

# A hybrid tabu search evaluation of holding point entrance allocation methods for departures at London Heathrow airport

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## 1 Introduction And Problem Description

London Heathrow Airport is an extremely busy two runway airport. In this paper we will consider the departure system at Heathrow and, in particular, the departure runway as it is the main bottleneck for the system.

When aircraft take off, a minimum separation time must be enforced between them. The separation required between each pair of aircraft depends upon the weight classes, speed groups and departure routes. This ensures that wake vortices have time to dissipate, in-flight separations are maintained and congestion is controlled. Some separations may need to be increased in bad weather. The take-off order has a large effect on throughput and delay.

Departing aircraft leave their stands and taxi towards the runway under the control of a Ground Movement Controller. When they approach the end of the runway they enter holding points, within which they are reordered by the Runway Controller to improve the take-off order. The variability in ready times and taxiing means that the timetable or daily schedule for the airport cannot be a helpful guide to ordering aircraft. The high level of demand and the small physical size of Heathrow airport combine to ensure that it is most practical to perform the reordering within the holding points at the ends of the runway rather than on the taxiways. The structure of the holding points, which differs at each runway end, constrains the reordering that can be performed. Some reordering is not physically possible and some is prohibitively difficult or time consuming to perform.

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Some aircraft are assigned a Calculated Time Of Take-off (CTOT), which specifies a fifteen minute timeslot within which the aircraft can take off. Special rules for Heathrow allow some of these to be missed by up to five minutes but it is important to miss as few as possible.

The Runway Controller aims to ensure as many aircraft as possible take off within their CTOT and also to keep the throughput of the runway as high as possible, thus reducing the delay aircraft suffer in the holding point. The aim of our research is to develop the scheduling algorithms that would underpin any decision support system to help the controllers with this difficult task.

Previous academic research has considered the problem of obtaining the best order for take-off from an airport. Trivizas presented a dynamic programming based approach in [10] and van Leeuwen et al. presented a constraint satisfaction model in [9]. Anagnostakis et al. suggested a search tree to solve the problem in [1] and a two-stage heuristic/integer program approach in [2]. The arrivals problem has some similarities to the departure problem and has been more heavily studied, for example in [4], [5] and [6]. The major effect that the holding point structures at specific airports have upon the reordering has not been considered in the previous literature so the relevance of this previous research to the real world situation at airports such as Heathrow is limited.

We presented a solution method for a simplified problem in [3], including the effects of the holding point structures. We have subsequently investigated the effects of also including the CTOT constraints and found that, by giving a decision support system using this solution method the information about the aircraft under the control of the Ground Movement Controller, results predict that substantial improvements in the holding point delay become available. We will now present an improved model for use within a decision support system, incorporating the complex constraints of the real world situation and considering the effects of the uncertainty involved in various aspects. We use our solution method, incorporating the improved model, to evaluate the benefits of providing feedback to the Ground Movement Controllers so they can simplify the reordering task within the holding points. We will present our results and conclusions in the full paper.

## 2 Solution Method

A decision support system for the Runway Controller must be able to find solutions swiftly to be of use in a real time system. Any time spent performing the search will be seen as a lag between the situation changing and the advice changing. There is not time for an exhaustive search and the difficulty caused by the holding point structure ensures that an approach such as branch and bound or dynamic programming is impractical as the feasibility can be changed by the addition or removal of a single aircraft.

Our method is intended for an online decision support system, using only the information available to a real system at the time. At any time, the system has information only about those aircraft currently at the holding point or already taxiing towards it. Some information can only be estimated. The estimation accuracy often increases as the aircraft move through the system. It is imperative that this scheduling does not have an adverse effect upon the scheduling of aircraft that may arrive later so our objective function and heuristics include

criteria to avoid this. This rolling planning horizon approach has been successfully applied to other online problems, for example [7] and [8].

We developed a hybrid metaheuristic approach to address the take-off scheduling problem. A tabu search algorithm is used to find schedules for the aircraft in the system at that time. Each potential schedule is tested using a heuristic and network method to ensure that the reordering can be achieved within the holding point. A mathematical model is used to predict the take-off times for aircraft in a schedule and then calculate the final cost of the schedule. The algorithm used for each search is summarised below:

1. Apply random moves to the current solution to generate 50 candidate solutions. Each move is of one of three types: swap two aircraft, shift a set of aircraft forwards or backwards or randomly reorder a set of aircraft.
2. A cache is maintained with details of the feasibility and cost of previously evaluated solutions. For each candidate solution not in the cache, heuristically assign paths through the holding point then test the feasibility of the reordering using the network model.
3. Evaluate each feasible solution not in the cache using the mathematical model, predicting take off times and determining a cost for the schedule. If the cost is better than the lowest cost schedule found so far then the solution is recorded as the new best solution.
4. A tabu list is maintained with details of the last 10 moves made, recording moved aircraft and where they were moved from. Future moves that reverse the effects of a move on the tabu list will be rejected. If there is one, the non-tabu feasible schedule of lowest cost is adopted as the new current solution for the next iteration.
5. If the algorithm has completed 200 iterations, then return the best solution found, otherwise return to step 1 and perform another iteration.

Our system was evaluated using real, historic data provided by National Air Traffic Services. We employed a simulation which stepped through each test dataset in one minute increments, including the information a real decision support system would have had at that time, with estimation errors to simulate any inaccuracy at that point. One search was executed at each step, with each search completing in less than one second on a 2.4GHz PC. Decisions made in one step could affect future steps. At the end of the simulation, a take-off order had been determined for every aircraft in the dataset. The final, resulting order was then evaluated.

### 3 Formal Model

Let  $n$  be the number of aircraft in the current schedule. Let  $i \in \{1, \dots, n\}$  be an integer to represent an aircraft under consideration. Let  $a_i$  represent the position in the arrival order at the holding point and  $c_i$  be the position in the take-off order for aircraft  $i$ . Let  $h_i$  be the time aircraft  $i$  entered the holding point and  $d_i$  be its real or predicted take-off time. Let  $v_i$ ,  $s_i$  and  $r_i$  be integers representing the weight category, speed group and departure route respectively for aircraft  $i$ . Let the path through the holding point that has been assigned to aircraft  $i$  be

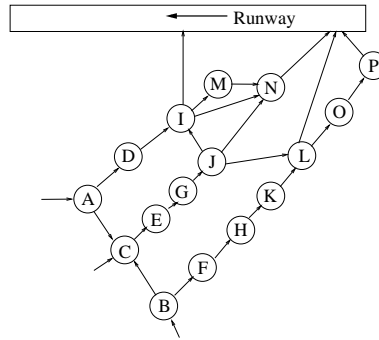


Figure 1: 27R Holding Point Network.

denoted by  $t_i$ . Where aircraft have a CTOT time allocated then let  $b_i$  and  $l_i$  be the earliest and latest times respectively at which aircraft  $i$  can take off while complying with the CTOT.

For any aircraft  $j$  and following aircraft  $i$  we define functions  $V(v_j, t_j, v_i, t_i)$  to determine the required minimum wake vortex separation and  $R(r_j, s_j, r_i, s_i)$  to determine the required separation to ensure compliance with the departure path separation rules. Both  $V(v_j, t_j, v_i, t_i)$  and  $R(r_j, s_j, r_i, s_i)$  are asymmetric and  $R(r_j, s_j, r_i, s_i)$  does not obey the triangle inequality.

The earliest time,  $e_i$ , at which aircraft  $i$  could take off and comply with all of the separation rules can be determined from equation 1. We define a function  $T(t_i)$  to determine the time an aircraft would take to traverse the holding point along path  $t_i$ . At busy times aircraft take off as soon as they can while obeying all separation rules so take-off times for aircraft which have not yet taken off can be predicted using equation 2.

$$e_i = \begin{cases} 0 & \text{if } c_i = 1 \\ \max_{j \in \{1, \dots, n\} | c_j < c_i} (d_j + \max(V(v_j, t_j, v_i, t_i), R(r_j, s_j, r_i, s_i))) & \text{if } c_i \geq 2 \end{cases} \quad (1)$$

$$d_i = \max(e_i, h_i + T(t_i), b_i) \quad (2)$$

The evaluation of the schedule is described in detail in the full paper. It contains four terms which represent delay, non-compliance with CTOT, difficulty of reordering and whether it is likely to be difficult to add future aircraft efficiently.

## 4 Holding Point Model For Testing Feasibility

The feasibility of the reordering depends upon the structure of the holding point. A network model was created for each holding point in order to test the reordering. The nodes represent the possible locations for aircraft and the arcs represent the movement between locations. An example model for the *27R holding point* is given in figure 1. The model deliberately gives a conservative view of feasibility, to ensure that its schedules can be realised in practice.

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The holding point can be considered as one or more entrances from the taxiways, one or more exits to the runway and some manoeuvring space between them. Some paths through the holding point are more complicated to achieve than others. Some take longer to traverse than others. Holding point specific heuristics are used to assign aircraft to paths. These ensure that the longer paths are only given to aircraft that are overtaken and that only good paths are ever used. Once paths have been assigned to all aircraft the aircraft are fed through the network model ensuring that at any time any node can have a maximum of one aircraft.

## 5 The Contribution Of The Ground Movement Controller

As explained above, the control of departing aircraft is split between Ground Movement Controllers and the Runway Controller. We present, above, a solution method that could be used by a decision support system for the Runway Controller. It is important to recognise the role that the Ground Movement Controller also has to play in deciding where to deliver aircraft. The entrance at which each aircraft is released can greatly influence which take-off orders are achievable as well as how easy they are to achieve and how robust they are to the real world uncertainties in taxi speeds, ready times or taxiway congestion.

Simple heuristics the Ground Movement Controller could use include ‘northbounds on the left’, ‘closest entrance to stand’ or ‘all heavy aircraft to one entrance’. All of these consider only characteristics of the aircraft rather than what is already happening in the holding point. More complicated methods would consider the potential take-off orders and ensure that aircraft are allocated to entrances that would make good take-off orders possible or easier to achieve. The job of the Ground Movement Controller is very intensive so any allocation method the Ground Movement Controller uses must be easy and fast to apply.

The solution method suggested for a decision support system can be used to simulate the role of the Runway Controller. This makes it possible to evaluate the effects of other parts of the system upon the Runway Controller. In particular, it becomes possible to evaluate the effects of the Ground Movement Controller’s decisions upon the possibility and ease of reordering in the holding point and to evaluate different methods for deciding where to deliver aircraft. Our results show the significant effect that the Ground Movement Controller’s decisions can have on the reordering. We use the results to determine the value of different heuristics and of obtaining information from the Runway Controller’s decision support system to aid the Ground Movement Controller in the decision making. We will discuss why some heuristics work better than others and identify the characteristics of those that work best.

## 6 Concluding Comments

When developing a decision support system to help with the take-off scheduling at London Heathrow, the Runway Controller is the natural user for the system as that is where the rescheduling is currently performed. We produced a model of the system and a solution method using a hybrid meta-heuristic approach to solve the problem very quickly, including the complex but vital constraints imposed by the physical holding point that are usually ignored in the existing academic literature. We have predicted a reduction of up to 25% in the

delay suffered by aircraft, while also improving CTOT compliance, if a decision support system for the Runway Controller is provided with information about aircraft under the control of the Ground Movement Controller.

Using the proposed model and search algorithm for a decision support system for the Runway Controller, we have evaluated the effects of the Ground Movement Controller's decisions upon the reordering that could be expected of the Runway Controller. We have found that the method used by the Ground Movement Controller to allocate aircraft to the holding point entrances can have a substantial effect upon the ease and ability of desirable reordering within the holding point. Finally, we will identify the advantages and disadvantages of each of the evaluated allocation methods and justify our suggestion for a mechanism to use at Heathrow.

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