ITU FRP 2010

Lecture 2: Yampa: Arrows-based FRP

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Outline

- CFRP issues
- Introduction to Yampa
- Arrows
- A closer look at Yampa

CFRP issues: Sharing

Consider:

```
let x = 1 + integral (x * x) in x
```

The recursively defined behavior, a ***function***, is applied over and over to the ***same*** stream of sample times.

- Causes recomputation
- Laziness does ***not*** help
- Memoization needed to get acceptable performance. But with care to avoid memory leaks.

CFRP issues: Restart (1)

Consider:

```
let
    c = hold 0 (count (repeatedly 0.5))
in
    c ‘until‘ after 5 -=> c * 2
```

What happened at the time of the switch?

- CFRP behaviors and events are ***signal generators***: they will start from scratch when swicthched in.
- But what if we just want to continue observing an evolving signal?

CFRP issues: Restart (2)

- A version of `until` that starts new behaviors from time 0.
  Time and space leak!
- Support signals as well, e.g. through some variant of `runningIn`:

  ```
  runningIn ::
          B a -> (B a -> B b) -> B b
  ```

  Idea: apply behavior to start time once and for all, then wrap up the resulting signal as a signal generator that ignores its starting time.

CFRP issues: Restart (3)

Problems with `runningIn`

- No type-level distinction between signals and signal generators: a “running behavior” is a signal masquerading as a signal generator. (But could be fixed though other designs.)
- Difficult to implement; requires imperative techniques, implies certain overhead.

An alternative

By adopting ***signal functions*** as the central notion, these problems are side stepped:

- Sharing amounts to sharing computations of signal samples: lazy evaluation handles that just fine.
- Observation of externally originating signals is inherent in the notion of a signal function.
- Implementation is straightforward.

Yampa

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- Discrete-time signals represented by continuous-time signal carrying option type ***`Event`***.

Yampa

What is ***Yampa***?

- FRP implementation structured using ***arrows***.
- Realised as an ***Embedded Domain-Specific Language*** (EDSL), i.e. a combinator library.
- ***Continuous-time*** signals (conceptually)
- Discrete-time signals represented by continuous-time signal carrying option type ***Event***.
- Functions on signals, ***Signal Functions***, is the central abstraction, forming the arrows.

Yampa

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- Signal functions are first-class entities, signals a secondary notion, only existing indirectly through the signal functions.
- Advanced ***switching constructs*** to describe systems with highly dynamic structure.
- People:
  - Antony Courtney
  - Paul Hudak
  - Henrik Nilsson
  - John Peterson

Yampa?

Yampa?

Yet

Another

Mostly

Pointless

Acronym

Yampa?

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???

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No . . .

Yampa?

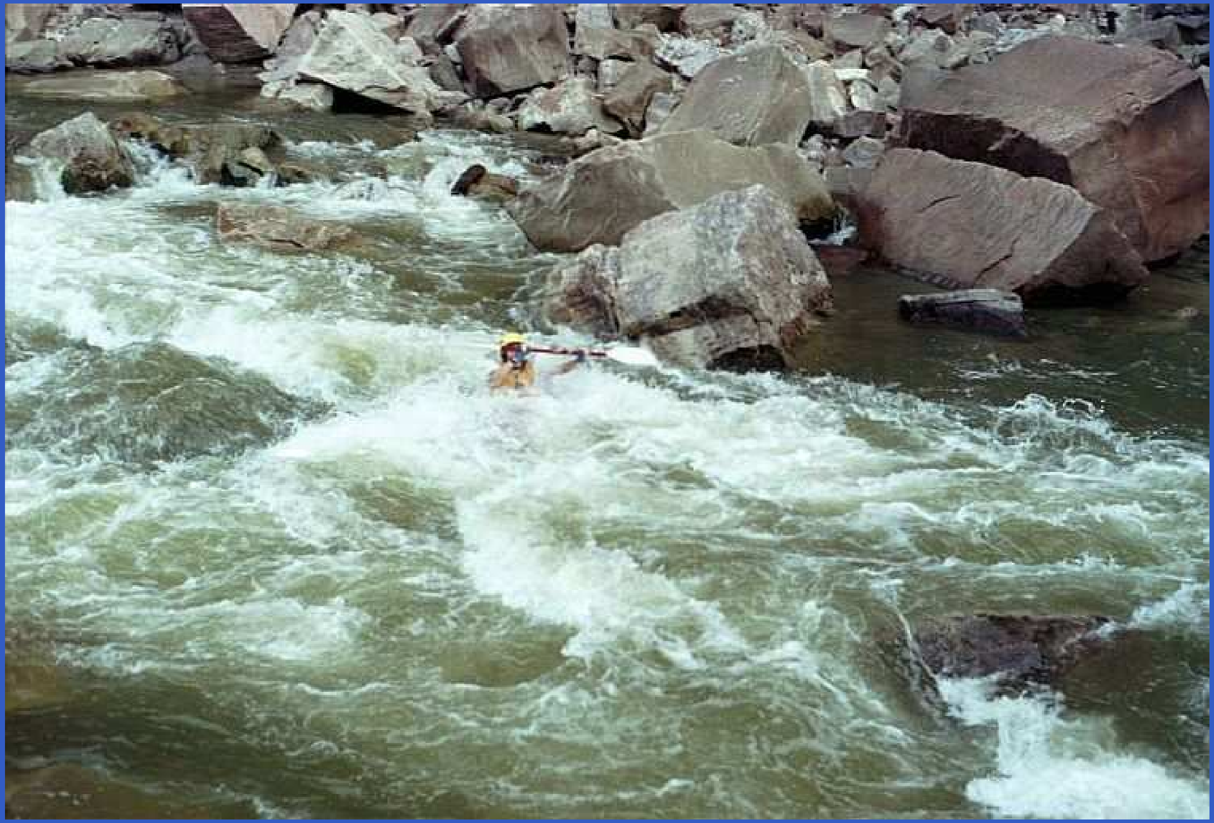
Yampa is a river . . .

Yampa?

. . . with long calmly flowing sections . . .

Yampa?

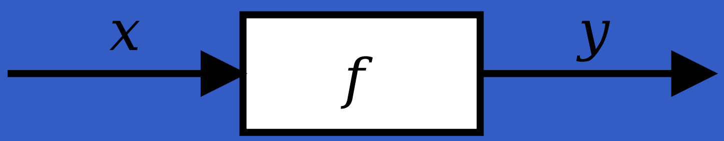
. . . and abrupt whitewater transitions in between.



A good metaphor for hybrid systems!

Signal functions (1)

Key concept: ***functions on signals***.



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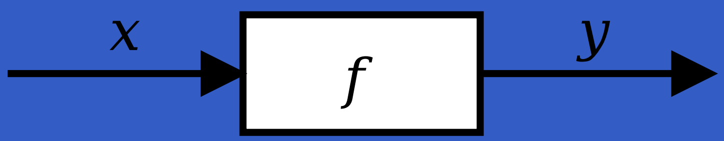

Intuition:

$$\texttt{Signal}\ \alpha \approx \texttt{Time} \rightarrow \alpha$$
$$\texttt{SF}\ \alpha\ \beta \approx \texttt{Signal}\ \alpha \rightarrow \texttt{Signal}\ \beta$$

x `:: Signal T1`
y `:: Signal T2`
f `:: SF T1 T2`

Signal functions (2)

Additionally, ***causality*** required: output at time t must be determined by input on interval $[0, t]$.

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Signal functions are said to be

- ***pure*** or ***stateless*** if output at time t only depends on input at time t
- ***impure*** or ***stateful*** if output at time t depends on input over the interval $[0, t]$.

Signal functions and state

Alternative view:

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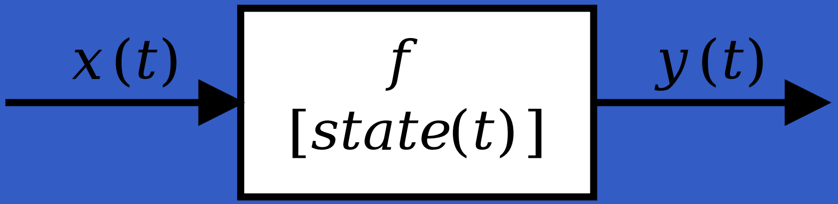
Signal functions can encapsulate ***state***.



$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.
Thus, really a kind of ***process***.

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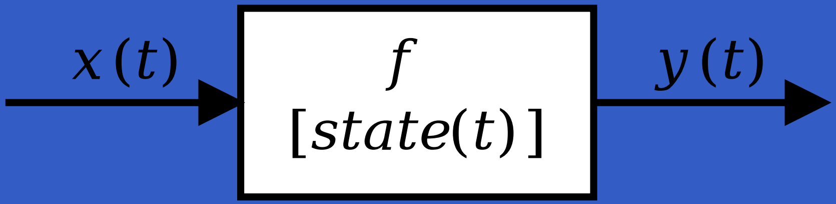

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Thus, really a kind of ***process***.

From this perspective, signal functions are:

- ***stateful*** if $y(t)$ depends on $x(t)$ and $state(t)$
- ***stateless*** if $y(t)$ depends only on $x(t)$

Yampa and arrows (1)

In Yampa, systems are described by combining signal functions (forming new signal functions).

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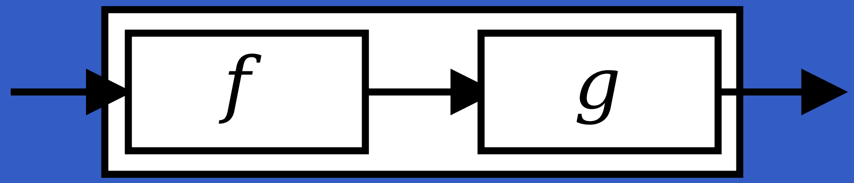
For example, serial composition:



Yampa and arrows (1)

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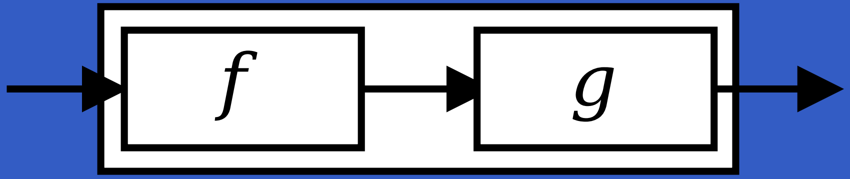
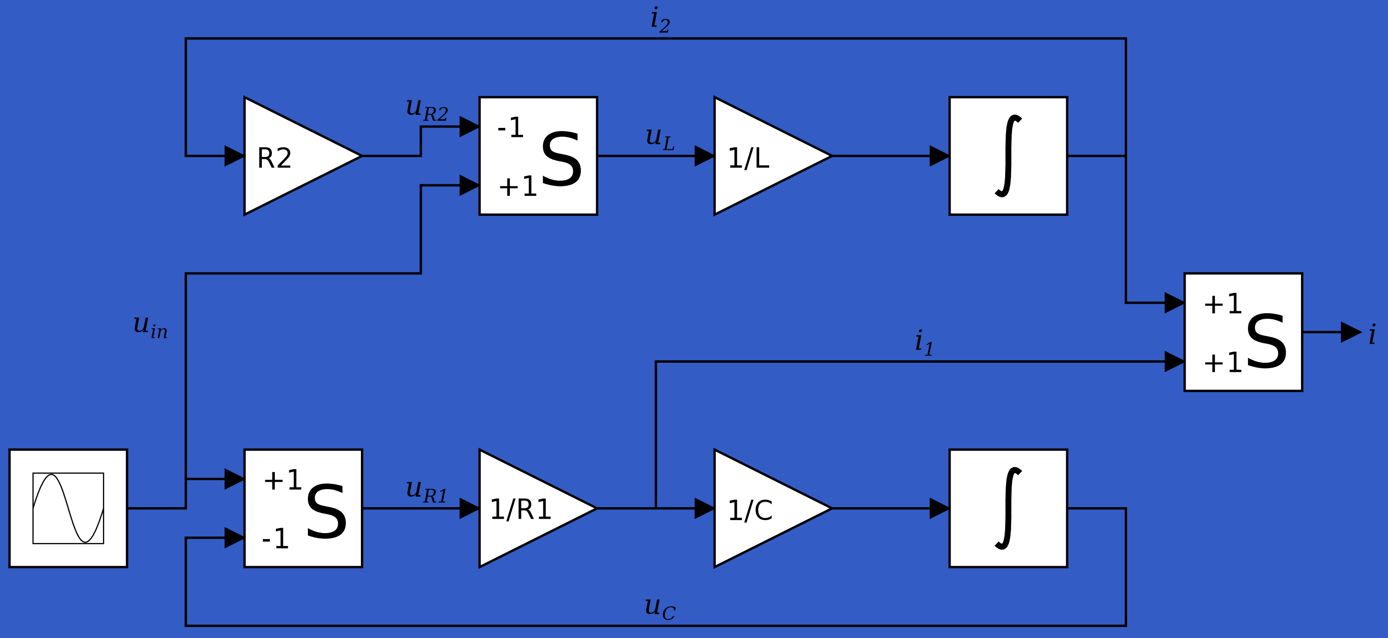


A *combinator* can be defined that captures this idea:

```
(>>>) :: SF a b -> SF b c -> SF a c
```

Yampa and arrows (2)

But systems can be complex:



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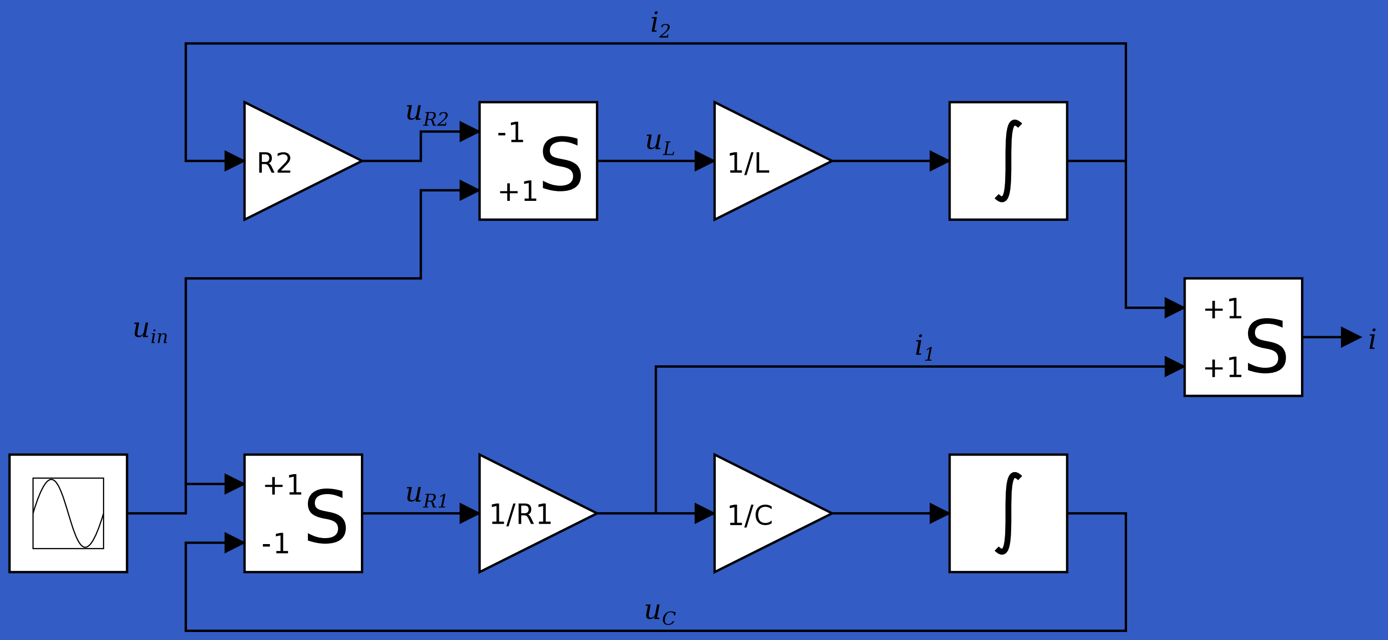

How many and what combinators do we need to be able to describe arbitrary systems?

Yampa and arrows (3)

John Hughes' ***arrow*** framework:

- Abstract data type interface for function-like types.

Yampa and arrows (3)

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- Particularly suitable for types representing process-like computations.
- Related to ***monads***, since arrows are (effectful) computations, but more general: any monad m induces an arrow, the Kleisli arrow, $\alpha \rightarrow m\ \beta$, but not vice versa.

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John Hughes' ***arrow*** framework:

- Abstract data type interface for function-like types.
- Particularly suitable for types representing process-like computations.
- Related to ***monads***, since arrows are (effectful) computations, but more general: any monad m induces an arrow, the Kleisli arrow, $\alpha \rightarrow m\ \beta$, but not vice versa.
- Provides a minimal set of "wiring" combinators.

What is an arrow? (1)

- A ***type constructor*** a of arity two.

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  - ***lifting***:

```
arr :: (b->c) -> a b c
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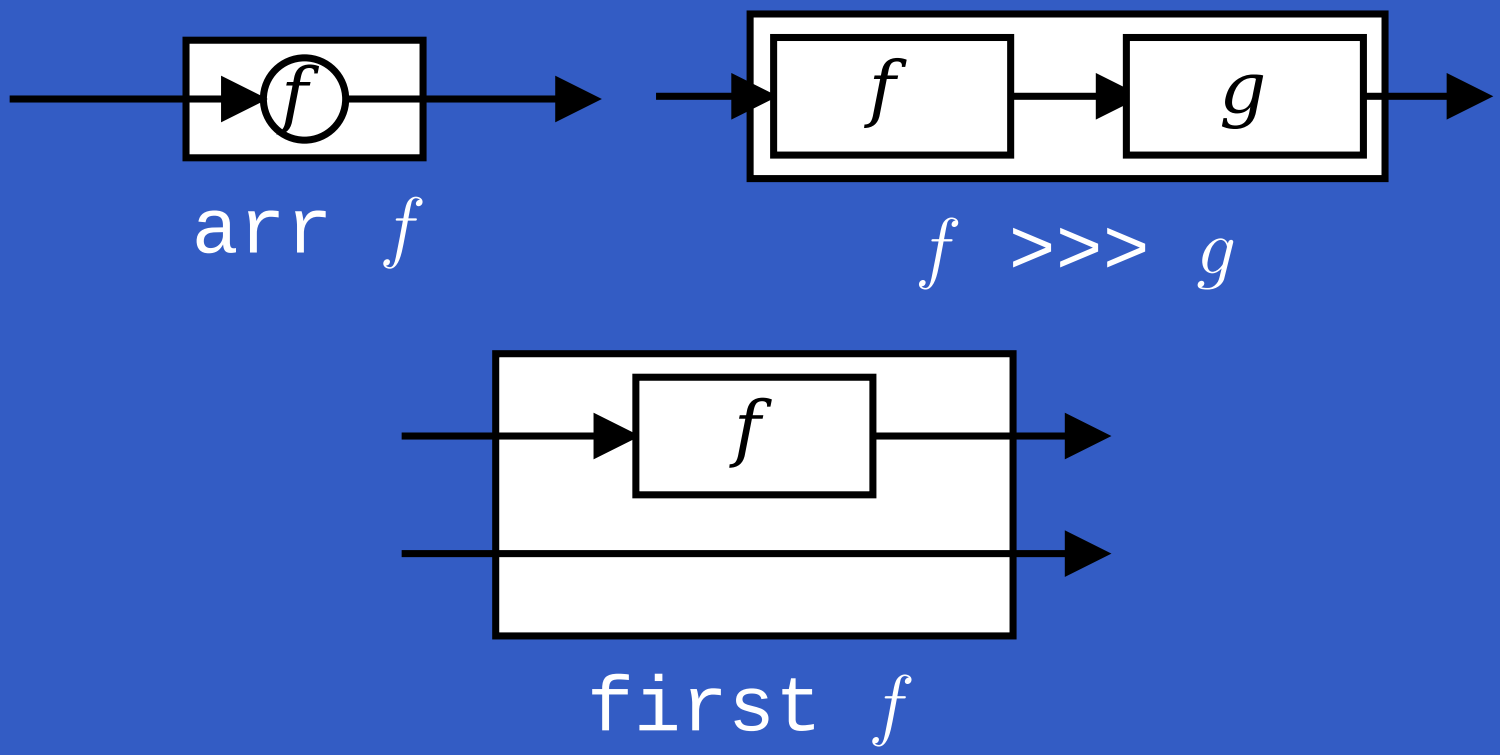
    `first :: a b c -> a (b,d) (c,d)`
- A set of ***algebraic laws*** that must hold.

What is an arrow? (2)

These diagrams convey the general idea:



The `Arrow` class

In Haskell, a ***type class*** is used to capture these ideas (except for the laws):

```
class Arrow a where
    arr   :: (b -> c) -> a b c
    (>>>) :: a b c -> a c d -> a b d
    first :: a b c -> a (b,d) (c,d)
```

Arrow laws

```
(f >>> g) >>> h  =  f >>> (g >>> h)
```

Arrow laws

```
(f >>> g) >>> h  =  f >>> (g >>> h)
    arr (g . f)  =  arr f >>> arr g
```

Arrow laws

```
(f >>> g) >>> h  =  f >>> (g >>> h)
    arr (g . f)  =  arr f >>> arr g
  arr id >>> f   =  f
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Arrow laws

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(f >>> g) >>> h  =  f >>> (g >>> h)
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   arr id >>> f  =  f
              f  =  f >>> arr id
```

Arrow laws

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    arr (g . f)  =  arr f >>> arr g
   arr id >>> f  =  f
              f  =  f >>> arr id
  first (arr f)  =  arr (f × id)
```

Arrow laws

$$
\begin{aligned}
\texttt{(f >>> g) >>> h} &= \texttt{f >>> (g >>> h)} \\
\texttt{arr (g . f)} &= \texttt{arr f >>> arr g} \\
\texttt{arr id >>> f} &= \texttt{f} \\
\texttt{f} &= \texttt{f >>> arr id} \\
\texttt{first (arr f)} &= \texttt{arr (f} \times \texttt{id)} \\
\texttt{first (f >>> g)} &= \texttt{first f >>> first g}
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\texttt{first f >>> arr (id} \times \texttt{g)} &= \texttt{arr (id} \times \texttt{g) >>> first f} \\
\texttt{first f >>> arr fst} &= \texttt{arr fst >>> f}
\end{aligned}
$$

Arrow laws

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\begin{array}{rcl}
\texttt{(f >>> g) >>> h} & = & \texttt{f >>> (g >>> h)} \\
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\texttt{first f >>> arr fst} & = & \texttt{arr fst >>> f} \\
\texttt{first (first f) >>> arr assoc} & = & \texttt{arr assoc >>> first f}
\end{array}
$$

Functions are arrows (1)

Functions are a simple example of arrows. The arrow type constructor is just `(->)` in that case.

Exercise 1: Suggest suitable definitions of

- `arr`
- `(>>>)`
- `first`

for this case!

Functions are arrows (2)

Solution:

- `arr = id`

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  To see this, recall

  ```
  id :: t -> t
  arr :: (b->c) -> a b c
  ```

Functions are arrows (2)

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  To see this, recall

  ```
  id :: t -> t
  arr :: (b->c) -> a b c
  ```

  Instantiate with

  $$\texttt{a} = \texttt{(->)}$$
  $$\texttt{t} = \texttt{b->c} = \texttt{(->) b c}$$

Functions are arrows (3)

- `f >>> g = \a -> g (f a)`

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- `f >>> g = \a -> g (f a)`   ***or***
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Functions are arrows (3)

- `f >>> g = \a -> g (f a)` ***or***
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- `(>>>) = flip (.)`
- `first f = \(b,d) -> (f b,d)`

Functions are arrows (4)

`Arrow` instance declaration for functions:

```
instance Arrow (->) where
    arr     = id
    (>>>)   = flip (.)
    first f = \(b,d) -> (f b,d)
```

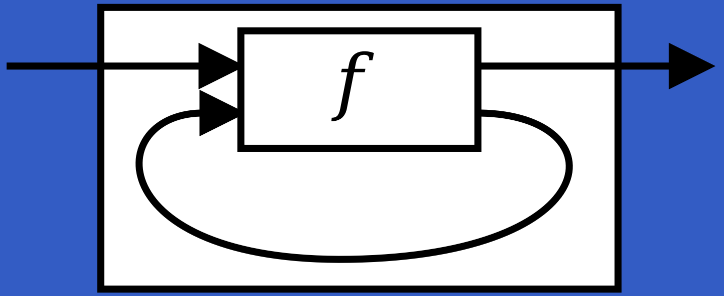
The arrow laws reformulated

Exploiting that functions are arrows, some of the laws can be formulated more neatly. E.g:

$$\mathtt{arr\ (f >>> g)} = \mathtt{arr\ f >>> arr\ g}$$
$$\mathtt{first\ (arr\ f)} = \mathtt{arr\ (first\ f)}$$

The `loop` combinator (1)

Another important operator is `loop`: a fixed-point operator used to express recursive arrows or ***feedback***:



`loop` f

The `loop` combinator (2)

Not all arrow instances support `loop`. It is thus a method of a separate class:

```
class Arrow a => ArrowLoop a where
    loop :: a (b, d) (c, d) -> a b c
```

Remarkably, the four combinators `arr`, `>>>`, `first`, and `loop` are sufficient to express any conceivable wiring!

Some more arrow combinators (1)

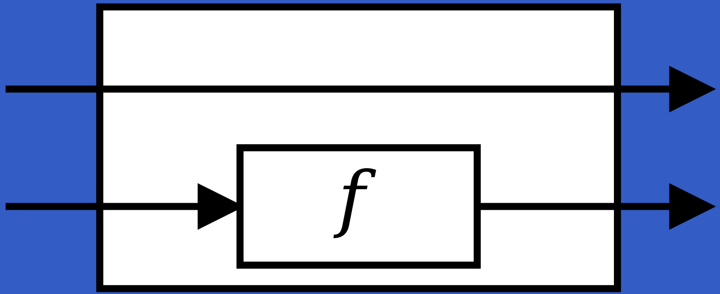
```
second :: Arrow a =>
    a b c -> a (d,b) (d,c)

(***) :: Arrow a =>
    a b c -> a d e -> a (b,d) (c,e)

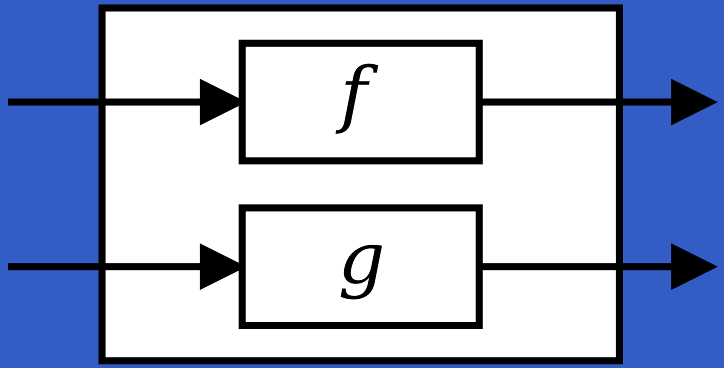
(&&&) :: Arrow a =>
    a b c -> a b d -> a b (c,d)
```

Some more arrow combinators (2)

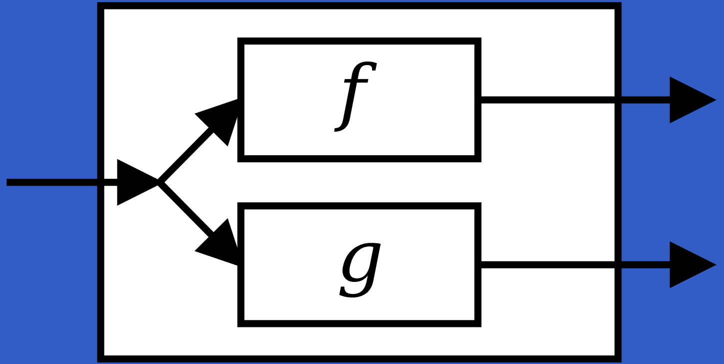
As diagrams:



second f



f *** g



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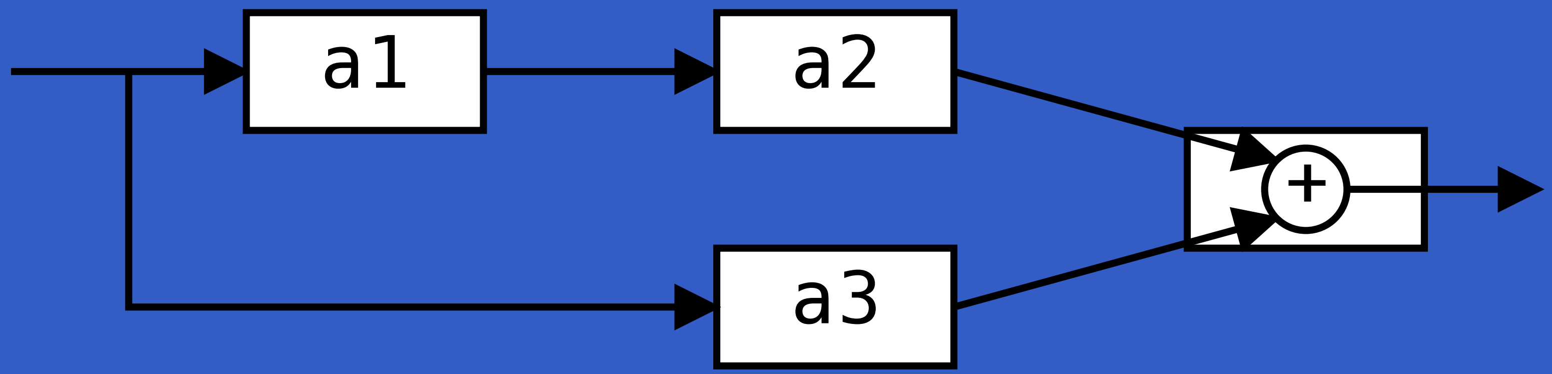
Some more arrow combinators (3)

Exercise 2: Describe the following circuit using arrow combinators:



`a1, a2, a3 :: A Double Double`

Exercise 3: The combinators `second`, `(***)`, and `(&&&)` are not primitive, but defined in terms of `arr`, `(>>>)`, and `first`. Suggest suitable definitions!

Exercise 2: One solution

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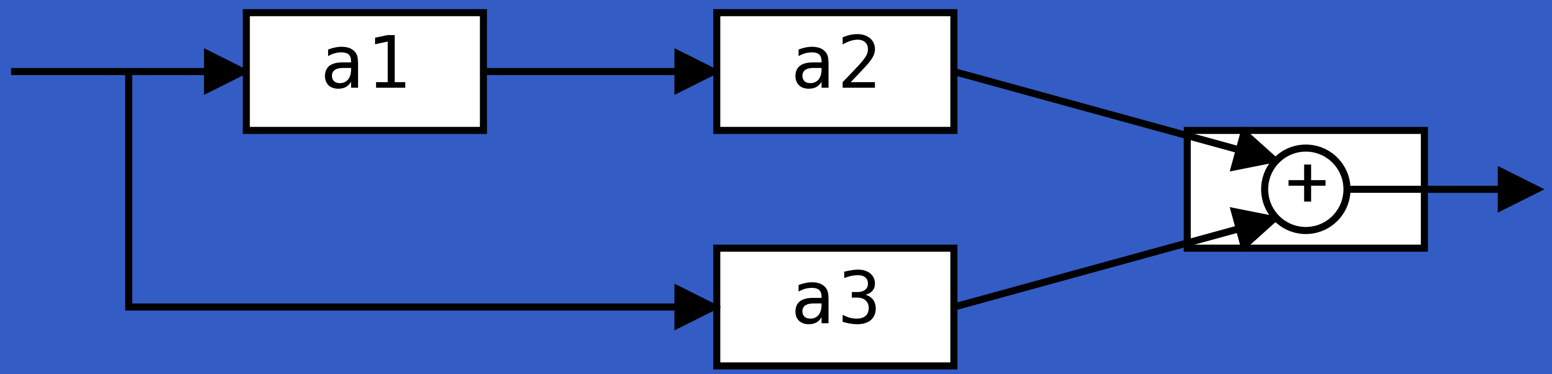

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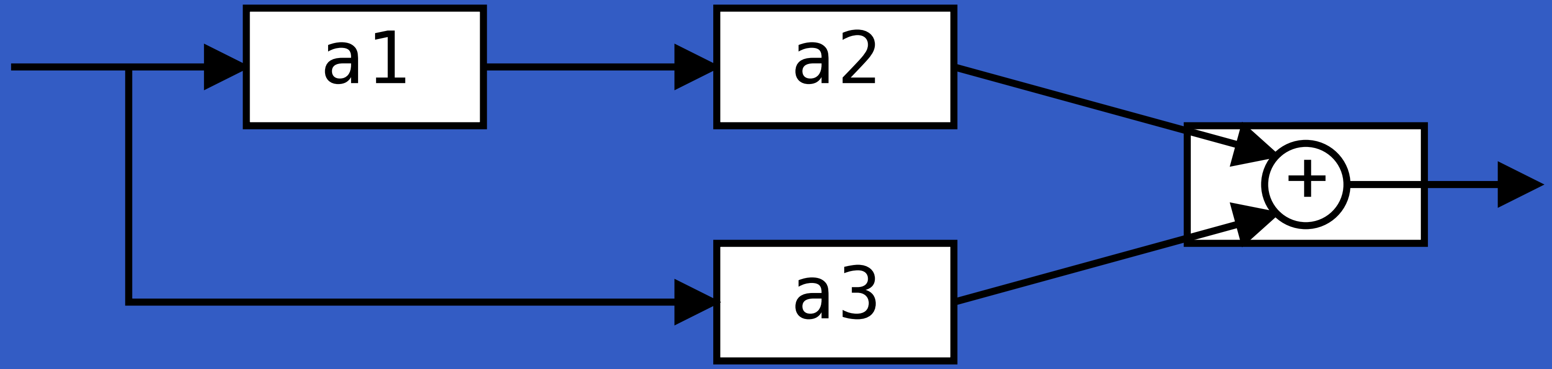


```
a1, a2, a3 :: A Double Double

circuit_v1 :: A Double Double
circuit_v1 = (a1 &&& arr id)
             >>> (a2 *** a3)
             >>> arr (uncurry (+))
```

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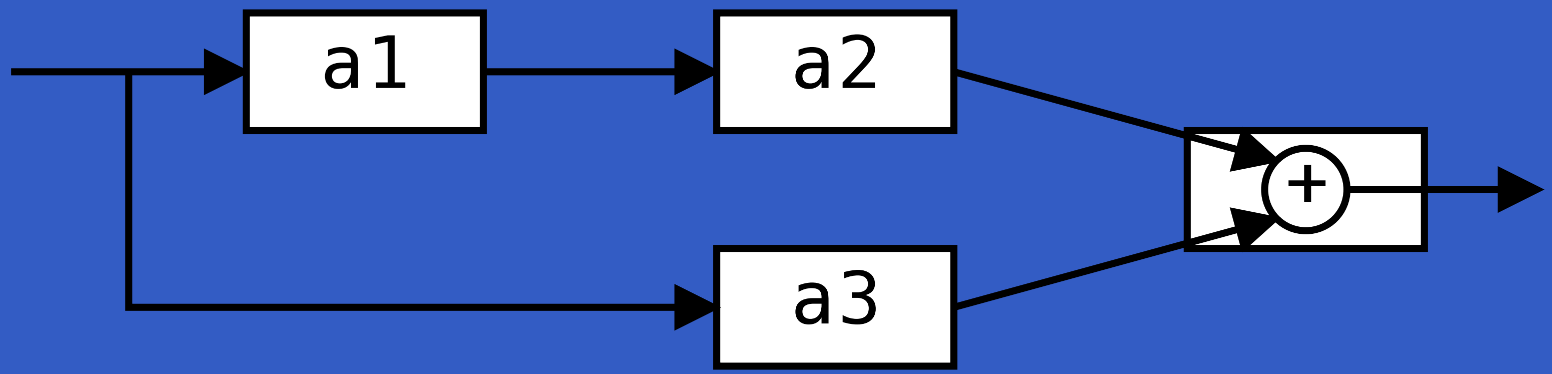


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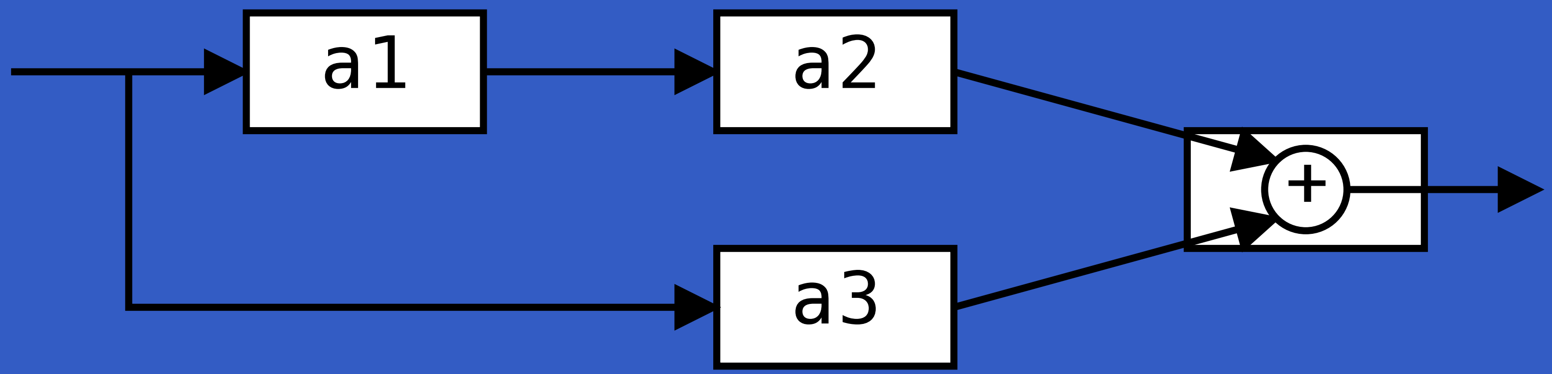


```
a1, a2, a3 :: A Double Double

circuit_v2 :: A Double Double
circuit_v2 = arr (\x -> (x,x))
             >>> first a1
             >>> (a2 *** a3)
             >>> arr (uncurry (+))
```

Exercise 3: Solution

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second :: Arrow a => a b c -> a (d,b) (d,c)
second f = arr swap >>> first f >>> arr swap
swap (x,y) = (y,x)
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f *** g = first f >>> second g
```

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```
(&&&) :: Arrow a => a b c -> a b d -> a b (c,d)
f &&& g = arr (\x->(x,x)) >>> (f *** g)
```

Note on the definition of (`***`) (1)

Are the following two definitions of (`***`) equivalent?

- `f *** g = first f >>> second g`
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Note on the definition of `(***)` (1)

Are the following two definitions of `(***)` equivalent?

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No, in general

$$\texttt{first}\ f \ \texttt{>>>}\ \texttt{second}\ g \neq \texttt{second}\ g\ \texttt{>>>}\ \texttt{first}\ f$$

since the ***order*** of the two possibly effectful computations f and g are different.

Note on the definition of (***) (2)

Similarly

$$(f *** g) >>> (h *** k) \neq (f >>> h) *** (g >>> k)$$

since the order of f and g differs.

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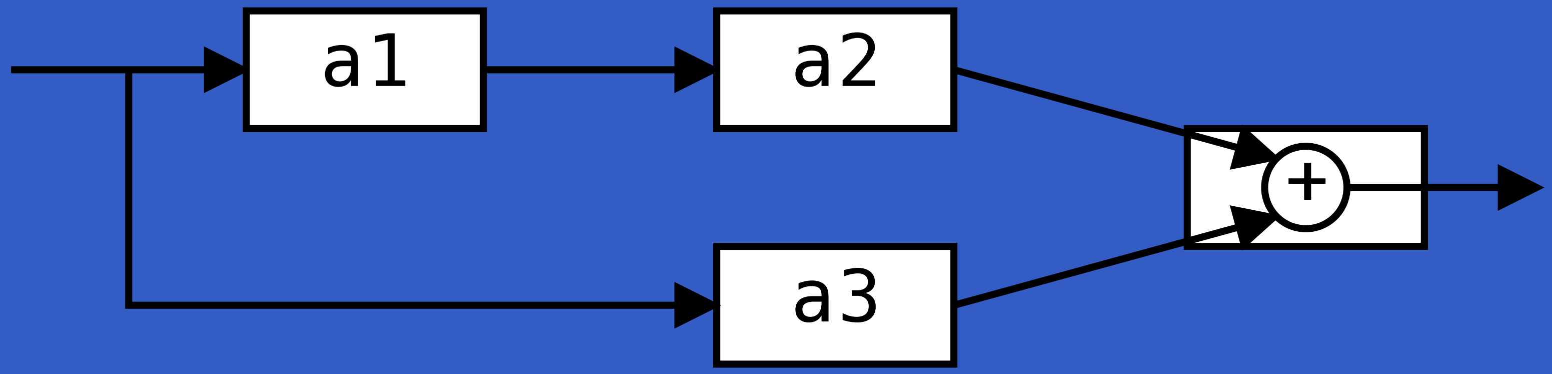
However, Yampa's signal functions have no effectful interaction: they are Causal Commutative Arrows (Liu, Cheng, Hudak 2009) ***Both*** considered identities actually hold.

Yet another attempt at exercise 2

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circuit_v3 :: A Double Double
circuit_v3 = (a1 &&& a3)
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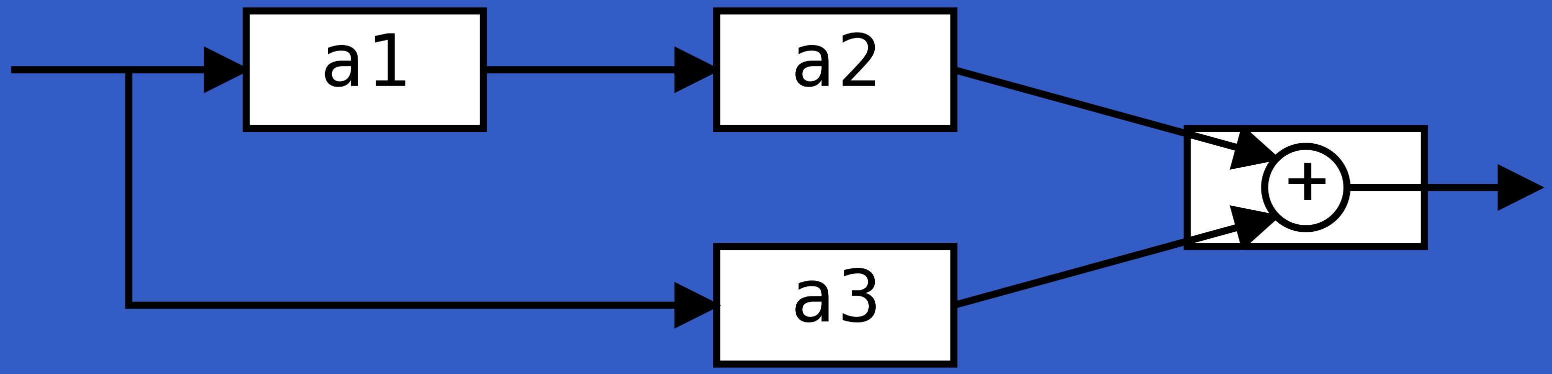


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```

Are `circuit_v1`, `circuit_v2`, and `circuit_v3` all equivalent?

Point-free vs. pointed programming

What we have seen thus far is an example of ***point-free*** programming: the values being manipulated are not given any names.

Point-free vs. pointed programming

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Point-free vs. pointed programming

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This is often appropriate, especially for small definitions, and it facilitates equational reasoning as shown by Bird & Meertens (Bird 1990).

However, large programs are much better expressed in a ***pointed*** style, where names can be given to values being manipulated.

The arrow do notation (1)

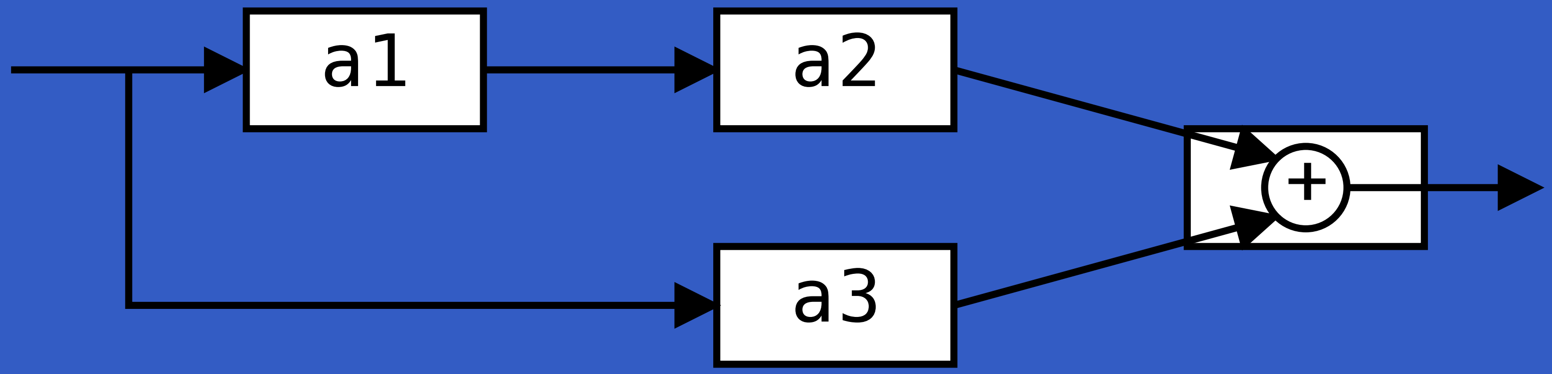
Ross Paterson's do-notation for arrows supports ***pointed*** arrow programming. Only ***syntactic sugar***.

$$
\begin{array}{l}
\texttt{proc}\ \mathit{pat}\ \texttt{->}\ \texttt{do}\ [\ \texttt{rec}\ ] \\
\qquad \mathit{pat}_1\ \texttt{<-}\ \mathit{sfexp}_1\ \texttt{-<}\ \mathit{exp}_1 \\
\qquad \mathit{pat}_2\ \texttt{<-}\ \mathit{sfexp}_2\ \texttt{-<}\ \mathit{exp}_2 \\
\qquad \texttt{...} \\
\qquad \mathit{pat}_n\ \texttt{<-}\ \mathit{sfexp}_n\ \texttt{-<}\ \mathit{exp}_n \\
\qquad \texttt{returnA}\ \texttt{-<}\ \mathit{exp}
\end{array}
$$

Also: $\texttt{let}\ \mathit{pat} = \mathit{exp} \quad \equiv \quad \mathit{pat}\ \texttt{<-}\ \texttt{arr id}\ \texttt{-<}\ \mathit{exp}$

The arrow do notation (2)

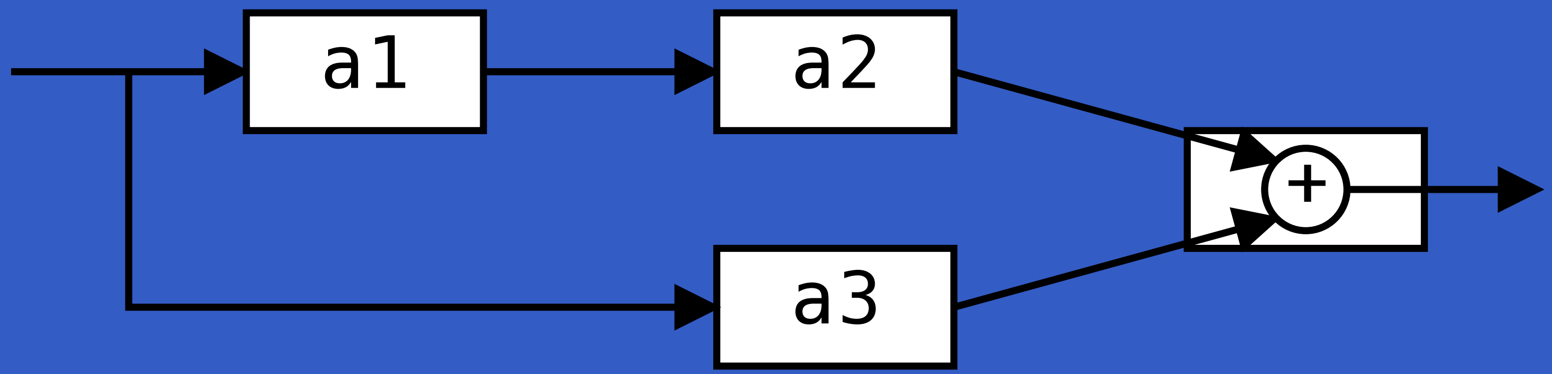
Let us redo exercise 3 using this notation:




```
circuit_v4 :: A Double Double
circuit_v4 = proc x -> do
    y1 <- a1 -< x
    y2 <- a2 -< y1
    y3 <- a3 -< x
    returnA -< y2 + y3
```

The arrow do notation (3)

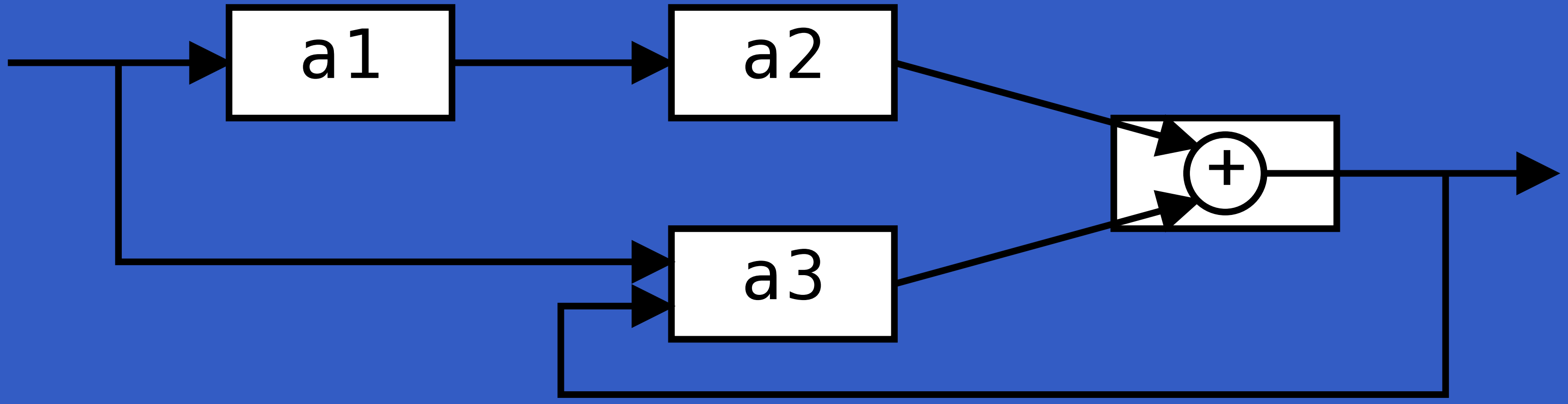
We can also mix and match:




```
circuit_v5 :: A Double Double
circuit_v5 = proc x -> do
    y2 <- a2 <<< a1 -< x
    y3 <- a3        -< x
    returnA -< y2 + y3
```

The arrow do notation (4)

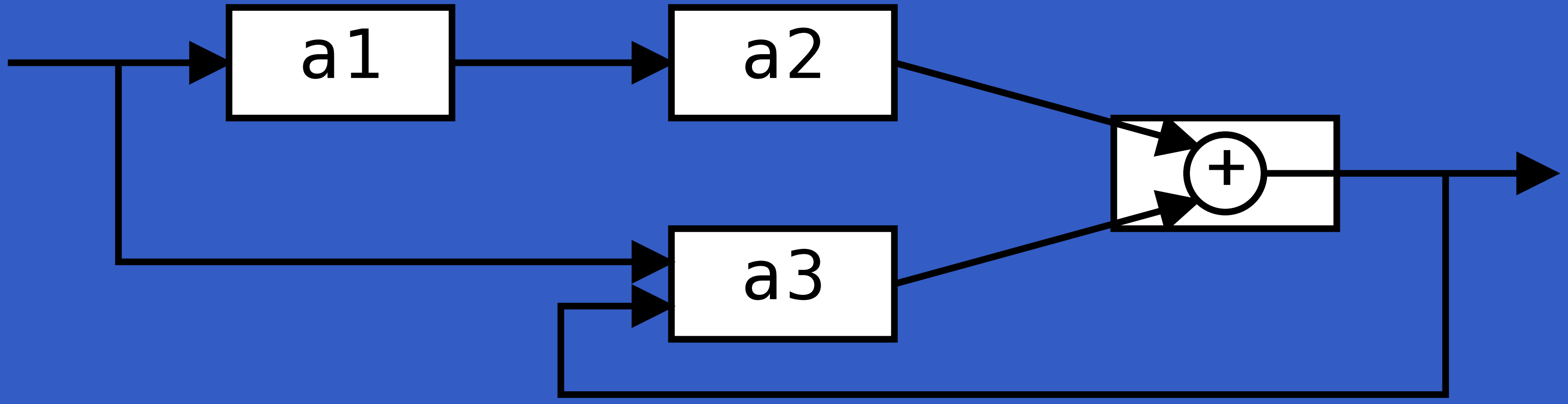
Exercise 4: Describe the following circuit using the arrow do-notation:




```
a1, a2 :: A Double Double
a3 :: A (Double,Double) Double
```

The arrow do notation (4)

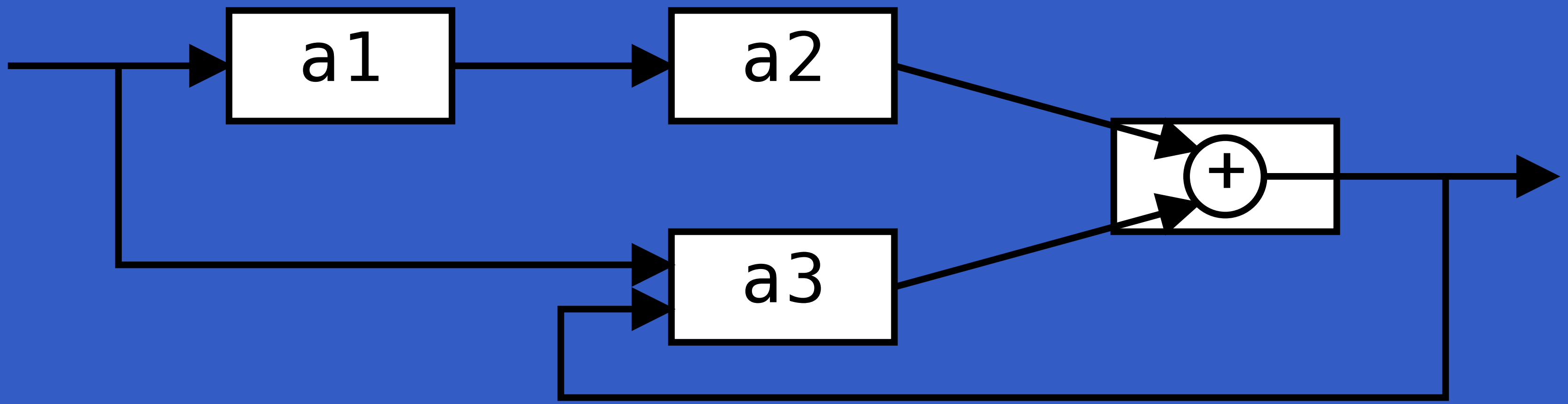
Exercise 4: Describe the following circuit using the arrow do-notation:




```
a1, a2 :: A Double Double
a3 :: A (Double,Double) Double
```

Exercise 5: As 4, but directly using only the arrow combinators.

Solution exercise 4




```
circuit = proc x -> do
    rec
        y1 <- a1 -< x
        y2 <- a2 -< y1
        y3 <- a3 -< (x, y)
        let y = y2 + y3
    returnA -< y
```

Some basic signal functions (1)

- ```
  identity :: SF a a
  identity = arr id
  ```
  ```

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  ```

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  identity :: SF a a
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- ```
  constant :: b -> SF a b
  constant b = arr (const b)
  ```
- `integral :: VectorSpace a s=>SF a a`
  It is defined through:

$$y(t) = \int_0^t x(\tau)\,\mathrm{d}\tau$$

Some basic signal functions (2)

- `iPre :: a -> SF a a`

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- `iPre :: a -> SF a a`
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  Quick Exercise: Define time!

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  `time = constant 1.0 >>> integral`

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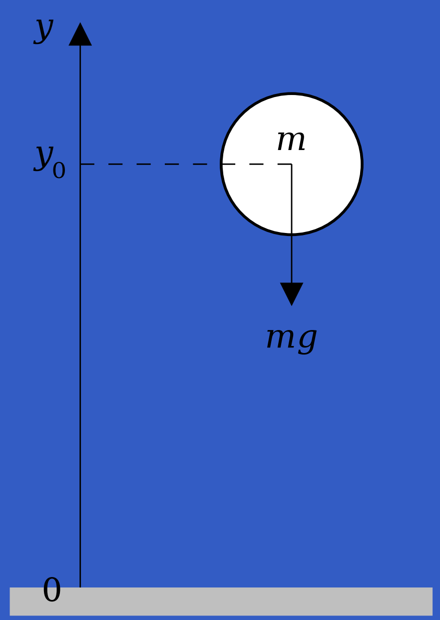
  Quick Exercise: Define time!

  ```
  time = constant 1.0 >>> integral
  ```

Note: there is ***no*** built-in notion of global time in Yampa: time is always ***local***, measured from when a signal function started.

A bouncing ball



$$
\begin{aligned}
y &= y_0 + \int v \, \mathrm{d}t \\
v &= v_0 + \int -9.81
\end{aligned}
$$

On impact:

$$v = -v(t-)$$

(fully elastic collision)

Modelling the bouncing ball: part 1

Free-falling ball:

```
type Pos = Double
type Vel = Double

fallingBall ::
    Pos -> Vel -> SF () (Pos, Vel)
fallingBall y0 v0 = proc () -> do
    v <- (v0 +) ^<< integral -< -9.81
    y <- (y0 +) ^<< integral -< v
    returnA -< (y, v)
```

Events

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Yampa models discrete-time signals by lifting the ***range*** of continuous-time signals:

```
data Event a = NoEvent | Event a
```

Discrete-time signal $= \texttt{Signal}\,(\texttt{Event}\,\alpha)$.

Associating information with an event occurrence:

```
tag :: Event a -> b -> Event b
```

Some basic event sources

- `never :: SF a (Event b)`
- `now :: b -> SF a (Event b)`
- `after :: Time -> b -> SF a (Event b)`
- ```
  repeatedly ::
      Time -> b -> SF a (Event b)
  ```
- `edge :: SF Bool (Event ())`
  ```

Stateful event suppression

- `notYet :: SF (Event a) (Event a)`
- `once :: SF (Event a) (Event a)`

Modelling the bouncing ball: part 2

Detecting when the ball goes through the floor:

```
fallingBall' ::
    Pos -> Vel
    -> SF () ((Pos,Vel), Event (Pos,Vel))
fallingBall' y0 v0 = proc () -> do
    yv@(y, _) <- fallingBall y0 v0 -< ()
    hit       <- edge              -< y <= 0
    returnA -< (yv, hit `tag` yv)
```

Switching

Q: How and when do signal functions “start”?

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- ***Switchers*** “apply” a signal functions to its input signal at some point in time.
- This creates a “running” signal function ***instance***.
- The new signal function instance often replaces the previously running instance.

Switchers thus allow systems with ***varying structure*** to be described.

The basic switch (1)

Idea:

- Allows one signal function to be replaced by another.
- Switching takes place on the first occurrence of the switching event source.

```
switch ::
    SF a (b, Event c)
   -> (c -> SF a b)
   -> SF a b
```

The basic switch (1)

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Initial SF with event source

The basic switch (1)

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- Allows one signal function to be replaced by another.
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```
switch ::
    SF a (b, Event c)
   -> (c -> SF a b)
   -> SF a b
```

Function yielding SF to switch into

The basic switch (2)

Exercise 6: Define an event counter `countFrom`

```
countFrom ::
    Int -> SF (Event a) Int
```

using

```
switch   :: SF a (b, Event c)
            -> (c -> SF a b)
            -> SF a b
constant :: b -> SF a b
notYet   :: SF (Event a) (Event a)
```

and any other basic combinators you might need.

Solution exercise 6

```
countFrom :: Int -> SF (Event a) Int
countFrom n =
  switch
    (constant n &&& identity
    (const (notYet >>> countFrom (n+1)))
```

Solution exercise 6

Another version that ignores any event at time 0 also from the very start:

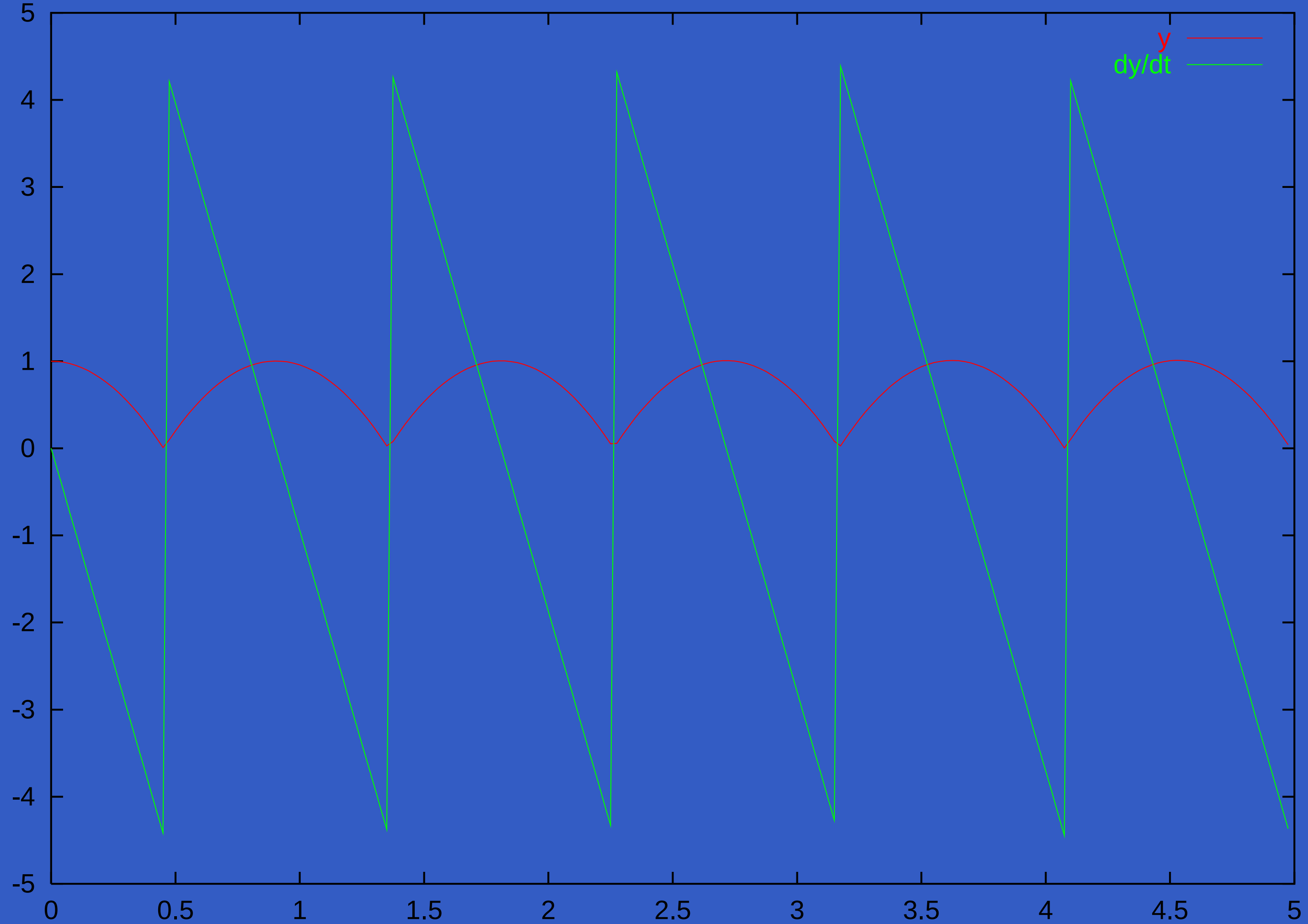
```
countFrom :: Int -> SF (Event a) Int
countFrom n =
  switch
    (constant n &&& notYet)
    (const (countFrom (n+1)))
```

Modelling the bouncing ball: part 3

Making the ball bounce:

```
bouncingBall :: Pos -> SF () (Pos, Vel)
bouncingBall y0 = bbAux y0 0.0
  where
    bbAux y0 v0 =
      switch (fallingBall’ y0 v0) $ \(y,v) ->
      bbAux y (-v)
```

Simulation of bouncing ball



Modelling using impulses (1)

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Modelling using impulses (1)

From a modelling perspective, using a device like `switch` to model the interaction between the ball and the floor may seem rather unnatural.

A more appropriate account of what is going on is that an ***impulsive*** force is acting on the ball for a short time.

This can be abstracted into ***Dirac Impulses***: impulses that act instantaneously. See

> Henrik Nilsson. Functional Automatic Differentiation with Dirac Impulses. In *Proceedings of ICFP 2003*.

Modelling using impulses (2)

However, Yampa does provide a derived version of `integral` capturing the basic idea:

```
impulseIntegral ::
    VectorSpace a k =>
        SF (a, Event a) a
```

The decoupled switch

```
dSwitch ::
        SF a (b, Event c)
        -> (c -> SF a b)
        -> SF a b
```

- Output at the point of switch is taken from the old subordinate signal function, ***not*** the new residual signal function.
- This means the ***output*** at the current point in time is ***independent*** of whether or not the ***switching event*** occurs at that point in time. Hence decoupled.

The recurring switch

```
rSwitch, drSwitch ::
    SF a b -> SF (a,Event (SF a b)) b
```

- Switching events received on the signal function input, carrying signal function to switch into.
- Switching occurs whenever an event occurs, not just once.
- Can be defined in terms of `switch`/`dSwitch`.

Reading (1)

- John Hughes. Generalising monads to arrows. *Science of Computer Programming*, 37:67–111, May 2000
- John Hughes. Programming with arrows. In *Advanced Functional Programming*, 2004. Springer Verlag.
- Ross Paterson. A New Notation for Arrows. In *Proceedings of the 2001 ACM SIGPLAN International Conference on Functional Programming*, pp. 229–240, Firenze, Italy, 2001.

Reading (2)

- Henrik Nilsson, Antony Courtney, and John Peterson. Functional reactive programming, continued. In *Proceedings of the 2002 Haskell Workshop*, pp. 51–64, October 2002.
- Paul Hudak, Antony Courtney, Henrik Nilsson, and John Peterson. Arrows, robots, and functional reactive programming. In *Advanced Functional Programming*, 2002. LNCS 2638, pp. 159–187.

Reading (3)

- Hai Liu, Eric Cheng and Paul Hudak. Causal Commutative Arrows and Their Optimization. In *The 14th ACM SIGPLAN International Conference on Functional Programming (ICFP 2009)*, Edinburgh, Scotland, September, 2009
- Richard S. Bird. A calculus of functions for program derivation. In *Research Topics in Functional Programming*, Addison-Wesley, 1990.