Calculating Programs

Lecture 1: Fold/Unfold Calculation

MGS Spring School 2019

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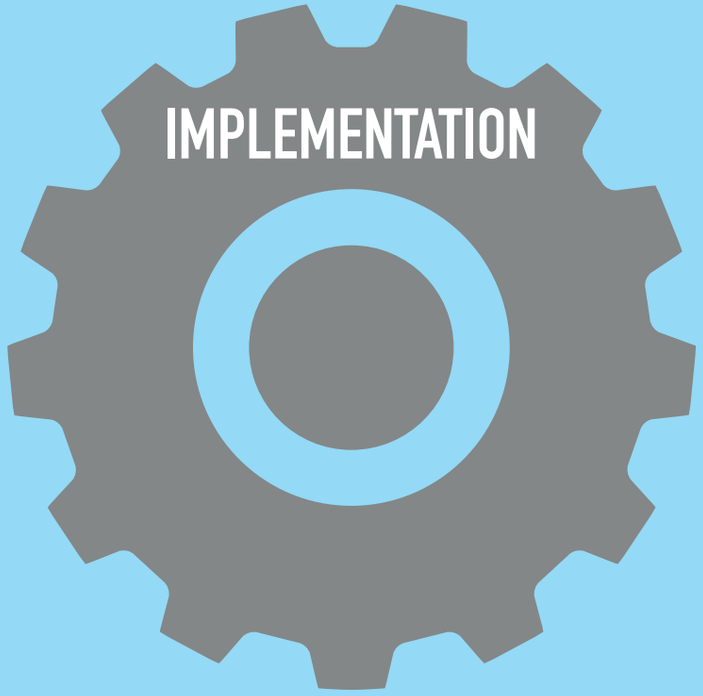

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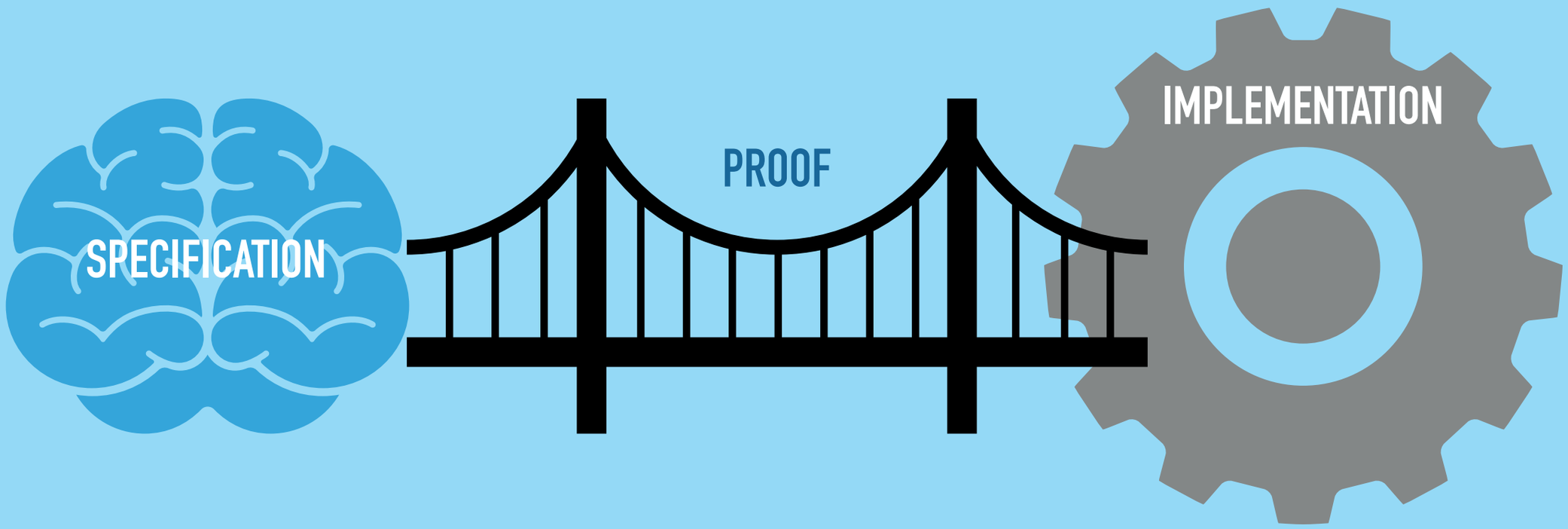

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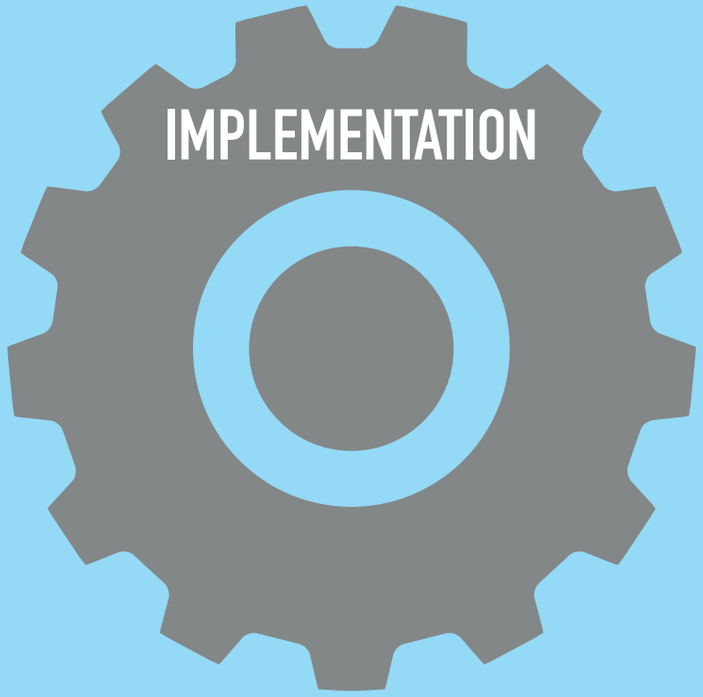

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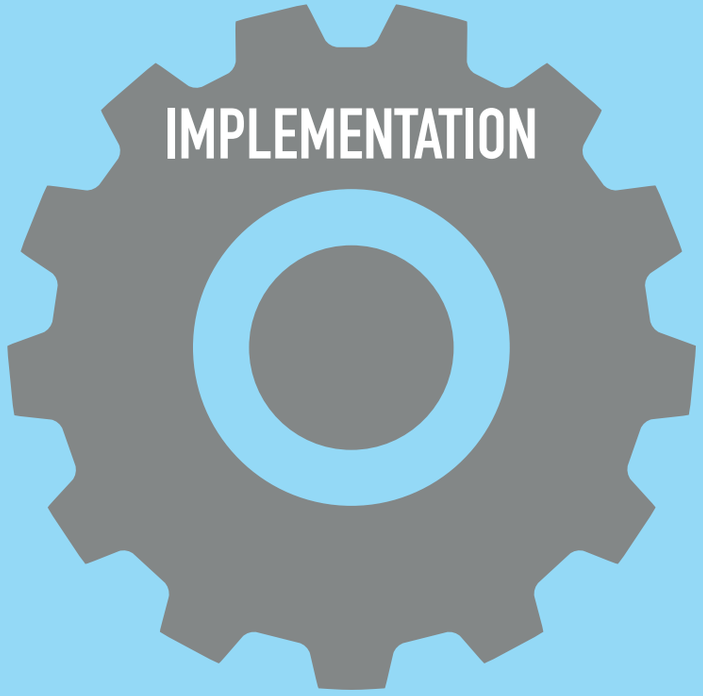

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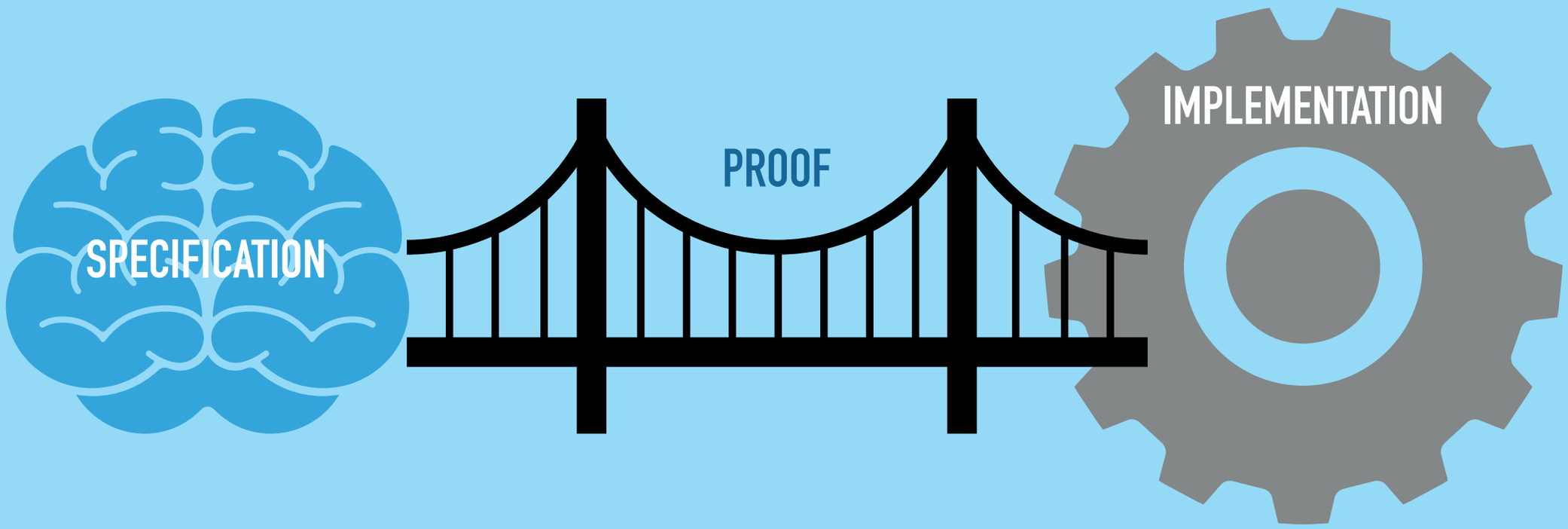

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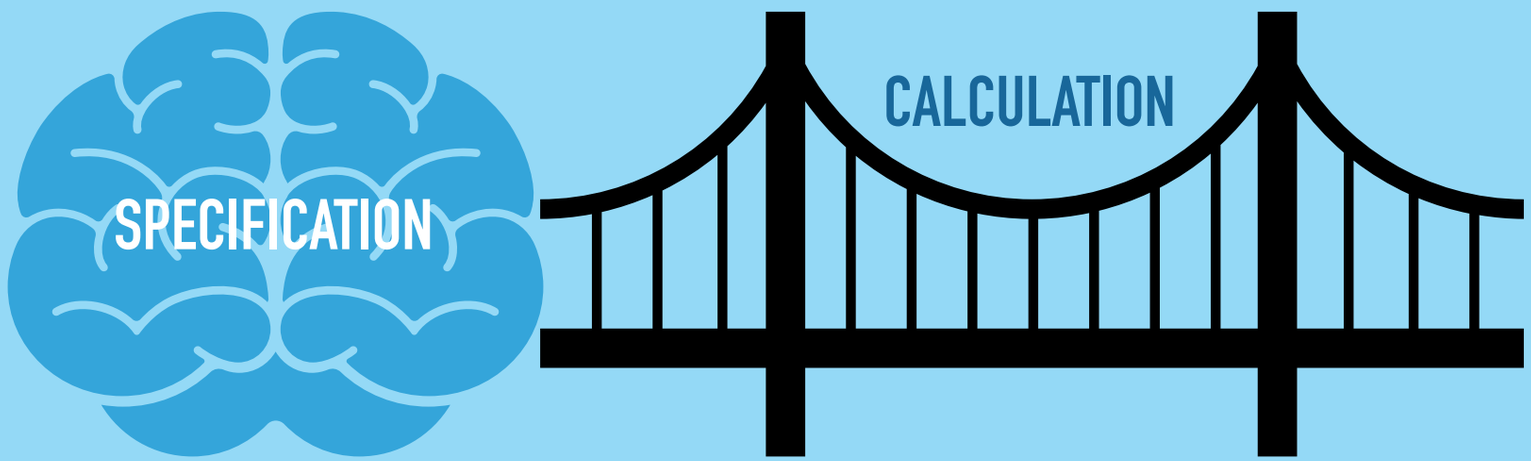

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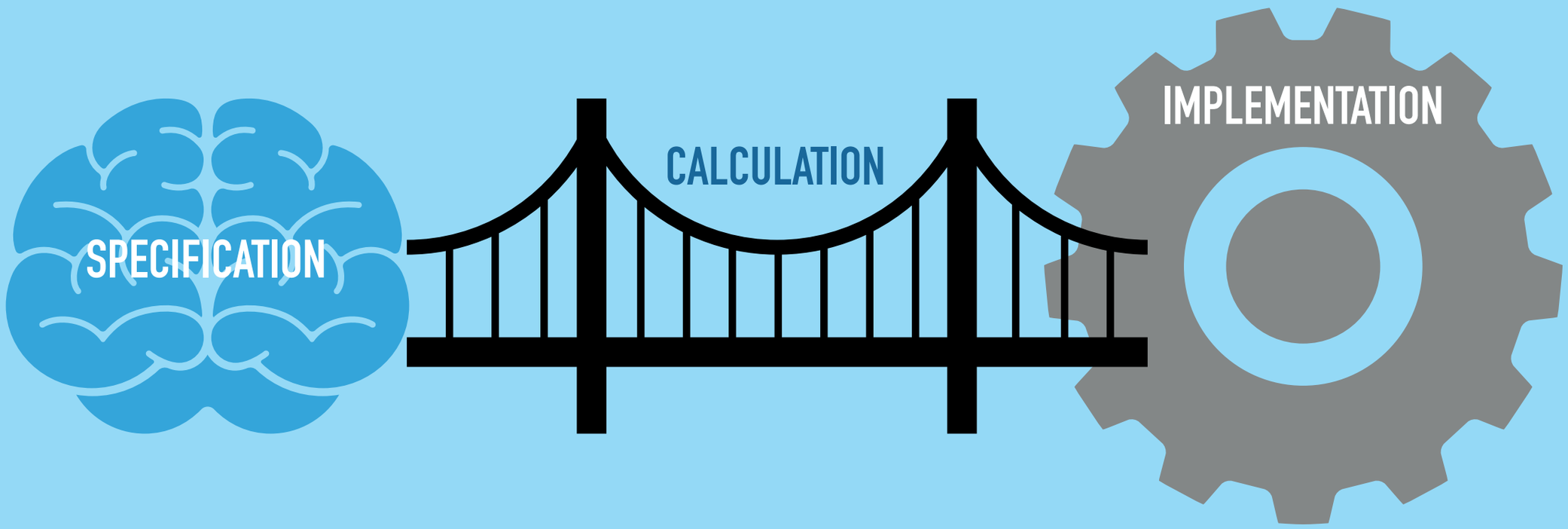

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  3. Fixed-point calculus
- The final lecture will go through a long worked example

A Note on Notation...

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- In this course, I will write programs in a Haskell-like notation:

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- Function application requires no brackets and associates to the left, functions defined by pattern-matching on the input, operators can be infix…

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  - Might lose *total* correctness, which requires same termination behaviour
- See *A Transformation System for Developing Recursive Programs* — Burstall & Darlington, 1977

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- Suppose we want a new program that does the same thing, but using an accumulating parameter and tail-recursion…

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- The resulting program may not be totally correct, but it will be partially correct
- Exercise: Explain, informally, why these five steps will preserve partial correctness.

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  (fib (n+1), fib (n+1) + fib n)
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  (fib (n+1), fib (n+1) + fib n)
= { introducing a let-binding }
  let (f0,f1) = (fib n, fib (n+1))
        in (f1, f0+f1)
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= { unfolding fib }
  (1,1)
```

```
  twofib (n+1)
= { unfolding twofib }
  (fib (n+1), fib ((n+1)+1))
= { arithmetic }
  (fib (n+1), fib (n+2))
= { unfolding fib }
  (fib (n+1), fib (n+1) + fib n)
= { introducing a let-binding }
  let (f0,f1) = (fib n, fib (n+1))
        in (f1, f0+f1)
= { folding twofib }
```

Example: Fibonacci

```
fib 0     = 1
fib 1     = 1
fib (n+2) = fib (n+1) + fib n
```

Spec: `twofib n = (fib n, fib (n+1))`

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  twofib 0
= { unfolding twofib }
  (fib 0, fib (0+1))
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- …which will run in *linear* time!

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1. Explain, informally, why the five steps allowed in fold/unfold calculations preserve partial correctness.

2. Consider the following function that reverses a list:

```
reverse [] = []
reverse (x:xs) = reverse xs ++ [x]
```

Calculate a new version of this function that uses an accumulating parameter.

Why will this function be more efficient? Remember the definition of ++:

```
[]     ++ ys = ys
(x:xs) ++ ys = x:(xs ++ ys)
```

Exercises! (cont.)

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3. Consider the following simple expression evaluator:

```
data Expr = Num Int | Add Expr Expr
eval (Num n)       = n
eval (Add e1 e2)   = eval e1 + eval e2
```

Calculate a new version of the program that takes as an additional argument a function that tells it what to do with the final result. The specification should be as follows:

```
evalCont e k = k (eval e)
```

(Programs written in this form are said to be in *continuation-passing style*.)