed a triangular fuzzy entropy and complexity-based discrete Choquet integral for the evaluation of emergency logistics support capability. They presented a case study to validate the proposed model. Beiki et al. [55] addressed a sustainable supplier selection and order allocation problem using a novel integration of language entropy weight method and multi-objective programming. The proposed model has three objectives such as the total cost and the carbon emissions. Examples of other studies are as follows: Nenadic [56], Kushwaha et al. [57], Mihajlovic et al., [58], and Ramakrishnan and Chakraborty [59], Ebadi Torkayesh [60], and Ebadi Torkayesh [61].

When compared with other approaches, WASPAS based method separates final ranking values more sharply that especially MABAC and TODIM gives closer and similar values to all alternatives. COPRAS also selects the most appropriate alternative but on the other hand, third, fourth and fifth alternatives are not clearly selected.

To the best of our knowledge, none of the previous studies considers the aircraft type selection using interval type-2 hesitant fuzzy sets based MCDM methods. Our study presents one of the first applications of IT2HFSs using both intervals and WASPAS integration that intensifies more degrees of freedom in the decision making process.

### 3. Criteria for Aircraft Type Selection

Airline services and products, such as, in-flight entertainment, seat pitch and widths could vary between airlines, sometimes even aircraft types for the same airline. Such services and products can influence passenger demand. It is critical to match the demand providing high quality service and products, at the same time, maximizing the revenue. Hence, it is important to cope with the operational cost of the selected aircraft type on a given route. In particular, trip cost and optimal capacity of the aircraft type play a major role in profitability. More capacity derives more revenue; however, it also generates higher costs. Higher capacity aircraft have a lower cost per seat [62]. In order to be profitable, airlines must select their aircrafts for the routes based on the route characteristics and capacity needs.

[63] defined the operating profit of an airline based on the following equation, where yield is the unit revenue:

$$\text{Operating Profit} = (\text{Revenue Passenger Kilometer (RPK)} \times \text{Passenger Revenue Yield}) - (\text{Available Seat Kilometer} \times \text{Unit Cost})$$

In our approach, the operating profit formulated as follows:

$$\text{Operating Profit} = (\text{Demand} \times \text{Yield} + \text{Cargo Revenue}) - (\text{Variable} - \text{Fixed Cost of Route Operation})$$

The criteria influencing aircraft selection on a route can be classified into 5 main and 10 sub-criteria. Each criterion is shown in Fig. 2. While cost, revenue and capacity are commercial indicators for operating profit of an airline. The competition and customer expectations are mostly qualitative criteria for airline route profitability. The description of each of the five main criteria is given in the following subsections.

### 3.1. Revenue

The revenue is the main factor that an airline intends to fly on a specific route. If an airline cannot make money on a route, that route will eventually be closed. However, before closing the route, airlines are likely to change their aircraft type first or weekly frequency in order to find a better match for the cost and demand for the route. The total revenue given in the following equation can be computed relying on demand, unit passenger revenue and cargo revenue.

$$Total\ Revenue = Total\ PassengerRevenue + CargoRevenue$$

- *Demand* is the total number of passengers travelling on a specific route and time period
- *Unit Passenger Revenue* is the total passenger revenue divided by the total number of passengers. Demand and unit revenue are significant indicators of profitability as they determine the total passenger revenue:

$$Total\ Passenger\ Revenue = Demand \times Unit\ Passenger\ Revenue$$

Higher demand and yields indicate that the route is most likely profitable.

- *Cargo Revenue* is total cargo revenue obtained by the cargo transported in the belly of the aircraft increasing the total revenue on a route.

### 3.2. Capacity

Capacity of an aircraft determines the total number of available business and economy seats in the aircraft that an airline is able to sell. Availability does not mean that every seat in the aircraft can be sold, as there could be seat limitations based on the route range, and maximum take-off weight.

- *Economy Cabin Seat Capacity* is usually much higher in economy aircrafts with more seats than the aircraft that have a business cabin.
- *Business Cabin Seat Capacity* derives more revenue per seat for a given route while the economy cabin seat capacity derives gains from demand.

### 3.3. Customer Expectation

As the customer expectations in a flight are increasing, airlines are investing in their services and products in order to satisfy the customer needs. In the past, a meal was the only difference between airline offerings; however, nowadays, lounge, Wi-Fi, in-flight entertainment, newspaper service appear as the main competition areas between airlines. Especially, high yield passengers and frequent flyers prefer to know the aircraft type before making their reservations on a certain route in their airline choice [64]. For example, travellers from gulf countries are known for their high-level expectation in services. When it comes to airline industry, seat configuration, in-flight entertainment, lounges, aircraft cabins have significant impact on passenger choice between airlines served on the same route.

The airline cabin products are classified into four major group in industry: First Class, Business, Premium Economy, Economy. In this study, we consider two cabin products, Economy Cabin Product, and Business Cabin Product, respectively. The following Table 1 summarizes main differences between airline cabin products.

Table 1: Business and economy cabin product differences.

Cabin Features	Business Cabin Product	Economy Cabin Product
Wi-Fi	* (Mostly Free)	*(In Charge)
Meal	**	*
Inflight entertainment	**	*
Lounge access	*	*(In Charge)
Leg space	**	*
Amenity kits	*	

Note:\*Currently Exist, \*\*Better

### 3.4. Cost

The total cost of a route comprises of direct and indirect costs. The indirect costs include marketing, salary, general management etc. costs that are not directly related with the operation on the specific route, which is not considered in this study. The direct costs can be classified into variable and fixed costs.

- *Fixed Costs of Operation* are the ownership cost of an aircraft, crew costs, maintenance cost of the aircraft and insurance costs.
- *Variable Costs of Operation* are directly related with the operation output. The important variable costs of the airlines include the fuel costs, handling, passenger service costs, airport service charges, commissions etc.

Every aircraft type has its own characteristic cost structure and is important for the route profitability. The airline operating cost breakdown is given in Table 2.

Table 2: Cost breakdown example of an airline.

Direct Variable	Fixed	Indirect
Fuel	Aircraft Ownership (Lease)	General Management
Commission	Crew	Marketing
Handling	Maintenance	Distribution Costs
Passenger Service Cost	Insurance	
Airport Fees		
Overflight Fees		
Carbon Emissions		

### 3.5. Competition

The airlines compete on a specific route could be based on promotions, loyalty programs, schedules, price and cabin product. Aircraft type and in-flight experience can be assessed as airlines' product and also be evaluated as competition indicator.

- *Competitors' Aircraft Type* directly affects the demand and unit revenue.

## 4. Preliminaries

### 4.1. Hesitant Fuzzy Set

Classical fuzzy sets are inadequate when decision makers hesitate to make one preference. For this reason, Torra [37] defined hesitant fuzzy sets (HFS). After that, Rodriguez et al. [65] examined those sets and proposed hesitant fuzzy linguistic term sets (HFLTS) to enrich the content of the linguistic terms. By this way, experts can be more flexible in expressing their preferences when comparing two alternatives [66]. Some of the related definitions are given in the following [37, 67].

Hesitant fuzzy sets are defined as a function returning a set of membership values for each element in the domain. For example, when two decision makers are discussing the membership of  $x$  into  $A$ , and one wants to assign 0.4 and the other wants to assign 0.7. Hence, the uncertainty over the possible values of the membership function is limited to a certain extent.

### 4.2. Interval Type-2 Fuzzy Set

Type-2 fuzzy set concept was first introduced by Zadeh [33] as an extension of type-1 fuzzy sets [17]. In type-1 fuzzy sets, membership degrees of the elements belonging to those sets take values in the interval  $[0, 1]$ . On the other hand, type-2 fuzzy sets are represented with membership functions where the membership degrees of each element in those sets are also fuzzy sets. In this way, type-2 fuzzy sets the uncertainties can be better represented depending on the additional degree of freedom. If there is an uncertainty in membership degrees



of those elements belonging to the sets defining the membership functions, in other words, if a membership function cannot be determined precisely, type-2 fuzzy sets are used [11].

In general notation, type-2 fuzzy sets are shown with  $\tilde{A}$ . The definitions regarding type-2 fuzzy sets are given, drawing from the studies of [11].

For type-2 fuzzy systems, a membership function (the degree of membership) denoted as  $\mu_{\tilde{A}}(x, u)$  characterize a fuzzy set shown as  $\tilde{A}$  where  $x \in X$  in  $\tilde{A}$  and  $u \in J_x \subseteq [0, 1]$  [68]. It is also shown as:

$$\tilde{A} = \{(x, u), \mu_{\tilde{A}}(x, u) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \quad (1)$$

An  $\tilde{A}$  fuzzy set, having the interval  $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$ , is defined with type-2 membership function as in the following:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad (2)$$

Here,  $x$  denotes the primary variable in domain  $X$  and  $u$  denotes the secondary variable for each  $x \in X$  in interval  $J_x$ .  $J_x$  is defined as the primary membership of variable  $x$  and  $\mu_{\tilde{A}}(x, u)$  shows the secondary membership values of set  $\tilde{A}$ . The expression  $\int \int$  shows all possible combinations of elements,  $x$  and  $u$ .

When all  $\mu_{\tilde{A}}(x, u) = 1$  for  $\forall x \in X$  and  $u \in J_x \subseteq [0, 1]$ , then  $\tilde{A}$  is named as an interval type-2 fuzzy set [11]. It is denoted as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1 / (x, u) \quad J_x \subseteq [0, 1] \quad (3)$$

Although, membership functions may have different shapes like triangular, trapezoidal, Gaussian, etc., in this study, a trapezoidal interval type-2 fuzzy sets (IT2FSs)  $\tilde{A}_i$  is used and some definitions are given as in the followings [69]:

$$\begin{aligned} \tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = & ((a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u; h_1(\tilde{A}_i^U), h_2(\tilde{A}_i^U)), \\ & (a_{i1}^l, a_{i2}^l, a_{i3}^l, a_{i4}^l; h_1(\tilde{A}_i^L), h_2(\tilde{A}_i^L))) \end{aligned} \quad (4)$$

In addition, algebraic operations used in this work are addition and multiplication. In the following, they are given as an example for the trapezoidal interval type-2 fuzzy sets  $\tilde{A}_1$  and  $\tilde{A}_2$ ;

The addition between trapezoidal interval type-2 fuzzy sets are shown as in the following:

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 = & (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_2^U, \tilde{A}_2^L) \\ = & ((a_{11}^u + a_{21}^u, a_{12}^u + a_{22}^u, a_{13}^u + a_{23}^u, a_{14}^u + a_{24}^u; \\ & \min(h_1(\tilde{A}_1^U), h_1(\tilde{A}_2^U)), \min(h_2(\tilde{A}_1^U), h_2(\tilde{A}_2^U))), \\ & (a_{11}^l + a_{21}^l, a_{12}^l + a_{22}^l, a_{13}^l + a_{23}^l, a_{14}^l + a_{24}^l; \\ & \min(h_1(\tilde{A}_1^L), h_1(\tilde{A}_2^L)), \min(h_2(\tilde{A}_1^L), h_2(\tilde{A}_2^L)))) \end{aligned} \quad (5)$$

The subtraction between trapezoidal interval type-2 fuzzy sets are shown as in the following:

$$\begin{aligned}
\tilde{A}_1 \ominus \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \ominus (\tilde{A}_2^U, \tilde{A}_2^L) \\
&= ((a_{11}^u - a_{24}^u, a_{12}^u - a_{23}^u, a_{13}^u - a_{22}^u, a_{14}^u - a_{21}^u; \\
&\quad \min(h_1(\tilde{A}_1^U), h_1(\tilde{A}_2^U)), \min(h_2(\tilde{A}_1^U), h_2(\tilde{A}_2^U))), \\
&\quad (a_{11}^l - a_{24}^l, a_{12}^l - a_{23}^l, a_{13}^l - a_{22}^l, a_{14}^l - a_{21}^l; \\
&\quad \min(h_1(\tilde{A}_1^L), h_1(\tilde{A}_2^L)), (h_2(\tilde{A}_1^L), h_2(\tilde{A}_2^L)))
\end{aligned} \tag{6}$$

The multiplication between trapezoidal interval type-2 fuzzy sets are shown as in the following:

$$\begin{aligned}
\tilde{A}_1 \otimes \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_2^U, \tilde{A}_2^L) \\
&= ((a_{11}^u \times a_{21}^u, a_{12}^u \times a_{22}^u, a_{13}^u \times a_{23}^u, a_{14}^u \times a_{24}^u; \\
&\quad \min(h_1(\tilde{A}_1^U), h_1(\tilde{A}_2^U)), \min(h_2(\tilde{A}_1^U), h_2(\tilde{A}_2^U))), \\
&\quad (a_{11}^l \times a_{21}^l, a_{12}^l \times a_{22}^l, a_{13}^l \times a_{23}^l, a_{14}^l \times a_{24}^l; \\
&\quad \min(h_1(\tilde{A}_1^L), h_1(\tilde{A}_2^L)), (h_2(\tilde{A}_1^L), h_2(\tilde{A}_2^L))).
\end{aligned} \tag{7}$$

The arithmetic operations between trapezoidal interval type-2 fuzzy sets and scalar  $s$  are shown as in the following:

$\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u; h_1(\tilde{A}_i^U), h_2(\tilde{A}_i^U)), (a_{i1}^l, a_{i2}^l, a_{i3}^l, a_{i4}^l; h_1(\tilde{A}_i^L), h_2(\tilde{A}_i^L)))$  and the crisp value  $s$  is defined as follows:

$$\begin{aligned}
s\tilde{A}_1 &= ((s \times a_{11}^u, s \times a_{12}^u, s \times a_{13}^u, s \times a_{14}^u \times a_{24}^u; h_1(\tilde{A}_1^U), h_2(\tilde{A}_1^U)), \\
&\quad (s \times a_{11}^l, s \times a_{12}^l, s \times a_{13}^l, s \times a_{14}^l; h_1(\tilde{A}_1^L), h_2(\tilde{A}_1^L))).
\end{aligned} \tag{8}$$

### 4.3. Interval Type-2 Hesitant Fuzzy Set

Interval type-2 hesitant fuzzy sets method was proposed by [13]. In many real-life problems, there are generally ambiguous or uncertain information and decision makers have a hard time in expressing their decisions with precise and accurate values. IT2HFS is proposed based on IT2FS in the hesitant fuzzy linguistic domain which not only facilitates computation process of the hesitant fuzzy linguistic set, but also models the uncertainty with better accuracy. The steps of interval type-2 hesitant fuzzy sets are described by [13]:

*Step 1.* Define the MCDM problem by identifying criteria set as  $C = c_1, c_2, \dots, c_n$  and alternatives set as  $A = a_1, a_2, \dots, a_m$  with the criteria weight vector  $W = w_1, w_2, \dots, w_n$  and  $\sum_{j=1}^n w_j = 1$ .

*Steps 2-3.* Identify appropriate the linguistic terms set and their corresponding interval type-2 hesitant fuzzy sets.

*Step 4.* Collect experts preference relations ( $R^l$ ) for the criteria, sub-criteria and alternatives by  $k$  number of experts where  $l \in \{1, 2, \dots, k\}$  and express IT2HF linguistic term set (LTS) according to the lower and upper bounds as  $[r_{ij}^l, r_{ij}^{l+}]$ .

*Step 5.* Convert the hesitant fuzzy linguistic evaluations to IT2HFSs to acquire the corresponding ratings  $\tilde{h}_{ij}$  where  $i$  denotes alternatives ( $A_i$ ) and  $j$  denotes criterion ( $c_j$ ) and IT2HFS based  $H$  matrix is obtained as follows for  $\forall_j$ :

$$\tilde{h} = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u; h_1(\tilde{A}_i^U), h_2(\tilde{A}_i^U)), (a_{i1}^l, a_{i2}^l, a_{i3}^l, a_{i4}^l; h_1(\tilde{A}_i^L), h_2(\tilde{A}_i^L)))$$

*Step 6.* Gather optimistic and pessimistic preferences as interval type 2 hesitant fuzzy set according to Table 5. Pessimistic presentation for each alternative  $i$  is denoted as  $r_i^-$  and optimistic tendency is denoted as  $r_i^+$ .

#### 4.4. Entropy-based WASPAS Method

WASPAS (Weighted aggregated sum product assessment) method was developed by [6], which is a type of MCDM method that integrates weighted sum (WS) and weighted product (WP) in order to cope with the limitations of each approach. Assume that  $w_j$  denotes the weight of  $j^{th}$  criterion, and  $h_{ij}$  represents the performance value of  $i^{th}$  alternative according to  $j^{th}$  criterion ( $i=1, \dots, n$  and  $j=1, \dots, m$ ). The flowchart of the proposed methodology is provided as in the Fig. 1. The process of evaluating the alternatives using the WASPAS method is summarized as follows:

*Step 7.* Obtain linear normalization of performance values as follows:

$$\bar{h}_{ij} = \begin{cases} \frac{\min_i \tilde{h}_{ij}}{\tilde{h}_{ij}} & \text{if } j^{th} \text{ criterion is a cost criterion} \end{cases} \quad (9)$$

$$\bar{h}_{ij} = \begin{cases} \frac{\tilde{h}_{ij}}{\max_i \tilde{h}_{ij}} & \text{if } j^{th} \text{ criterion is a benefit criterion} \end{cases} \quad (10)$$

where  $\tilde{h}_{ij}$  is the non-normalized value for performance values.

*Step 8.* Calculate the entropy values ( $e_{ij}$ ) and degree of divergence ( $k_{ij}$ ) for normalized aggregated preference values as follows.

$$e_{ij} = \sum_{i=1, p_{.j} \neq 0}^n p_{.j} \cdot \ln p_{.j} \quad (11)$$

$$k_{ij} = 1 + e_{ij} \quad (12)$$

where  $p_{.j}$  is given as the pairwise aggregated preference values for criteria and includes pre normalized  $\bar{h}_{ij}$  values. For instance,  $p_{.j}$  can be realized as overall aggregated value when considering entire criteria for the first criteria (Revenue). We now generalized the equation as  $p_{.j}$  in the text as well.

*Step 9.* Determine the criterion weights according to degree of divergence ( $k_{ij}$ ):

$$w_j = \frac{k_{ij}}{\sum k_{ij}} \quad (13)$$

where  $w$  denotes criterion weight.

Aggregate the individual preferences using interval type 2 hesitant fuzzy weighted average (IT2HFWA) to achieve optimistic and pessimistic preference relations. This operator is proposed by [13] and presented as in following:

$$\begin{aligned}
\tilde{h}_i &= IT2HFWA(\tilde{h}_{i1}, \tilde{h}_{i2}, \dots, \tilde{h}_{in}) = \sum_{j=1}^n w_j \tilde{h}_{ij}, \quad i=1, 2, \dots, m \\
&= \cup_{\tilde{A}_{i1} \in \tilde{h}_{i1}, \tilde{A}_{i2} \in \tilde{h}_{i2}, \dots, \tilde{A}_{in} \in \tilde{h}_{in}} \left\{ \left( \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij1}^u) \right), \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij2}^u) \right), \right. \right. \right. \\
&\quad \left. \left. \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij3}^u) \right), \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij4}^u) \right); \min(H_1(A_{ij}^U), \min(H_2(A_{ij}^U)), \right. \right. \right. \\
&\quad \left. \left. \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij1}^l) \right), \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij2}^l) \right), \right. \right. \right. \\
&\quad \left. \left. \left. \left. \left. \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij3}^l) \right), \left( l^{-1} \left( \sum_{j=1}^n w_j l(a_{ij4}^l) \right); \min(H_1(A_{ij}^L), \min(H_2(A_{ij}^L)) \right) \right) \right) \right) \right) \right\}
\end{aligned}$$

*Step 10.* This step utilizes a distance matrix as defined for IT2HFSs as an extended type of Euclidean distance. Calculate the distances between each pair of alternatives before identifying  $\lambda_i$ .

$$d_{ij}^2(\tilde{y}_n, \tilde{y}_{n'}) = [(\| l_{1n} - l_{1n'} \|^2 + \| m_{1n} - m_{1n'} \|^2 + \| k_{1n} - k_{1n'} \|^2 + \| r_{1n} - r_{1n'} \|^2)] \quad (14)$$

where  $\tilde{y}_n = ([l_{nj}, m_{nj}], [k_{nj}, r_{nj}])$  and  $\tilde{y}_{n'} = ([l_{nj'}, m_{nj'}], [k_{nj'}, r_{nj'}])$  represents the interval type 2 hesitant fuzzy data vector of  $n$ th object  $l_{1n} \equiv (l_{1j1}, \dots, l_{1jn}, \dots, l_{1jN})'$ ,  $m_{1n} \equiv (m_{1j1}, \dots, m_{1jn}, \dots, m_{1jN})'$ ,  $k_{1n} \equiv (k_{1j1}, \dots, k_{1jn}, \dots, k_{1jN})'$ ,  $r_{1n} \equiv (r_{1j1}, \dots, r_{1jn}, \dots, r_{1jN})'$  are the components of intervals and squared Euclidean distance between two interval type 2 hesitant fuzzy number is denoted as  $\| l_{1n} - l_{1n'} \|^2$ ,  $\| m_{1n} - m_{1n'} \|^2$ ,  $\| k_{1n} - k_{1n'} \|^2$ , and  $\| r_{1n} - r_{1n'} \|^2$  respectively.

As mentioned in the text, distance is calculated as the difference between each alternative scores in such a way that lower limits and upper limits are subtracted each other. For instance, if first alternative is compared with second alternative, lower bounds are subtracted to each other and then upper bounds are subtracted. This calculation presents distance between evaluated alternatives.

*Step 11.* Gather degree of divergence values again considering the distances as follows:

$$d'_{ij} = \sum_{i=1}^n d_{ij}(\tilde{y}_n, \tilde{y}_{n'}) \quad \text{that } i \neq j \quad (15)$$

and

$$t_{ij} = 1 - \frac{d_{ij}}{\max d_{ij}} \quad (16)$$

*Step 12.* Determine  $\lambda_i$  by dividing the variance of a specific alternative to the total variance of alternatives based on the degree of divergence given in the previous step.

$$\lambda_i = \frac{\sigma_i^2(t_{ij})}{\sum_{l=1}^n \sigma_l^2(t_{ij})} \quad \text{and} \quad 1 - \lambda_i = \lambda_{i'}. \quad (17)$$

where  $t_{ij}$  includes the interval type 2 hesitant fuzzy number components with respect to  $l_{nj}, m_{nj}, k_{nj}$  and  $r_{nj}$ .

*Step 13.* Combine the measures of WS ( $Q_i^1$ ) and WP ( $Q_i^2$ ) for each alternative:

$$Q_i^1 = \sum_{j=1}^m w_j \bar{h}_{ij} \quad (18)$$

and

$$Q_i^2 = \prod_{j=1}^m (\bar{h}_{ij})^{w_j} \quad (19)$$

*Step 14.* Compute the aggregate measure of the WASPAS method for each alternative as follows:

$$Q_i = \lambda_i Q_i^1 + (\lambda_{i'}) Q_i^2 \quad (20)$$

where  $\lambda$  in  $[0, 1]$  is the parameter of the WASPAS method. For  $\lambda = 1$ , WASPAS method is transformed into WS, whereas  $\lambda = 0$  leads to WP.

*Step 15.* Calculate the scores  $s(h_i)$  for aggregated  $\tilde{h}_i (i = 1, 2, \dots, m)$  using score function of [70] as follows:

$$s(h_i) = \sum_{\tilde{h} \in \tilde{h}} \text{score} \left[ \frac{a_i^u + a_i^l}{2} + \frac{H_1(\tilde{W}_i^U) + H_1(\tilde{W}_i^L) + H_2(\tilde{W}_i^U) + H_2(\tilde{W}_i^L)}{4} \right] \times \frac{a_i^u + a_i^l + a_i^u + a_i^l + a_i^l + a_i^u + a_i^l + a_i^u}{8} \quad (21)$$

where  $s(h_i)$  is a crisp score and  $h_i$  is the number of IT2HF element.  $W$  denoted as upper and lower bound values of WASPAS aggregated preferences gathered in *Step 14*.

*Step 16.* Rank the alternatives with respect to the decreasing values of scores.

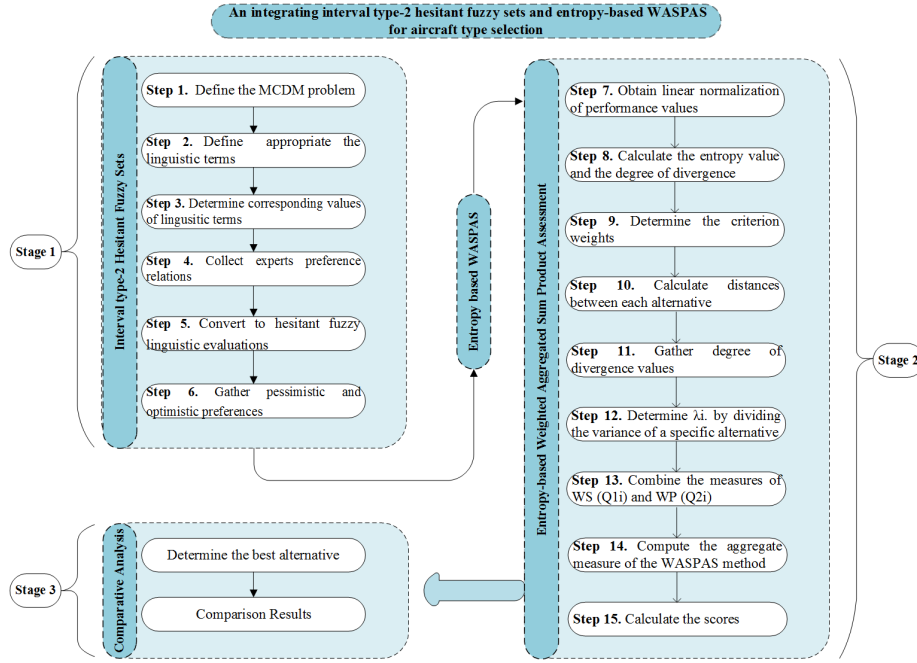


Figure 1: The flowchart diagram of the proposed approach.

## 5. Case Study

Aircrafts are selected for the routes from the current fleet based on utilization targets, range, expected demand, market trends, and cost of the operation. This process is conducted by mostly network planning and medium term planning departments. These studies should be carried out approximately 6-8 months before the scheduling seasons on short haul routes. While network and schedule planners try to find optimal utilization of the sources, namely aircrafts, they need to match the demand of market with the supply of capacity. In order to find optimal mix, they re-schedule the aircrafts, re-assign to the routes based on cost, demand and revenue potential.

Three expert decision makers (DMs) from an airline company have been selected, each from the planning and scheduling, revenue management and sales departments, respectively. A survey based on questionnaires are conducted with the decision makers. Related criteria are determined by the airline route experts.

The aircraft type selection problem considering the specific route of Istanbul-Kuwait is investigated, which is illustrated in Fig. 3. There are five *main* criteria and ten *sub*-criteria identified and synthesized for seven types of narrow body alternative aircrafts that could be selected from. Each criterion is explained in Section 3 and shown as in the Fig. 2.

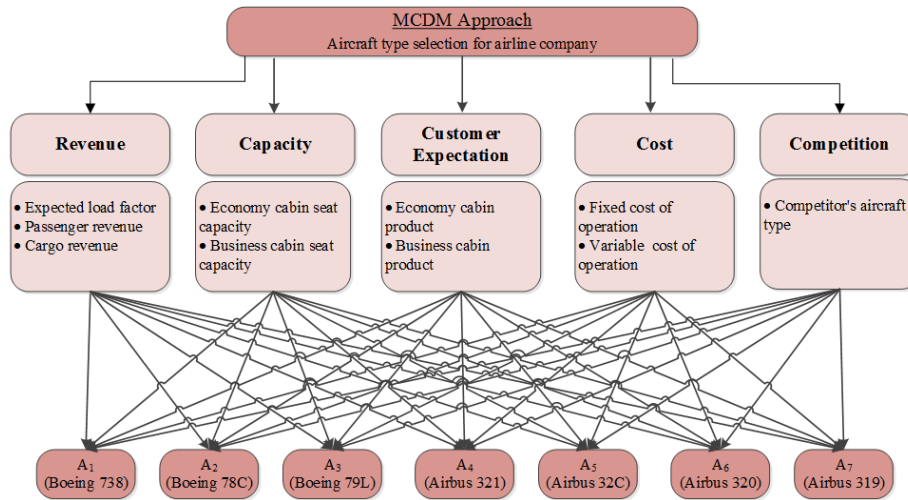


Figure 2: The decision hierarchy of the route analysis problem.



Figure 3: Map of Middle East region showing the route in between Kuwait and Istanbul airports.

Since the route is short haul, all of the aircraft types are narrow body aircrafts. Three of the aircraft types has separate business class cabin. Capacity of the aircrafts are between 126 and 180. Besides that, competition between airlines is very critical. There are 53 weekly flights between Istanbul-Kuwait. The comparison of weekly frequency between *Istanbul (IST)* and *Kuwait (KWI)* is summarized in Table 3. The alternative aircraft types and features are presented in Table 4. Pricing policy is also another important subject however it is not considered due to its unique nature to each airline and confidentiality.

Table 3: August 2017 weekly frequency between Istanbul (IST) and Kuwait (KWI) as of March 2017.

Airline	Weekly Frequency
Turkish Airlines	28
Kuwait Airways	12
Atlasglobal	7
Jazeera Airways	6

Table 4: Alternative aircraft types and features (5: Best match and 1: Worst match).

Alternative	Economy Seat Capacity	Business Seat Capacity	Fixed Business Cabin	Description	Competitors Product Match
A1 Boeing 738	12	147	No	Old, Business cabin is separated by a curtain	2
A2 Boeing 78C	16	135	Yes	Separate Business Cabin Class, IFE exist	5
A3 Boeing 79L	16	135	Yes	Separate Business Cabin Class, IFE exist, Extended Range version of 738	4
A4 Airbus 321	12	168	No	Old, Business cabin is separated by a curtain	2
A5 Airbus 32C	20	156	Yes	Separate Business Cabin Class, IFE exist	5
A6 Airbus 320	12	147	No	Old, Business cabin is separated by a curtain	2
A7 Airbus 319	12	114	No	Old, Business cabin is separated by a curtain	1

### 5.1. Experimental Results

We propose the following method for computing and identifying the best aircraft alternative for a given route, in particular, Kuwait-Istanbul.

*Step 1.* The main criteria and sub criteria are taken from the airline route experts as presented in Fig. 2. Seven alternative aircraft are evaluated for aircraft type selection on the short haul route between Turkey - Middle East.

*Step 2 and Step 3.* Define linguistic term set and semantics according to Table 5.

Table 5: Hesitant linguistic term set [13]

Label	Linguistic terms	Corresponding IT2HFNs
AL	Absolutely low	(0.0, 0.0, 0.0, 0.0;1,1) (0.0, 0.0, 0.0, 0.0;1,1)
VL	Very low	(0.0075, 0.0075, 0.015, 0.0525; 0.8,0.8),(0,0,0.02,0.07; 1.0,1.0)
L	Low	(0.0875, 0.12, 0.16, 0.1825;0.8,0.8),(0.04,0.10,0.18,0.23;1.0,1.0)
ML	Slightly low	(0.2325, 0.255, 0.325, 0.3575;0.8,0.8),(0.17,0.22,0.36,0.42;1.0,1.0)
M	Middle	(0.4025, 0.4525, 0.5375, 0.5675;0.8,0.8),(0.32,0.41,0.58,0.65;1.0,1.0)
MH	Slightly high	(0.65, 0.6725, 0.7575, 0.79;0.8,0.8),(0.58,0.63,0.80,0.86;1.0,1.0)
H	High	(0.7825, 0.815, 0.885, 0.9075;0.8,0.8),(0.72,0.78,0.92,0.97;1.0,1.0)
VH	Very high	(0.9475, 0.985, 0.9925, 0.9925;0.8,0.8),(0.93,0.98,1.0,1.0;1.0,1.0)
AH	Absolutely high	(1.0, 1.0, 1.0, 1.0;1.0,1.0), (1.0, 1.0, 1.0, 1.0;1.0,1.0)

*Step 4.* The alternative in terms of criteria evaluated by experts are collected. The collected preferences for major criteria are given in Table 6.



Table 6: Pairwise comparison matrix according to three linguistic evaluation categories.

Expert 1	Revenue	Capacity	Customer Expectation	Cost	Competition
Revenue	-	[H, AH]	[M, VH]	[H, AH]	[H, AH]
Capacity	[AL, L]	-	[AL, ML]	[ML, MH]	[ML, MH]
Customer Expectation	[VL, M]	[MH, AH]	-	[MH, H]	[M, MH]
Cost	[AL, L]	[ML, MH]	[L, ML]	-	[VL, ML]
Competition	[AL, L]	[ML, MH]	[ML, M]	[MH, VH]	-
Expert 2	Revenue	Capacity	Customer Expectation	Cost	Competition
Revenue	-	[MH, VH]	[ML, MH]	[H, AH]	[MH, VH]
Capacity	[VL, ML]	-	[AL, L]	[ML, MH]	[ML, M]
Customer Expectation	[ML, MH]	[H, AH]	-	[MH, H]	[M, H]
Cost	[AL, L]	[ML, MH]	[L, ML]	-	[AL, ML]
Competition	[VL, ML]	[M, MH]	[L, M]	[MH, AH]	-
Expert 3	Revenue	Capacity	Customer Expectation	Cost	Competition
Revenue	-	[MH, AH]	[M, VH]	[H, AH]	[MH, H]
Capacity	[AL, ML]	-	[VL, L]	[ML, MH]	[ML, M]
Customer Expectation	[VL, M]	[H, VH]	-	[H, VH]	[M, MH]
Cost	[AL, L]	[ML, MH]	[VL, L]	-	[VL, ML]
Competition	[L, ML]	[M, MH]	[ML, M]	[MH, VH]	-

*Step 5.* Gather numerical representations of pairwise comparison matrices. After pairwise linguistic evaluations are obtained, evaluations should be transformed to numeric intervals according to the corresponding interval type-2 hesitant fuzzy linguistic term set (IT2HLTS) appeared in Table 5. For instance, the pairwise comparison of the evaluations for “Revenue” and “Capacity” is defined as [H, AH], [MH, VH] and [MH, AH]. For [H, AH], the corresponding IT2HFS is given as  $(0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8)$ ,  $(0.72, 0.78, 0.92, 0.97; 1.0, 1.0)$ ;  $(1.0, 1.0, 1.0, 1.0; 1.0, 1.0)$ ,  $(1.0, 1.0, 1.0, 1.0; 1.0, 1.0)$  and [MH, VH] is determined as  $(0.65, 0.6725, 0.7575, 0.79; 0.8, 0.8)$ ,  $(0.58, 0.63, 0.80, 0.86; 1.0, 1.0)$ ;  $(0.9475, 0.985, 0.9925, 0.9925; 0.8, 0.8)$ ,  $(0.93, 0.98, 1.0, 1.0; 1.0, 1.0)$ , and similarly, for [MH, AH] is determined as  $(0.65, 0.6725, 0.7575, 0.79; 0.8, 0.8)$ ,  $(0.58, 0.63, 0.80, 0.86; 1.0, 1.0)$ ;  $(1.0, 1.0, 1.0, 1.0; 1.0, 1.0)$ ,  $(1.0, 1.0, 1.0, 1.0; 1.0, 1.0)$ .

*Step 6.* Define pessimistic and optimistic preferences of decision makers for each alternative, sub criteria and main criteria.

The numerical representations of aggregated preference are determined. For aggregating three decision makers opinion as [H, AH], [MH, VH] and [MH, AH], pessimistic value is determined as H and optimistic value can be implied as AH. This presentation overcomes information loss. Here, corresponding numerical representations are illustrated as in Table 5.

*Step 7.* This step implies alternative fuzzy normalized values for each alternative. Here we divide preference value of  $A_1$  and  $A_2$  to maximum value of related lower or upper bound.  $A_1$  has values as 0.0038, 0.0038, 0.0075, 0.0266 for the upper bound and 0.5675, 0.5375, 0.4525, 0.4025 as the maximum value

for considering all alternatives as the upper bound. The values are taken from experts to consider alternative differences for each sub criterion and calculated. Normalization process is adapted by dividing these values to maximum value of all alternatives. The sub criterion calculations are not provided in the text but the expression given below is written for giving the main perspective to the reader.

Fuzzy preference values considering sub criteria for each alternative are also utilized in fuzzy normalization process. Repeating *Step 9* to all sub criteria to the alternative evaluations provides better understanding of indicating overall importance of related criterion. After *Step 9* is repeated, fuzzy normalization process is adapted of alternative  $A_1$  (*Boeing 738*) with respect to  $A_2$  (*Boeing 78C*) for criterion  $C_1$  (Expected load factor) is given by (benefit criteria) as an example:

$$\bar{h}_{12} = \left( \frac{0.0038}{0.5675}, \frac{0.0038}{0.5375}, \frac{0.0075}{0.4525}, \frac{0.0266}{0.4025}; 0.8, 0.8 \right), \left( \frac{0}{0.65}, \frac{0}{0.58}, \frac{0.01}{0.41}, \frac{0.04}{0.32}; 1, 1 \right).$$

$$\bar{h}_{12}^L \cong (0.0066, 0.0070, 0.0166, 0.0661; 0.8, 0.8), (0, 0, 0.0245, 0.1114; 1, 1)$$

as normalized lower limit and  $(0.0852, 0.1218, 0.1997, 0.2979; 0.8, 0.8)$   $(0.0311, 0.0885, 0.2526, 0.4805; 1, 1)$  as normalized upper bound for normalization process. Note that normalization process is adapted as dividing max values of relevant preferences.

The classical WASPAS method uses a linear normalization method in its process. When we have zero elements in the IT2FSs of the decision matrix, this method of normalization can lead to some normalized values with infinite elements related to the non-beneficial criteria. To overcome this issue, a modification has been performed in the normalization process by dividing maximum and minimum values of the component.

*Steps 8-9.* Calculate the entropy value ( $e_j$ ), degree of divergence ( $k_j$ ) for normalized aggregated preference values and  $w_j$  weight for criterion “*Revenue*” in main criteria evaluations as follows:

$$e_1 = \sum_{i=1, p_j \neq 0}^n p_{.j} \ln p_{.j} = 0.0038 \ln 0.0038 + \dots + 0.049 \ln 0.049 = -0.31$$

$$k_1 = 1 + e_{ij} = 1 - 0.31 = 0.69, k_{12} = -0.14, k_{13} = 0.83, k_{14} = -0.0022, \dots etc.$$

$$w_1 = \frac{0.69}{1.37} = 0.50$$

Note: IT2HFWA operator is applied for aggregation. For instance, aggregated preference relation for “*Revenue*” with respect to “*Capacity*” is calculated in the following according to the aggregation operator discussed in *Step 9*:

$$\tilde{h} = IT2HFWA(\tilde{h}_{i1}, \tilde{h}_{i2}, \dots, \tilde{h}_{in}) = \sum_{j=1}^n w_j \tilde{h}_{ij} \quad (i = 1, 2 \dots m)$$

$$= \frac{1}{3} [(0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8)(0.72, 0.78, 0.92, 0.97; 1, 1),$$

$$\begin{aligned}
& (1.0, 1.0, 1.0, 1.0; 1.0, 1.0), (1.0, 1.0, 1.0, 1.0; 1.0, 1.0)) \\
& \oplus \frac{1}{3} [(0.65, 0.6725, 0.7575, 0.79; 0.8, 0.8)(0.58, 0.63, 0.80, 0.86; 1, 1), \\
& \quad (0.9475, 0.985, 0.9925, 0.9925; 0.8, 0.8)(0.93, 0.98, 1.0, 1.0; 1, 1)] \\
& \oplus \frac{1}{3} [(0.65, 0.6725, 0.7575, 0.79; 0.8, 0.8)(0.58, 0.63, 0.80, 0.86; 1, 1), \\
& \quad (0.9475, 0.985, 0.993, 0.993; 0.8, 0.8)(0.93, 0.98, 1.0, 1.0; 1, 1)] \\
& = [(0.69, 0.72, 0.8, 0.83; 0.8, 0.8)(0.63, 0.68, 0.84, 0.9; 1, 1), \\
& \quad (0.98, 1.0, 1.0, 1.0; 0.8, 0.8)(0.98, 0.99, 1.0, 1.0; 1.0, 1.0)].
\end{aligned}$$

Then, this process is repeated for the sub-criteria.

*Steps 10-12.* Calculate distance between each alternative before the identification of  $\lambda_i$  for the "Revenue". The distance measure is adapted to Capacity - Revenue comparison as follows.

$$\begin{aligned}
d'_{12} &= [(\|l_{12} - l_{12'}\|^2 + \|m_{12} - m_{12'}\|^2 + \|k_{12} - k_{12'}\|^2 + \|r_{12} - r_{12'}\|^2)] \\
&= [|0.701 - 0.037| + \dots + |0.841 - 0.095|] = 2.97
\end{aligned}$$

Other components are calculated as (2.97, 2.27, 1.55, 1.38; 0.8, 0.8) and for upper bound (3.02, 2.39, 1.70, 1.22; 1, 1).

$$t_{12} = 1 - \frac{d_{ij}}{\max d_{ij}} = (0.99, 0.98, 1.00, 1.00; 0.8, 0.8)(1, 00, 1.00, 1.00; 1, 1)$$

$$\lambda_1 = \frac{\sigma^2(t_{ij})}{\sum \sigma^2(t_{ij})} = \frac{Var(0.99; \dots; 1)}{Var(0.99; \dots; 1) + \dots + Var(0.61; \dots; 0.76)} = 0.52$$

and  $\lambda_2 = 1 - \lambda_1 = 0.48$  respectively.

*Steps 13-15.* Calculate the values of  $Q_i^{(1)}$  and  $Q_i^{(2)}$  and aggregate the individual values of  $Q_1^{(1)}$  and  $Q_1^{(2)}$  for the "Revenue" criterion as follows:

$$\begin{aligned}
Q_i^{(1)} &= \sum_{j=1}^m w_1 \bar{h}_{1j} = 0.50 \sum_{j=1}^m \bar{h}_{ij} \\
&= \begin{bmatrix} (0, 0, 0, 0; 0.8, 0.8) & \dots & (0.13, 0.15, 0, 24, 0.28; 1, 1) \\ \dots & \dots & \dots \\ (0.054, 0.065, 0.097, 0.311; 0.8, 0.8) & \dots & (0, 0, 0, 0; 1, 1) \end{bmatrix} \\
Q_1 &= 0.52Q_i^{(1)} + 0.48Q_i^{(2)} \\
&= \begin{bmatrix} (0, 0, 0, 0; 0.8, 0.8) & \dots & (0.56, 0.60, 0, 76, 0.83; 1, 1) \\ \dots & \dots & \dots \\ (0.36, 0.39, 0.47, 0.89; 0.8, 0.8) & \dots & (0, 0, 0, 0; 1, 1) \end{bmatrix}
\end{aligned}$$

Then, the scores of pairwise comparison matrix are calculated. For example, score function results for the “*Revenue*” with respect to “*Capacity*” are calculated as in the following:

$$\begin{aligned}
score(h_{12}) &= \frac{1}{\#\bar{h}} \sum_{\bar{A} \in \bar{h}} score(Revenue \tilde{to} Capacity) \\
&= \frac{1}{\#\bar{h}} \sum_{\bar{A} \in \bar{h}} score \left[ \frac{(0.56 + 0.62)}{2} + \frac{(0.8 + 0.8 + 1 + 1)}{4} \right] \\
&\times \frac{(0.56 + 0.57 + 0.60 + 0.62 + 0.53 + 0.55 + 0.62 + 0.64)}{8} = 1.11
\end{aligned}$$

*Step 16.* Normalization of final weights effected from pairwise comparisons are conducted with the reflection of the distances to the finalized criteria weights. This provide the consideration of both pairwise comparisons and differences between the evaluations of the criteria and alternatives in the same baseline. Rankings of the criteria are given as “*Revenue*” with the weight of 0.17, “*Capacity*” with 0.24, “*Customer expectation*” 0.10, “*Cost*” with 0.26 and “*Competition*” with 0.23. Defuzzified values of each alternative is presented according to each criterion and sub criterion in 7. The score value of alternatives is also given in Table 7.

The steps are followed considering all criteria for each alternative The weights are determined as shown in Table 7. According to the interval type-2 hesitant fuzzy WASPAS based on entropy solution, *Airbus 32C* is the best alternative among the seven alternatives because it has the largest weights, while *Airbus 320* is the worst alternative.

## 5.2. Comparison Results

In this section, we report the results from a comparative study considering the existing decision making methods to demonstrate the validity of our methodology. For this reason, IT2HFS based Multi-Attributive Border Approximation area Comparison (MABAC), additive ratio assessment (ARAS), Complex Proportional Assessment (COPRAS) and TODIM (an acronym in Portuguese for iterative multi-criteria decision-making) methods are selected to facilitate the comparison to the interval type-2 hesitant fuzzy decision-making method. The IT2HFS based results are transformed to crisp values by using a scoring function when all the computational process is complete. The final rankings are given in Table 8.

As seen from Table 8, the results from the four methods appear to be very similar except MABAC. The ranking result obtained using the method in this paper is stated as *Airbus32C* > *Boeing78C* > *Boeing79L* > *Airbus321* > *Boeing738* > *Airbus319* > *Airbus320*. Regarding the alternatives *Airbus 32C*, *Boeing 79L*, *Boeing 78C*, *Airbus 319* and *Airbus 320*, ranking results appeared almost in the same order. On the other hand, the ranking results differ for

Table 7: Defuzzified values of alternatives and main and sub criteria.

Main and sub-criteria	Criteria weight	Sub-criteria weight	Global sub-criteria weight	Evaluation of alternative								
				Boeing 738	Boeing 78C	Boeing 79L	Airbus 321	Airbus 32C	Airbus 320	Airbus 319		
<b>Revenue</b>	0.166											
<i>Expected Load Factor</i>		0.288	0.048	0.122	0.064	0.085	0.072	0.090	0.433	0.134		
<i>Passenger Revenue</i>		0.451	0.075	0.118	0.063	0.052	0.092	0.086	0.411	0.133		
<i>Cargo Revenue</i>		0.260	0.043	0.139	0.058	0.045	0.095	0.089	0.426	0.138		
<b>Capacity</b>	0.238											
<i>Economy Cabin Seat</i>		0.414	0.099	0.068	0.091	0.116	0.082	0.079	0.436	0.128		
<i>Business Cabin Seat</i>		0.586	0.140	0.089	0.108	0.126	0.123	0.099	0.545	0.157		
<b>Customer Expectation</b>	0.101											
<i>Economy Cabin Product</i>		0.474	0.048	0.130	0.054	0.054	0.098	0.089	0.425	0.149		
<i>Business Cabin Product</i>		0.526	0.053	0.037	0.017	0.013	0.030	0.027	0.125	0.051		
<b>Cost</b>	0.261											
<i>Fixed Cost</i>		0.425	0.111	0.075	0.097	0.123	0.033	0.101	0.429	0.141		
<i>Variable Cost</i>		0.575	0.150	0.074	0.098	0.080	0.040	0.079	0.373	0.115		
<b>Competition</b>	0.233											
<i>Competitors's Aircraft Type</i>		1.000	0.233	0.149	0.030	0.089	0.068	0.096	0.431	0.137		
<b>Total score</b>				0.112	0.181	0.158	0.178	0.253	0.056	0.087		

Table 8: Ranking of alternatives using MABAC, TODIM, COPRAS, ARAS and the proposed entropy-based WASPAS method.

Methods	Alternatives							Final rankings
	Boeing 738 ( $A_1$ )	Boeing 78C ( $A_2$ )	Boeing 79L ( $A_3$ )	Airbus 321 ( $A_4$ )	Airbus 32C ( $A_5$ )	Airbus 320 ( $A_6$ )	Airbus 319 ( $A_7$ )	
MABAC	0.3204	0.3038	0.2771	0.2569	0.3472	0.2499	0.4311	$A_7 > A_2 > A_5 > A_1 > A_3 > A_4 > A_6$
TODIM	0.5182	0.5520	0.5299	0.5281	0.5605	0.5230	0.5200	$A_5 > A_2 > A_4 > A_1 > A_6 > A_7 > A_3$
COPRAS	0.0436	0.0175	0.0345	0.3660	0.5128	0.0200	0.0055	$A_5 > A_1 > A_1 > A_3 > A_6 > A_7 > A_4$
ARAS	0.2651	0.2747	0.2918	0.2958	0.3061	0.2112	0.2277	$A_5 > A_1 > A_3 > A_2 > A_1 > A_7 > A_6$
Entropy-based WASPAS	0.1120	0.1814	0.1383	0.1777	0.2528	0.0560	0.0870	$A_5 > A_2 > A_4 > A_3 > A_1 > A_7 > A_6$

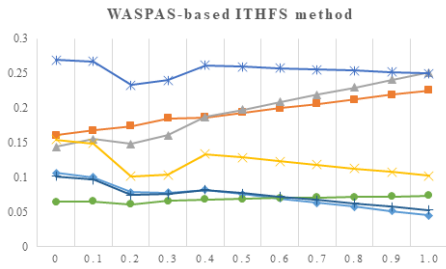
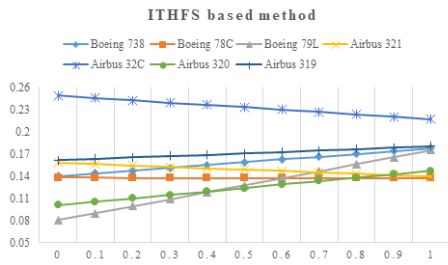
Boeing 738 and Airbus 321 when considering MABAC results. The main reason for this inconsistency mainly lie in the following two characteristics of the MABAC methodology: (i) scores are calculated just after normalization process, not in the final step and this situation can cause information loss, (ii) maximum distance measure and entropy measure for better representation of alternatives are implemented. In addition to that, COPRAS and ARAS results indicate the most preferred alternatives but they do not differentiate the other alternatives. According to the TODIM results, related scores appear close to each other which prevents the distinction of the alternatives. In other words, out-ranked alternatives can be identified, but it might not be possible to distinguish the priorities of certain alternatives. On the other hand, the proposed approach obtains a distinctive ranking of the alternatives with proper scores, which has been validated and confirmed by the experts from the airline company.

### 5.3. Sensitivity Analysis

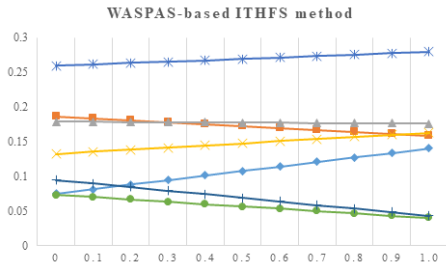
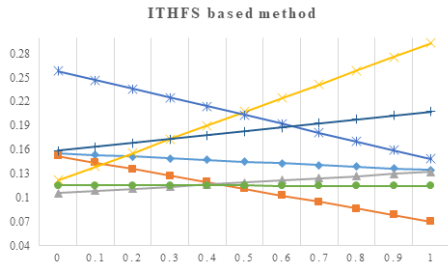
In this section, the results of the proposed approach are validated by comparing the results gathered from interval type 2 hesitant fuzzy (IT2HF) decision making approach which is conducted by Hu et al. [13]'s paper. For this reason, one at a time sensitivity analysis is applied to observe the effects of the changes of criteria weights. In other words, possible changes in terms of ranking the alternatives are determined for main criteria by changing the related criterion from 0.1 to 1 while other criteria weights are fixed in one at a time sensitivity analysis. For instance, if *Revenue* changes from the remaining value to 0.2, then *Capacity* should be updated by retaining the importance level using the following calculation  $\frac{0.1772}{(1-0.318)} \times (1 - 0.2) = 0.2081$ . The similar calculations are performed for the other criteria. All sensitivity analysis results are presented in Fig. 4.

According to the criterion, *Revenue*, *Airbus 32C* still remains as the favourite alternative. On the other hand, if *Revenue's* criterion weight increases, *Airbus 319*, *Boeing 738* and *Boeing 79L* become as second alternatives. Additionally, *Airbus 321* falls as third and fourth alternative and *Boeing 79L's* alternative score is dramatically increasing while criterion weight increases according to Hu et al. [13]'s IT2HF based decision making procedure. In entropy-based WASPAS decision-making process, *Airbus 32C* still remains the best alternative and the ranking of the alternatives acutely separated with other alternatives.

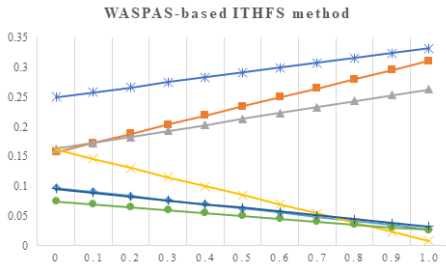
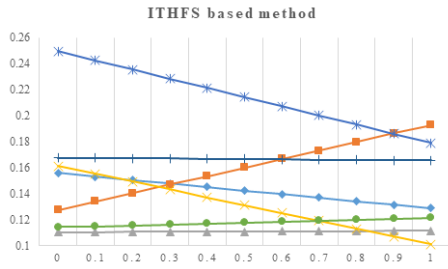
In the similar manner, if the weight of capacity increases, *Airbus 321* becomes the best alternative and the ranking of *Airbus 320* and *Boeing 79L* are aroused the same when in Hu et al. [13]'s IT2HF based decision making process is adopted. On the other hand, *Airbus 320* and *Boeing 79L* are separated clearly.



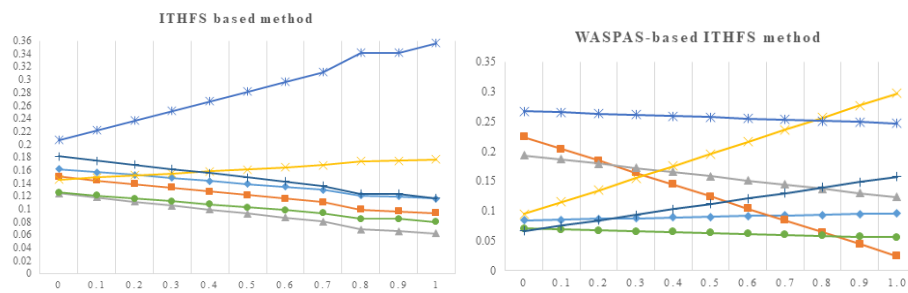
(a) Revenue



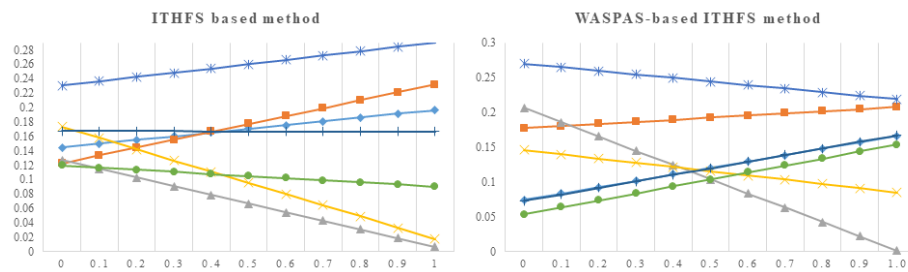
(b) Capacity



(c) Customer expectation



(d) Cost



(e) Competition

Figure 4: One at a time sensitivity analysis of each main criteria while its weight changes from 0.0 to 1.0 (x-axis) illustrating how its membership value varies (y-axis) considering each fuzzy method, where IT2HFN is used to obtain the plots on left column while entropy-based interval type-2 hesitant fuzzy WASPAS is used on the right column.



As seen from the figure, customer expectation weight changes, *Airbus 32C* fall into disfavor and *Boeing 78C* is selected as the best alternative in Hu et al. [13]’s IT2HF based decision making procedure. In entropy-based WASPAS method, *Airbus 32C* is still the best alternative but *Boeing 78C* is selected as the second best alternative which demonstrates the validity of the proposed methodology.

If cost criterion is considered for the comparison, *Airbus 32C* still appears as the best alternative but in higher criteria weights, *Airbus 321* becomes the best alternative in entropy-based WASPAS method. The similar appearance could be realized in Hu et al. [13]’s IT2HF based decision-making process, which demonstrates the validity of the proposed methodology.

Finally, if competition criterion increases, *Airbus 32C* is selected as the most preferred alternative and *Boeing 78C* appears as second best alternative both for Hu et al. [13]’s IT2HF based decision making method and proposed methodology.

In conclusion, if criteria weights vary between 0 and 1, same alternative *Airbus 32C* appears as the best alternative which validates the proposed decision making methodology according to IT2HF based decision making process mentioned in Hu et al. [13]’s paper. Additionally, normalization process adopted in entropy-based WASPAS integrated IT2HF methodology assists the evaluation process by eliminating the differences caused by scalability.

In the second part, the differences caused by  $\theta$  value is detected for different criteria weights. Four values of  $\theta$  (0, 0.2, 0.5 and 0.8) and eleven values of  $\beta$  from 0 to 1 are chosen for this analysis. Varying the values of  $\lambda$  enables us to evaluate the sensitivity of the proposed methodology by relocating the weighted sum model to weighted product model. Furthermore, varying the values of  $\beta$  (x-axis of right column values) is performed to analyze the sensitivity of the proposed methodology according to the quality of criteria weights. Note that if one of the criteria has the highest weight in each set and the other criteria have lower weights than it appears as the pattern of alternatives as discussed in the comparison part of this section. Using this pattern can help us to consider a wide extent of criteria weights for evaluating the sensitivity of the proposed method to changing the weights of criteria.

According to this table, all values of alternative scores generally remain stable while  $\theta$  value changes. Therefore, it can be indicated that the proposed approach has good stability with different criteria weights and different values of parameters. Also, one could be said that using more subjective weights (increasing the value of  $\beta$ ) can lead to increasing the sensitivity of the proposed methodology. According to this sensitivity analysis, it could be stated that using a combination of the weighted sum model to weighted product model could increase the stability of the proposed entropy-based interval type-2 hesitant fuzzy WASPAS approach.

## 6. Conclusion and Discussion

Interval type 2 hesitant fuzzy numbers (IT2HFNs) are favoured over other types of fuzzy sets due to the capability to reflect a pessimistic and optimistic membership with intervals. This fuzzy number form provides well representation of imprecise information or uncertainty in decision environment.

Aircraft type selection process matches the different aircraft types to the flights based on demand, ranges, capacity, aircraft availability, costs and revenues. It has high impacts on route revenues and route operational costs. Thus, application of the IT2HFS based methodology is illustrated by a case study of aircraft type selection case. Also, the same example is solved for the validation of the proposed entropy-based WASPAS-IT2HFs by comparing the results with simple IT2HFs based decision making procedure proposed by [13]. Additionally, validation of the proposed method is provided by a comparative study. From all of these studies, the integration of entropy based WASPAS in decision-making case demonstrates a good consistency and practical for real life applications.

The general conclusions from this study are:

- The comparative example presented above has demonstrated the applicability and effectiveness of the proposed approach for selecting the most suitable alternative among others.
- Sensitivity analysis and comparative studies indicate the robustness and validity of the proposed methodology.
- The impreciseness of decision-making process can be presented using IT2HFSs. Furthermore, both subjective reflections and objective data can be evaluated by the proposed approach.
- Considering entropy-based WASPAS-IT2HFs methodology, more explicable and decoupled ranking results of alternative aircrafts can be extracted, which facilitates the selection of aircraft type for marketing considerations including route characteristics and aircrafts types.
- Finally, *Airbus 32C* is the best alternative among other aircrafts. *Airbus 32C* aircraft type has fixed cabin class configuration, In-flight entertainment (IFE) systems and high match for competitors' product. The selected aircraft type is consistent with the expectations of a typical potential gulf country passenger. It has high standards compared to the alternatives.

The main reasons for using IT2HFSs and contribution of the proposed methodology are summarized as follows:

1. Exact membership degrees cannot be defined for some real life cases especially when subjective evaluations are gathered. Interval type-2 hesitant fuzzy set based decision making can facilitate decision making procedure using primary and secondary memberships.

2. In some instances, it is not easy to proceed with hesitant fuzzy set based formed data. In our case, criterion 1 is slightly stronger than criterion 2 can be shown using Type-2 fuzzy sets as [71] mentioned in their paper. For this reason, interval type 2 based hesitant notation provides better explanations of linguistic terms by considering optimistic and pessimistic perspective.
3. Therefore, computational easiness, a clear logic of integrated weighted sum average and weighted product average, and balance with these methods can be pointed out as the main advantages of the proposed methodology.
4. Entropy based WASPAS-IT2HFSs methodology provides disjunction of ranking results compared with different MCDM methods by adapting de-fuzzification process at the end of the decision making process.

For future improvements, the same study should be applied to domestic flights in order to compare the regional results. Different fuzzy decision making techniques such as interval valued intuitionistic fuzzy sets can be adapted for improving the proposed methodology and also, the results can be compared with the results that are found in this paper. Besides these extensions, For future research, extending IT2HFSs based WASPAS model by adding other characteristic aggregation operators and Choquet integral can be conducted. In addition, the proposed approach in this paper can be utilized for solving other aircraft type selection problems to further show its robustness and efficiency.

### Acknowledgement

The authors would like to thank Prof. Bob John for his valuable comments.

### References

- [1] Tyler T. Iata annual review 2016. 2016.
- [2] Çiftçi ME, Şevkli M. A new hub and spoke system proposal: A case study for turkey's aviation industry. *Journal of Air Transport Management* 2015;47:190–8.
- [3] Deveci M, Demirel NC, Ahmetoglu E. Airline new route selection based on interval type-2 fuzzy mcdm: A case study of new route between turkey-north american region destinations. *Journal of Air Transport Management* 2017;59:83–99.
- [4] Dursun ME, O'Connell JF, Lei Z, Warnock-Smith D. The transformation of a legacy carrier—a case study of turkish airlines. *Journal of air transport management* 2014;40:106–18.
- [5] Warburg V, Bhat C, Adler T. Modeling demographic and unobserved heterogeneity in air passengers' sensitivity to service attributes in itinerary choice. *Transportation Research Record: Journal of the Transportation Research Board* 2006;(1951):7–16.

- [6] Zavadskas E, Turskis Z, Antucheviciene J, Zakarevicius A. Optimization of weighted aggregated sum product assessment. *Elektronika ir elektrotechnika* 2012;122(6):3–6.
- [7] Pamučar D, Čirović G. The selection of transport and handling resources in logistics centers using multi-attributive border approximation area comparison (mabac). *Expert Systems with Applications* 2015;42(6):3016–28.
- [8] Zavadskas EK, Turskis Z. A new additive ratio assessment (aras) method in multicriteria decision-making. *Technological and Economic Development of Economy* 2010;16(2):159–72.
- [9] Gomes L, Lima M. Todim: Basics and application to multicriteria ranking of projects with environmental impacts. *Foundations of Computing and Decision Sciences* 1992;16(4):113–27.
- [10] Zavadskas EK, Kaklauskas A, Sarka V. The new method of multicriteria complex proportional assessment of projects. *Technological and economic development of economy* 1994;1(3):131–9.
- [11] Mendel JM, John RI, Liu F. Interval type-2 fuzzy logic systems made simple. *IEEE transactions on fuzzy systems* 2006;14(6):808–21.
- [12] Mendel JM, John RB. Type-2 fuzzy sets made simple. *IEEE Transactions on fuzzy systems* 2002;10(2):117–27.
- [13] Hu J, Xiao K, Chen X, Liu Y. Interval type-2 hesitant fuzzy set and its application in multi-criteria decision making. *Computers & Industrial Engineering* 2015;87:91–103.
- [14] Oner SC, Oztaysi B. An interval type 2 hesitant fuzzy mcdm approach and a fuzzy c means clustering for retailer clustering. *Soft Computing* 2018;;1–17.
- [15] Ghorabae MK, Zavadskas EK, Amiri M, Esmaeili A. Multi-criteria evaluation of green suppliers using an extended waspas method with interval type-2 fuzzy sets. *Journal of Cleaner Production* 2016;137:213–29.
- [16] Zavadskas EK, Antucheviciene J, Hajiagha SHR, Hashemi SS. Extension of weighted aggregated sum product assessment with interval-valued intuitionistic fuzzy numbers (waspas-ivif). *Applied soft computing* 2014;24:1013–21.
- [17] Zadeh LA. Information and control. *Fuzzy sets* 1965;8(3):338–53.
- [18] Belbag S, Deveci M, Uludag AS. Comparison of two fuzzy multi criteria decision methods for potential airport location selection. In: *ICORES*. 2013, p. 270–6.
- [19] Chang YH, Yeh CH. Evaluating airline competitiveness using multiattribute decision making. *Omega* 2001;29(5):405–15.

- [20] Aydoğan EK. Performance measurement model for turkish aviation firms using the rough-ahp and topsi methods under fuzzy environment. *Expert Systems with Applications* 2011;38(4):3992–8.
- [21] Barros CP, Wanke P. An analysis of african airlines efficiency with two-stage topsi and neural networks. *Journal of Air Transport Management* 2015;44:90–102.
- [22] Wanke P, Barros CP, Chen Z. An analysis of asian airlines efficiency with two-stage topsi and mcmc generalized linear mixed models. *International Journal of Production Economics* 2015;169:110–26.
- [23] Rezaei J, Fahim PB, Tavasszy L. Supplier selection in the airline retail industry using a funnel methodology: Conjunctive screening method and fuzzy ahp. *Expert Systems with Applications* 2014;41(18):8165–79.
- [24] Chiu MC, Hsieh MC. Latent human error analysis and efficient improvement strategies by fuzzy topsi in aviation maintenance tasks. *Applied ergonomics* 2016;54:136–47.
- [25] Sevklı M, Oztekin A, Uysal O, Torlak G, Turkyilmaz A, Delen D. Development of a fuzzy anp based swot analysis for the airline industry in turkey. *Expert systems with Applications* 2012;39(1):14–24.
- [26] Dožić S, Kalić M. An ahp approach to aircraft selection process. *Transportation Research Procedia* 2014;3:165–74.
- [27] Kiracı K, Akan E. Aircraft selection by applying ahp and topsi in interval type-2 fuzzy sets. *Journal of Air Transport Management* 2020;89:101924.
- [28] Dožić S, Kalić M. Three-stage airline fleet planning model. *Journal of air transport management* 2015;46:30–9.
- [29] Liou JJ, Tzeng GH, Tsai CY, Hsu CC. A hybrid anp model in fuzzy environments for strategic alliance partner selection in the airline industry. *Applied Soft Computing* 2011;11(4):3515–24.
- [30] Liou JJ. Developing an integrated model for the selection of strategic alliance partners in the airline industry. *Knowledge-Based Systems* 2012;28:59–67.
- [31] Garg CP. A robust hybrid decision model for evaluation and selection of the strategic alliance partner in the airline industry. *Journal of Air Transport Management* 2016;52:55–66.
- [32] Pandey MM. Evaluating the strategic design parameters of airports in thailand to meet service expectations of low-cost airlines using the fuzzy-based qfd method. *Journal of Air Transport Management* 2020;82:101738.

- [33] Zadeh L. The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences* 1975;8(3):199 – 249. URL: <http://www.sciencedirect.com/science/article/pii/0020025575900365>. doi:[https://doi.org/10.1016/0020-0255\(75\)90036-5](https://doi.org/10.1016/0020-0255(75)90036-5).
- [34] Sambuc R. *Functions  $\phi$ -fous. application a laide au diagnostic en pathologie thyroïdienne*. Ph.D. thesis; Ph. D. thesis, Université de Marseille, France. Google Scholar; 1975.
- [35] Gorzaczany MB. A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems* 1987;21(1):1 – 17. URL: <http://www.sciencedirect.com/science/article/pii/0165011487901485>. doi:[https://doi.org/10.1016/0165-0114\(87\)90148-5](https://doi.org/10.1016/0165-0114(87)90148-5).
- [36] Turksen I. Interval valued fuzzy sets based on normal forms. *Fuzzy Sets and Systems* 1986;20(2):191 – 210. URL: <http://www.sciencedirect.com/science/article/pii/0165011486900771>. doi:[https://doi.org/10.1016/0165-0114\(86\)90077-1](https://doi.org/10.1016/0165-0114(86)90077-1).
- [37] Torra V. Hesitant fuzzy sets. *International Journal of Intelligent Systems* 2010;25(6):529–39. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/int.20418>. doi:10.1002/int.20418. arXiv:<https://onlinelibrary.wiley.com/doi/pdf/10.1002/int.20418>.
- [38] Atanassov KT. Intuitionistic fuzzy sets. *Fuzzy sets and Systems* 1986;20(1):87–96.
- [39] Alcantud JCR, Torra V. Decomposition theorems and extension principles for hesitant fuzzy sets. *Information Fusion* 2018;41:48–56.
- [40] Chen N, Xu Z. Hesitant fuzzy electre ii approach: a new way to handle multi-criteria decision making problems. *Information Sciences* 2015;292:175–97.
- [41] Peng J, He H, Xiong R. Study on energy management strategies for series-parallel plug-in hybrid electric buses. *Energy Procedia* 2015;75:1926–31.
- [42] Liao HC, Yang LY, Xu ZS. Two new approaches based on electre ii to solve the multiple criteria decision making problems with hesitant fuzzy linguistic term sets. *Applied Soft Computing* 2018;63:223–34.
- [43] Ren Z, Xu Z, Wang H. Dual hesitant fuzzy vikor method for multi-criteria group decision making based on fuzzy measure and new comparison method. *Information Sciences* 2017;388:1–16.
- [44] Dong Jy, Yuan Ff, Wan Sp. Extended vikor method for multiple criteria decision-making with linguistic hesitant fuzzy information. *Computers & Industrial Engineering* 2017;112:305–19.

- [45] Gou X, Xu Z, Liao H. Hesitant fuzzy linguistic entropy and cross-entropy measures and alternative queuing method for multiple criteria decision making. *Information Sciences* 2017a;388:225–46.
- [46] Gou X, Liao H, Xu Z, Herrera F. Double hierarchy hesitant fuzzy linguistic term set and multimooora method: A case of study to evaluate the implementation status of haze controlling measures. *Information Fusion* 2017b;38:22–34.
- [47] Lan J, Jin R, Zheng Z, Hu M. Priority degrees for hesitant fuzzy sets: Application to multiple attribute decision making. *Operations Research Perspectives* 2017;4:67–73.
- [48] Liang D, Xu Z. The new extension of topsis method for multiple criteria decision making with hesitant pythagorean fuzzy sets. *Applied Soft Computing* 2017;60:167–79.
- [49] Liao H, Xu Z. Approaches to manage hesitant fuzzy linguistic information based on the cosine distance and similarity measures for hftss and their application in qualitative decision making. *Expert Systems with Applications* 2015;42(12):5328–36.
- [50] Liao H, Xu Z, Zeng XJ, Merigó JM. Qualitative decision making with correlation coefficients of hesitant fuzzy linguistic term sets. *Knowledge-Based Systems* 2015;76:127–38.
- [51] Narayanamoorthy S, Ramya L, Baleanu D, Kureethara JV, Annapoorani V. Application of normal wiggly dual hesitant fuzzy sets to site selection for hydrogen underground storage. *International Journal of Hydrogen Energy* 2019;44(54):28874–92.
- [52] Mardani A, Saraji MK, Mishra AR, Rani P. A novel extended approach under hesitant fuzzy sets to design a framework for assessing the key challenges of digital health interventions adoption during the covid-19 outbreak. *Applied Soft Computing* 2020;96:106613.
- [53] Gong JW, Liu HC, You XY, Yin L. An integrated multi-criteria decision making approach with linguistic hesitant fuzzy sets for e-learning website evaluation and selection. *Applied Soft Computing* 2021;:107118.
- [54] Xu K, Gong H. Emergency logistics support capability evaluation model based on triangular fuzzy entropy and choquet integral. *Journal of Industrial and Production Engineering* 2016;33(7):435–42.
- [55] Beiki H, Mohammad Seyedhosseini S, V. Ponkratov V, Olegovna Zekiy A, Ivanov SA. Addressing a sustainable supplier selection and order allocation problem by an integrated approach: a case of automobile manufacturing. *Journal of Industrial and Production Engineering* 2021;:1–15.

- [56] Nenadić D. Ranking dangerous sections of the road using mcdm model. *Decision Making: Applications in Management and Engineering* 2019;2(1):115–31.
- [57] Kushwaha DK, Panchal D, Sachdeva A. Risk analysis of cutting system under intuitionistic fuzzy environment. *Reports in Mechanical Engineering* 2020;1(1):162–73.
- [58] Mihajlović J, Rajković P, Petrović G, Ćirić D. The selection of the logistics distribution center location based on mcdm methodology in southern and eastern region in serbia. *Operational Research in Engineering Sciences: Theory and Applications* 2019;2(2):72–85.
- [59] Ramakrishnan KR, Chakraborty S. A cloud topsis model for green supplier selection. *Facta Universitatis, Series: Mechanical Engineering* 2020;18(3):375–97.
- [60] Torkayesh AE, Zolfani SH, Kahvand M, Khazaelpour P. Landfill location selection for healthcare waste of urban areas using hybrid bwm-grey marcos model based on gis. *Sustainable Cities and Society* 2021;67:102712.
- [61] Torkayesh AE, Malmir B, Asadabadi MR. Sustainable waste disposal technology selection: The stratified best-worst multi-criteria decision-making method. *Waste Management* 2021;122:100–12.
- [62] Park Y, OKelly ME. Fuel burn rates of commercial passenger aircraft: variations by seat configuration and stage distance. *Journal of transport geography* 2014;41:137–47.
- [63] Belobaba PP. Airline pricing theory and practice. *The Global Airline Industry* 2015;:75.
- [64] Clark P. *Buying the big jets*. Farnham: Ashgate 2007;.
- [65] Rodriguez RM, Martinez L, Herrera F. Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems* 2012;20(1):109–19.
- [66] Ayhan MB. Yaz spor okulu seçiminde entegre bir tereddütlü-bulanık ahp ve topsis yaklaşımı. *Sakarya University Journal of Science* 2018;22(2):1–.
- [67] Torra V, Narukawa Y. On hesitant fuzzy sets and decision. In: *Fuzzy systems, 2009. FUZZ-IEEE 2009. IEEE international conference on. IEEE;* 2009, p. 1378–82.
- [68] Karnik N, Mendel J. Introduction to type-2 fuzzy logic systems. In: *Fuzzy Systems Proceedings, 1998. IEEE World Congress on Computational Intelligence.; vol. 2. 1998,* p. 915–20.



- [69] Lee LW, Chen SM. Fuzzy multiple attributes group decision-making based on the extension of topsis method and interval type-2 fuzzy sets. In: Machine Learning and Cybernetics, 2008 International Conference on; vol. 6. IEEE; 2008, p. 3260–5.
- [70] Chen SM, Yang MW, Lee LW, Yang SW. Fuzzy multiple attributes group decision-making based on ranking interval type-2 fuzzy sets. *Expert Systems with Applications* 2012;39(5):5295–308.
- [71] Cevik Onar S, Oztaysi B, Kahraman C. Strategic decision selection using hesitant fuzzy topsis and interval type-2 fuzzy ahp: a case study. *International Journal of Computational intelligence systems* 2014;7(5):1002–21.