CONSTRUCTIVE OPERATIONAL SET THEORY

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We present a system of constructive sets which has similarities with Feferman's operational set theory and Beeson's intuitionistic set theory with rules ([2], [5], [6]).

The set theory is obtained by introducing a notion of operation or rule aside the notion of set. One may regard the sets as representing the mathematical domain, and the operations as clarifying the computational content of the theory. While Feferman considers classical operational set theory and Beeson introduces an impredicative intuitionistic set theory with rules, we here present a constructive, thus predicative, theory of sets. This may also be seen as a bridge between constructive set theory à la Aczel and Feferman's explicit mathematics ([1], [4]). In previous joint work we introduced an operational extension, COST, of Aczel's CZF, which resembled as much as possible this set theory ([3]). In particular, it had CZF's implicit principles of collection. We here introduce a fragment, EST, of COST characterised by its being fully explicit. The set theory introduced is proof–theoretically weak, indeed as strong as Peano Arithmetic.

We here present some of the properties of the system and hint at the proof technique utilised for determining its proof–theoretic strength.

References

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