

INTRODUCTION TO FUNCTIONAL PROGRAMMING

Take-Home Exam

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The aim of this exam is to write a Haskell program to crack the *Caesar cipher* using simple frequency analysis. Familiarity is assumed with the Haskell standard prelude and lectures 1-7.

Introduction

A *cipher* is a method for encoding a message by replacing each character of the message by another character. One of the simplest examples is the *Caesar cipher*, which is said to have been used for military purposes by Julius Caesar. To encode a message, Caesar simply replaced each letter in the message by the letter three places further down the alphabet, wrapping around at the end of the alphabet. For example, the message

```
haskell is fun
```

would be encoded as

```
kdvnhoo lv ixq
```

More generally, the shift factor of three used by Caesar can be replaced by any natural number between one and twenty-five, thereby giving twenty-five different ways to encode a message. For example, with a shift factor of ten, the original message above would be encoded as

```
rkcuovv sc pex
```

The aim of this exam is to write a Haskell program that can automatically decode such messages, by using letter frequencies to determine the most likely shift factor that was used to encode the message.

Encoding and Decoding

- **Exercise:** Define a function

$$\text{let2nat} \quad :: \quad \text{Char} \rightarrow \text{Int}$$

that converts a lower-case letter in the range 'a' to 'z' into the corresponding natural number in the range 0 to 25. For example:

```
> let2nat 'a'  
0
```

```
> let2nat 'z'  
25
```

- **Exercise:** Define a function

$$\text{nat2let} \quad :: \quad \text{Int} \rightarrow \text{Char}$$

that performs the inverse function to *let2nat*. For example:

```
> nat2let 0  
'a'
```

```
> nat2let 25  
'z'
```

- **Exercise:** Using *let2nat* and *nat2let*, define a function

$$\text{shift} \quad :: \quad \text{Int} \rightarrow \text{Char} \rightarrow \text{Char}$$

that applies a shift factor in the range 0 to 25 to a lower-case letter in the range 'a' to 'z'. Characters outside this range, such as upper-case letters and punctuation, should be returned unshifted. Take care to ensure that your function wraps around at the end of the alphabet. For example:

```
> shift 3 'h'  
'k'
```

```
> shift 3 'z'  
'c'
```

```
> shift 3 'H'  
'H'
```

- **Exercise:** Using *shift*, define a function

$$\text{encode} \quad :: \quad \text{Int} \rightarrow \text{String} \rightarrow \text{String}$$

that encodes a string using a given shift factor. For example:

```
> encode 3 "haskell_is_fun"
"kdvnhoo_lv_ixq"
```

- **Exercise:** Define a function

$$\text{decode} \quad :: \quad \text{Int} \rightarrow \text{String} \rightarrow \text{String}$$

that performs the inverse function to *encode*. For example:

```
> decode 3 "kdvnhoo_lv_ixq"
"haskell_is_fun"
```

Frequency Analysis

In English text, some letters are used more frequently than others. By analysing a large volume of text, one can derive the following table of approximate percentage frequencies of the twenty-six letters of alphabet:

$$\begin{aligned} \text{table} &:: [\text{Float}] \\ \text{table} &= [8.2, 1.5, 2.8, 4.3, 12.7, 2.2, 2.0, 6.1, 7.0, \\ &\quad 0.2, 0.8, 4.0, 2.4, 6.7, 7.5, 1.9, 0.1, 6.0, \\ &\quad 6.3, 9.1, 2.8, 1.0, 2.4, 0.2, 2.0, 0.1] \end{aligned}$$

For example, this table shows that 'e' occurs most often, with a frequency of 12.7 percent, while 'q' and 'z' occur least often, with a frequency of 0.1 percent. You will need to type this table into your Haskell script.

- **Exercise:** Define a function

$$\text{lowers} \quad :: \quad \text{String} \rightarrow \text{Int}$$

calculates the number of lower-case letters in a string. For example:

```
> lowers "haskell_is_fun"
12
```

- **Exercise:** Define a function

$$\text{count} \quad :: \quad \text{Char} \rightarrow \text{String} \rightarrow \text{Int}$$

that calculates the number of a given character in a string. For example:

```
> count 's' "haskell_is_fun"
2
```

- **Exercise:** Define a function

$$\text{percent} \quad :: \quad \text{Int} \rightarrow \text{Int} \rightarrow \text{Float}$$

that calculates the percentage of one integer with respect to another, returning the result as a floating-point number. For example:

```
> percent 2 12
16.6667
```

Note: the library function $\text{fromInt} :: \text{Int} \rightarrow \text{Float}$ converts an integer into the corresponding floating-point number.

- **Exercise:** Using lowers , count and percent , define a function

$$\text{freqs} \quad :: \quad \text{String} \rightarrow [\text{Float}]$$

that returns the list of percentage frequencies of each of the lower-case letters 'a' to 'z' in a string of characters. For example:

```
> freqs "haskell_is_fun"
[8.33333, 0.0, 0.0, 0.0, 8.33333, 8.33333, 0.0, 8.33333,
8.33333, 0.0, 8.33333, 16.6667, 0.0, 8.33333, 0.0, 0.0,
0.0, 0.0, 16.6667, 0.0, 8.33333, 0.0, 0.0, 0.0, 0.0]
```

- **Exercise:** Define a function

$$\text{rotate} \quad :: \quad \text{Int} \rightarrow [a] \rightarrow [a]$$

that rotates a list n places to the left, wrapping around at the start of the list, and assuming n is in the range zero to the length of the list. For example:

```
> rotate 3 "haskell_is_fun"
"kell_is_funhas"
```

- **Exercise:** Define a function

$$\text{chisqr} \quad :: \quad [Float] \rightarrow [Float] \rightarrow Float \mid$$

that calculates the *chi square* statistic for a list of observed frequencies *os* with respect to a list of expected frequencies *es*, which is given by

$$\text{chisqr } os \ es \ = \ \sum_{i=0}^{n-1} \frac{(os_i - es_i)^2}{es_i}$$

where *n* is the length of the two lists *os* and *es*, and xs_i denotes the *i*th element of a list *xs*, counting from zero. For example:

```
> chisqr (freqs "haskell_is_fun") table
202.616
```

- **Exercise:** Define a function

$$\text{position} \quad :: \quad Eq \ a \Rightarrow \ a \rightarrow [a] \rightarrow Int$$

that returns the first position (counting from zero) at which a value occurs in a list, assuming that it occurs at least once. For example:

```
> position 5 [1, 3, 5, 7, 11]
2
```

- **Exercise:** Using the functions defined above, define a function

$$\text{crack} \quad :: \quad String \rightarrow String$$

that attempts to decode a string by first calculating the letter frequencies in the string, then calculating the chi square value of each rotation (in the range zero to twenty-five) of this list with respect to the table of expected frequencies, and finally using the position of the minimum value in this list as the shift factor to decode the original string. For example:

```
> crack (encode 3 "haskell_is_fun")
"haskell_is_fun"
```

Note that this method may not be successful if the message is short or contains an unusual distribution of letters. For example:

```
> crack (encode 3 "graham")  
"nyhoht"
```

```
> crack (encode 3 "the_five_boxing_wizards_jump_quickly")  
"dro_psfolyhsxqgsjkbnc_tewzaesmuvi"
```

For fun, try out your *crack* function on the following example:

```
myxqbkdevkdsyxc yx mywzvodsxq dro ohkw!
```

Further Reading

For a popular account of the Caesar cipher, and many other famous cryptographic methods, I recommend: Simon Singh, *The Code Book: The Secret History of Codes and Code-Breaking*, Fourth Estate, 1999.