Introduction to the Workshop on Logics for Resource-Bounded Agents

Natasha Alechina and Brian Logan

ESSLLI 2015, Barcelona
Topics of the workshop

- logics for modelling resource-bounded reasoners
  - epistemic logics for modelling resource-bounded reasoners
  - logics for modelling bounded memory, forgetting etc.
- logics for reasoning about resources
Timetable and brief introduction to the talks

Tuesday: Nils Bulling, *Verifying Resource-Bounded Agents*
Stéphane Demri, *Reversal-Bounded Counter Machines*

Wednesday: Fernando Velázquez-Quesada, *Forgetting Propositional Formulas*
Sophia Knight, *A Strategic Epistemic Logic for Bounded Memory Agents*

Thursday: Lasha Abzianidze, *A Logic of Belief with the Complexity Measure*
Igor Sedlár, *Substructural Epistemic Logics*

Friday: Dario Della Monica, *Model Checking Coalitional Games in Shortage Resource Scenarios*
Valentin Goranko, *Resource Bounded Reasoning in Concurrent Multi-Agent Systems*
Outline

- logics for modelling resource bounded reasoners
  - logical omniscience
  - Step Logic
  - Algorithmic Knowledge
  - Justification Logic
  - Dynamic Syntactic Epistemic Logic
- logics for reasoning about resources
  - RB-ATL
  - $\text{RB} \pm \text{ATL}$
- open problems
Logics for modelling resource-bounded reasoners

- this will be familiar to people who attended Fernando’s course last week

- often, in this approach knowledge and beliefs are modelled syntactically rather than using possible worlds semantics

- we will give a brief survey of this area

- the talks by Fernando Velázquez-Quesada, Sophia Knight, Lasha Abzianidze, and Igor Sedlár belong to this area
another area of the workshop is reasoning about actions that cost resources

at least from our point of view, the two areas are very connected

we started investigating syntactic epistemic logics where actions of deriving a formula and communicating had explicit costs, and storing formulas cost memory

we then generalised it to Coalition Logic (CL) and Alternating Time Temporal Logic (ATL) where action have costs (RB-CL, RB-ATL, RB±ATL)
Logics for reasoning about resources

- resource quantities are numerical, and in addition to states we get vectors of numbers (resource amounts) updated by transitions

- this is why model-checking of such systems is related to decision problems for counter machines and vector addition systems with state

- the talks by Nils Bulling, Stéphane Demri, Hoang Nga Nguyen, Dario Della Monica and Valentin Goranko belong to this area
Epistemic logic: logical omniscience

- epistemic logic studies belief and knowledge modalities

- it usually interprets ‘agent knows (believes) that $\phi$’ as ‘$\phi$ is true in all knowledge (belief)-accessible possible worlds’

- clearly, tautologies are all true in all accessible worlds, so the agent believes all tautologies

- also, if the agent believes $\phi$, and $\psi$ is a logical consequence of $\phi$, then $\psi$ is true in all $\phi$-worlds, so the agent believes $\psi$ as well

- so the agent believes all logical theorems and can derive infinitely many consequences infinitely fast (logical omniscience problem)
Logical omniscience: is this a problem?

- Hintikka 1975: philosophical problem (human reasoners)
- however, idealised reasoners *can be* considered logically omniscient (capable of arbitrary correct inferences)
- after all, not many people complain that epistemic logic does not account for logical mistakes
When logical omniscience is a problem

- logical modelling and verification of AI agents

- if we ascribe beliefs to the agent incorrectly (for example assume that it believes arbitrary logical consequences of its beliefs when it does not) then we may model its behaviour incorrectly

- so if we ascribe to the agent an ability to reason in logic, then:
  - either the agent should really be able to reason (and exactly to the extent that the logic predicts)
  - or, its internal belief language and belief tests in its action selection should be so trivial that it does not matter
Solutions to the logical omniscience problem

- impossible worlds (beliefs still closed under logical consequence but in a weaker logic)
- neighbourhood semantics (beliefs are closed under logical equivalence: if the agent believes one tautology, it believes them all)
- explicit knowledge defined using awareness (syntactic notion - ‘awareness set’ is an arbitrary set of formulas)
- algorithmic knowledge, syntactic knowledge/beliefs (beliefs are tokens to be manipulated rather than propositions corresponding to sets of possible worlds)
Step logic

- Elgot-Drapkin & Perlis 1990

- The idea is to represent stages in agent’s reasoning (corresponding to time points):

\[
\begin{align*}
  i : & \quad A, \ A \rightarrow B \\
  \hline
  i + 1 : & \quad B
\end{align*}
\]

- If at time $i$ the agent knows $A$ and $A \rightarrow B$, then at time $i + 1$ the agent will know $B$
Algorithmic Knowledge

- Halpern, Moses, and Vardi 1994: agents’ explicit knowledge is given by an algorithm they use to answer queries

- Pucella 2006: deductive algorithmic knowledge

- Explicit knowledge of agents comes from a logical theory expressed by a deductive system consisting of deduction rules

- Agents’ explicit knowledge is closed with respect to this set of rules (similar to Konolige 1986)
Logical omniscience as a complexity problem

- Artemov, Kuznets, Krupski since 2006, inspired by Justification Logic
- A proposition can be feasibly knowable if it is provable in polynomial time
- To be more precise:
  - A system weakly avoids logical omniscience, if for every provable $K A$, $A$ has a polynomial size proof
  - A system strongly avoids logical omniscience, if there is a polynomial algorithm such that which for every provable $K A$, produces a proof of $A$
Consider an agent reasoning in S4

- $K (A \rightarrow B) \rightarrow (K A \rightarrow K B)$
- $K A \rightarrow A$
- $K A \rightarrow K K B$
- Necessitation: if $A$ is an axiom, $\vdash_{S4} K A$
Feasible knowledge in $S4_\bullet$ (Artemov et al)

- $[k_1](A \rightarrow B) \rightarrow ([k_2]A \rightarrow [k_1 \cdot k_2]B)$
- $[k]A \rightarrow A$
- $[k]A \rightarrow [!k][k]B$
- If $A$ is an axiom, $\vdash_{S4_\bullet} [\bullet]A$
- $S4_\bullet$ (with $[k]$ read as knowledge operator) weakly avoids logical omniscience)
- Justification logic ($\bullet$ replaced by axiom names) strongly avoids logical omniscience
Dynamic syntactic epistemic logic

- Ho Ngoc Duc 1997: ‘φ is true after some train of thought of agent i’
- adds a generic operator \( \langle F_i \rangle \), for each agent \( i \), to the language
- \( \langle F_i \rangle K_i \phi \) means that agent \( i \) can get to know the formula \( \phi \) some time in the future
- Duc presents a formal logical system \( DES4_n \) for this language, intended to be a dynamic version of \( S4_n \)
- \( DES4_n \) describes agents who do not necessarily know any consequences of their knowledge now, but can get to know any such consequence in the future
- a sound and complete semantics for \( DES4_n \) is given in Ågotnes & Alechina 2006
More work on epistemic logics without omniscience

- Alechina & Logan 2001 (modal version of step logic)
- Ågotnes 2004 (PhD thesis on syntactic knowledge, knowing inference rules)
- Jago 2006 (PhD thesis on resource-bounded reasoning)
- Velázquez Quesada 2011 (PhD thesis on dynamics of information)
- ...
The basic idea of dynamic syntactic epistemic logic

Agent $a_1$'s epistemic state

$A$

$A \rightarrow B$

$B \rightarrow C$
The basic idea of dynamic syntactic epistemic logic

\[
K_{a_1} A \quad K_{a_1} (A \rightarrow B)
\]
Suppose the agent only knows Modus Ponens

\[ K_{a_1} A \quad K_{a_1} (A \rightarrow B) \]

\[ K_{a_1} B \]

\[ \text{Agent } a_1\text{'s epistemic state} \]

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Introduction to LRBA

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Eventually it can derive all consequences by MP

\[ K_{a_1} A \quad K_{a_1}(A \rightarrow B) \]

\[ K_{a_1} B \]

\[ K_{a_1} C \]
Resources required for reasoning

- so far, we only looked at the number of steps/proof length
- what about memory required for reasoning?
- what about communication (in a multi-agent setting)?
variants of Alternating-Time Temporal Logic (ATL) where transitions have costs (or rewards) and the syntax can express resource requirements of a strategy, e.g.:

agents $A$ can enforce outcome $\varphi$ if they have at most $b_1$ units of resource $r_1$ and $b_2$ units of resource $r_2$

various flavours of resource logics exist: RBCL, RB-ATL, RB±ATL (Alechina et al.), RAL (Bulling & Farwer), PRB-ATL (Della Monica et al.), QATL* (Bulling & Goranko)
Verification Using Resource Logic

- one of the main problems in resource logics is model-checking
- model-checking problem: given a structure, a state in the structure and a formula, does the state satisfy the formula?
- using model-checking, we can verify resource requirements of a multi-agent system (specify the system as a model, and write a formula expressing a system objective)
for most resource logics the model-checking problem is undecidable: in particular, various flavours of RAL, and QATL*

here, we present two resource logics with decidable model-checking problems:
- RB-ATL which allows only consumption of resources
- RB±ATL which allows unbounded production of resources
RB-ATL: syntax

- $Agt = \{a_1, \ldots , a_n\}$ a set of $n$ agents
- $Res = \{res_1, \ldots , res_r\}$ a set of $r$ resources,
- $\Pi$ a set of propositions
- $B = \mathbb{N}_\infty^r$ a set of resource bounds, where $\mathbb{N}_\infty = \mathbb{N} \cup \{\infty\}$
Formulas of RB-ATL are defined by the following syntax

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \lor \psi \mid \langle\langle A^b \rangle\rangle \varphi \mid \langle\langle A^b \rangle\rangle \varphi U \psi \mid \langle\langle A^b \rangle\rangle \Box \varphi \]

where \( p \in \Pi \) is a proposition, \( A \subseteq \text{Agt} \), and \( b \in B \) is a resource bound.
RB-ATL: meaning of formulas

- $\langle A^b \rangle \bigcirc \psi$ means that a coalition $A$ can ensure that the next state satisfies $\varphi$ under resource bound $b$

- $\langle A^b \rangle \psi_1 U \psi_2$ means that $A$ has a strategy to enforce $\psi$ while maintaining the truth of $\varphi$, and the cost of this strategy is at most $b$

- $\langle A^b \rangle \Box \psi$ means that $A$ has a strategy to make sure that $\varphi$ is always true, and the cost of this strategy is at most $b$
A RB-CGS is a tuple $M = (Agt, Res, S, \Pi, \pi, Act, d, c, \delta)$ where:

- $Agt$ is a non-empty set of $n$ agents, $Res$ is a non-empty set of $r$ resources and $S$ is a non-empty set of states;

- $\Pi$ is a finite set of propositional variables and $\pi : \Pi \rightarrow \wp(S)$ is a truth assignment

- $Act$ is a non-empty set of actions which includes $idle$, and $d : S \times Agt \rightarrow \wp(Act) \setminus \{\emptyset\}$ is a function which assigns to each $s \in S$ a non-empty set of actions available to each agent $a \in Agt$

- $c : S \times Agt \times Act \rightarrow \mathbb{Z}^r$ (the integer in position $i$ indicates consumption of resource $res_i$ by the action $a$)

- $\delta : (s, \sigma) \mapsto S$ for every $s \in S$ and joint action $\sigma \in D(s)$ gives the state resulting from executing $\sigma$ in $s$. 
for every $s \in S$ and $a \in Agt$, $idle \in d(s, a)$

- $c(s, a, idle) = \tilde{0}$ for all $s \in S$ and $a \in Agt$ where $\tilde{0} = 0'$

- we denote joint actions by all agents in $Agt$ available at $s$ by $D(s) = d(s, a_1) \times \cdots \times d(s, a_n)$

- for a coalition $A$, $D_A(s)$ is the set of all joint actions by agents in $A$

- $out(s, \sigma) = \{ s' \in S \mid \exists \sigma' \in D(s) : \sigma = \sigma'_A \land s' = \delta(s, \sigma')\}$

- $cost(s, \sigma) = \sum_{a \in A} c(s, a, \sigma_a)$
Example: dynamic syntactic epistemic logic in RB-ATL

Agent $a_1$'s epistemic state

$A$

$A \rightarrow B$

$B \rightarrow C$
Example: dynamic syntactic epistemic logic in RB-ATL

\[ K_{a_1} A \quad K_{a_1} (A \rightarrow B) \]

Agent \( a_1 \)’s epistemic state:

\[ K_{a_1} A \quad K_{a_1} (A \rightarrow B) \]

\[ \begin{align*}
A \\
A \rightarrow B \\
B \rightarrow C
\end{align*} \]
Example dynamic syntactic epistemic logic in RB-ATL

Application of MP is an action that costs 1 unit of time and 1 unit of memory

\[ \langle\{a_1\}^{time:1, memory:1}\rangle \Box K_{a_1} B \]

\[ K_{a_1} A \quad K_{a_1} (A \rightarrow B) \]

\[ K_{a_1} B \]

Agent \( a_1 \)'s epistemic state

\[ K_{a_1} A \quad K_{a_1} (A \rightarrow B) \]

\[ K_{a_1} B \]

\[ MP \]

\[ a_1 \]

\[ a_1 \]
Example: dynamic syntactic epistemic logic in RB-ATL

\[
\langle\langle \{a_1\}^{\text{time:1, memory:1}} \rangle \rangle \bigcirc K_{a_1} B \\
\langle\langle \{a_1\}^{\text{time:2, memory:2}} \rangle \rangle \bigvee U K_{a_1} C
\]

\[K_{a_1} A \quad K_{a_1} (A \rightarrow B)\]

\[K_{a_1} B\]

\[K_{a_1} C\]
Example: extending to multi-agent case

\[ \langle \{ a_1, a_2 \} \rangle \text{time:3, memory:3, energy:1} \top U K_{a_2} K_{a_1} C \]
Strategies and their costs

- A strategy for a coalition $A \subseteq \text{Agt}$ is a mapping $F_A : S^+ \rightarrow \text{Act}$ such that, for every $\lambda s \in S^+$, $F_A(\lambda s) \in D_A(s)$.

- A computation $\lambda \in S^\omega$ is consistent with a strategy $F_A$ iff, for all $i \geq 0$, $\lambda[i + 1] \in \text{out}(\lambda[i], F_A(\lambda[0, i]))$.

- $\text{out}(s, F_A)$ the set of all consistent computations $\lambda$ of $F_A$ that start from $s$.

- Given a bound $b \in B$, a computation $\lambda \in \text{out}(s, F_A)$ is $b$-consistent with $F_A$ iff, for every $i \geq 0$, $\sum_{j=0}^{i} \text{cost}(\lambda[j], F_A(\lambda[0, j])) \leq b$.

- $F_A$ is a $b$-strategy if all $\lambda \in \text{out}(s, F_A)$ are $b$-consistent.
Truth definition

- $M, s \models \langle A^b \rangle \bigcirc \phi$ iff $\exists$ $b$-strategy $F_A$ such that for all $\lambda \in \text{out}(s, F_A)$: $M, \lambda[1] \models \phi$

- $M, s \models \langle A^b \rangle \phi \bigcup \psi$ iff $\exists$ $b$-strategy $F_A$ such that for all $\lambda \in \text{out}(s, F_A)$, $\exists i \geq 0$: $M, \lambda[i] \models \psi$ and $M, \lambda[j] \models \phi$ for all $j \in \{0, \ldots, i - 1\}$

- $M, s \models \langle A^b \rangle \Box \phi$ iff $\exists$ $b$-strategy $F_A$ such that for all $\lambda \in \text{out}(s, F_A)$ and $i \geq 0$: $M, \lambda[i] \models \phi$
Model-checking RB-ATL

The model-checking problem for RB-ATL is the question whether, for a given RB-CGS structure $M$, a state $s$ in $M$ and an RB-ATL formula $\phi$, $M, s \models \phi$.

**Theorem (Alechina, Logan, Nguyen, Rakib 2010):**
The model-checking problem for RB-ATL is decidable.
Model-checking algorithm for RB-ATL

```
function RB-ATL-LABEL(M, φ)
    for φ' ∈ Sub⁺(φ) do
        case φ' = p, ¬ψ, ψ₁ ∧ ψ₂
            standard, see [Alur et al. 2002]
        case φ' = ⟨⟨A^b⟩⟩ ◯ψ
            [φ']_M ← Pre(A, [ψ]_M, b)
        case φ' = ⟨⟨A^b⟩⟩ψ₁ U ψ₂
            [φ']_M ← UNTIL-STRATEGY(M, ⟨⟨A^b⟩⟩ψ₁ U ψ₂)
        case φ' = ⟨⟨A^b⟩⟩□ψ
            [φ']_M ← BOX-STRATEGY(M, ⟨⟨A^b⟩⟩□ψ)

    return [φ]_M
```
subsi$m{(\phi_0)}$ includes all subformulas of $\phi_0$, $Sub(\phi_0)$, and in addition:
- if $\langle\langle A^b \rangle\rangle \Box \psi \in Sub(\phi_0)$, then $\langle\langle A^{b'} \rangle\rangle \Box \psi \in Sub^+(\phi_0)$ for all $b' < b$
- if $\langle\langle A^b \rangle\rangle \psi_1 \cup \psi_2 \in Sub(\phi_0)$, then $\langle\langle A^{b'} \rangle\rangle \psi_1 \cup \psi_2 \in Sub^+(\phi_0)$ for all $b' < b$

$Sub^+(\phi_0)$ is partially ordered in increasing order of complexity and of resource bounds (e.g., if $b' \leq b$, $\langle\langle A^{b'} \rangle\rangle \Box \psi$ precedes $\langle\langle A^b \rangle\rangle \Box \psi$)
$Pre(A, \rho, b)$ is a function which takes a coalition $A$, a set $\rho \subseteq S$ and a bound $b$, and returns the set of states $s$ in which $A$ has a joint action $\sigma_A$ with $cost(s, \sigma_A) \leq b$ such that $out(s, \sigma_A) \subseteq \rho$
UNTIL-STRATEGY (RB-ATL)

function UNTIL-STRATEGY($M, \langle A^b \rangle \psi_1 U \psi_2$)

case $\phi' = \langle A^0 \rangle \psi_1 U \psi_2$:
   $\rho \leftarrow [\text{false}]_M; \tau \leftarrow [\psi_2]_M$
   while $\tau \not\subseteq \rho$ do
      $\rho \leftarrow \rho \cup \tau; \tau \leftarrow \text{Pre}(A, \rho, \bar{0}) \cap [\psi_1]_M$
   return $\rho$

case $\phi' = \langle A^b \rangle \psi_1 U \psi_2$ where $b > \bar{0}$:
   $\rho \leftarrow [\text{false}]_M; \tau \leftarrow [\text{false}]_M$
   foreach $b' < b$ do
      $\tau \leftarrow \text{Pre}(A, \langle A^{b'} \rangle \psi_1 U \psi_2)_M, b - b') \cap [\psi_1]_M$
   while $\tau \not\subseteq \rho$ do
      $\rho \leftarrow \rho \cup \tau; \tau \leftarrow \text{Pre}(A, \rho, \bar{0}) \cap [\psi_1]_M$
   return $\rho$
BOX-STRATEGY (RB-ATL)

function BOX-STRATEGY(M, Ⰰ\rangle\langle A^b \rangle \Box \psi)

case φ' = Ⰰ\langle A^0 \rangle \Box \psi:
    ρ ← [\text{true}]_M; \tau ← [\psi]_M
    while ρ ∉ τ do
        ρ ← τ; τ ← Pre(A, ρ, \bar{0}) \cap [\psi]_M
    return ρ

case φ' = Ⰰ\langle A^b \rangle \Box \psi  \text{ where } b > \bar{0}:
    ρ ← [\text{false}]_M; \tau ← [\text{false}]_M
    foreach b' < b do
        τ ← Pre(A, [α\langle A^{b'} \rangle \Box \psi]_M, b - b') \cap [\psi]_M
    while τ ∉ ρ do
        ρ ← ρ \cup τ; τ ← Pre(A, ρ, \bar{0}) \cap [\psi]_M
    return ρ
RB±ATL considers only consumption of resources

A natural question is what happens if actions can produce as well as consume resources.

RB±ATL is a generalisation of RB-ATL where actions can produce resources.
RB±ATL: syntax and semantics

- Syntax and semantics are the same as RB-ATL, but production of resources is allowed.

- $c : S \times Agt \times Act \rightarrow \mathbb{Z}^r$ (the integer in position $i$ indicates consumption or production of resource $res_i$ by the action $a$).

- If one agent consumes 10 units of resource and another agent produces 10 units of resource, the cost of their joint action is 0.

- $b$-strategies are defined as before (the prefix of every computation generated by the strategy costs less than $b$).
Example: two agents $a_1, a_2$, two resources $r_1, r_2$

Actions available to the first agent:

$$d(s_1, a_1) = \{\alpha, \text{idle}\}, \ d(s, a_1) = \{\gamma, \text{idle}\}, \ d(s', a_1) = \{\text{idle}\}$$
Example: two agents $a_1, a_2$, two resources $r_1, r_2$

Actions available to the first agent:

$d(s_I, a_1) = \{\alpha, idle\}$, $d(s, a_1) = \{\gamma, idle\}$, $d(s', a_1) = \{idle\}$

Actions available to the second agent:

$d(s_I, a_2) = \{idle\}$, $d(s, a_2) = \{\beta, idle\}$, $d(s', a_2) = \{idle\}$
Example: two agents $a_1, a_2$, two resources $r_1, r_2$

Costs of actions:

$c(s_I, a_1, \alpha) = \langle -2, 1 \rangle$, 
$c(s, a_1, \gamma) = \langle 5, 0 \rangle$, 
$c(s, a_2, \beta) = \langle 1, -1 \rangle$
Example: strategy $F_1$ for $a_1$

$$s_I \mapsto \alpha$$
$$s_I s \mapsto \gamma$$
$$s_I s s' \ldots \mapsto \text{idle}$$

$$\text{out}(s_I, F_1) = \{s_I, s, s', s', \ldots\}$$

$$c(s_I, a_1, \alpha) = \langle -2, 1 \rangle$$
$$c(s, a_1, \gamma) = \langle 5, 0 \rangle$$
$$c(s, a_1, \text{idle}) = \langle 0, 0 \rangle$$
Example: strategy $F_1$ for $a_1$

$F_1$ is a $\langle 3, 1 \rangle$-strategy:

$$\langle -2, 1 \rangle \leq \langle 3, 1 \rangle$$
$$\langle -2, 1 \rangle + \langle 5, 0 \rangle \leq \langle 3, 1 \rangle$$
$$\langle -2, 1 \rangle + \langle 5, 0 \rangle + \langle 0, 0 \rangle \ldots \leq \langle 3, 1 \rangle$$

$$c(s_I, a_1, \alpha) = \langle -2, 1 \rangle$$
$$c(s, a_1, \gamma) = \langle 5, 0 \rangle$$
$$c(s, a_1, idle) = \langle 0, 0 \rangle$$
A strategy $F$ for $A = \{a_1, a_2\}$

$s_I \mapsto \langle \alpha, idle \rangle$

$s_I s \mapsto \langle idle, \beta \rangle$

$s_I s s_I \mapsto \langle \alpha, idle \rangle, \ldots$

$s_I s s_I s s_I s \mapsto \langle \gamma, idle \rangle$

$s_I s s_I s s_I s s' \ldots \mapsto \langle idle, idle \rangle$

$c(s_I, a_1, \alpha) = \langle -2, 1 \rangle$

$c(s, a_2, \beta) = \langle 1, -1 \rangle$

(repeat $s_I s s_I$ 4 times)

$c(s, a_1, \gamma) = \langle 5, 0 \rangle$
A strategy $F$ for $A = \{a_1, a_2\}$

\[
\text{out}(s_I, F) = \{s_I, s, s_I, s, s_I, s, s_I, s, s_I, s, s', s', \ldots\}
\]

$F$ is a $\langle 0, 1 \rangle$-strategy
The model-checking problem for $\text{RB}^\pm\text{ATL}$ is the question whether, for a given RB-CGS structure $M$, a state $s$ in $M$ and an $\text{RB}^\pm\text{ATL}$ formula $\phi$, $M, s \models \phi$.

**Theorem (Alechina, Logan, Nguyen, Raimondi 2014):**
The model-checking problem for $\text{RB}^\pm\text{ATL}$ is decidable.
Model-checking algorithm for RB±ATL

\begin{verbatim}
function RB±ATL-LABEL(M, φ) for φ' ∈ Sub(φ) do
    case φ' = p, ¬ψ, ψ₁ ∧ ψ₂
        standard, see [Alur et al. 2002]
    case φ' = ⟨⟨A⟩⟩ ∃ψ
        [φ']ₘ ← Pre(A, [ψ]ₘ, b)
    case φ' = ⟨⟨A⟩⟩ ψ₁ U ψ₂
        [φ']ₘ ← { s | s ∈ S ∧ UNTIL±STRATEGY(node₀(s, b), ⟨⟨A⟩⟩ ψ₁ U ψ₂) }
    case φ' = ⟨⟨A⟩⟩ □ψ
        [φ']ₘ ← { s | s ∈ S ∧ BOX±STRATEGY(node₀(s, b), ⟨⟨A⟩⟩ □ψ) }
return [φ]ₘ
\end{verbatim}
Search tree nodes

- UNTIL±STRATEGY and BOX±STRATEGY proceed by depth-first and-or search of $M$

- for each tree node $n$, $s(n)$ returns its state, $p(n)$ returns the nodes on the path to $n$ and $e_i(n)$ returns the resource availability on the $i$-th resource in $s(n)$ as a result of following $p(n)$

- $node_0(s, b)$ returns the root node ($s(n_0) = s$, $p(n_0) = []$ and $e_i(n_0) = b_i$ for all resources $i$)

- $node(n, \sigma, s')$ returns a node $n'$ where $s(n') = s'$, $p(n') = [p(n) \cdot n]$ and for all resources $i$, $e_i(n') = e_i(n) - cost_i(\sigma)$. 
UNTIL±STRATEGY (RB±ATL)

function UNTIL±STRATEGY(n, ≪A≫ψ₁ U ψ₂)
    if s(n) ⊭ ≪A≫ψ₁ U ψ₂ or
        ∃n' ∈ p(n) : s(n') = s(n) ∧ (∀j : e_j(n') ≥ e_j(n)) then
            return false
    for i ∈ {i ∈ Res | ∃n' ∈ p(n) : s(n') = s(n) ∧ (∀j : e_j(n') ≤ e_j(n)) ∧
                    e_i(n') < e_i(n)} do
        e_i(n) ← ∞
    if s(n) |= ψ₂ or e(n) = ∞ then
        return true
    for σ ∈ {σ ∈ DA(s(n)) | cost(σ) ≤ e(n)} do
        strat ← true
        for s' ∈ out(s(n), σ) do
            strat ← strat ∧ UNTIL±STRATEGY(node(n, σ, s'), ≪A≫ψ₁ U ψ₂)
        if strat then
            return true
    return false
**BOX±STRATEGY (RB±ATL)**

**function** BOX±STRATEGY\((n, \langle A^b \rangle \Box \psi)\)

1. if \(s(n) \not\models \langle A \rangle \Box \psi\) or
   1. \(\exists n' \in p(n) : s(n') = s(n) \land (\forall j : e_j(n') > e_j(n))\) then
     return false
   2. \(\exists n' \in p(n) : s(n') = s(n) \land (\forall j : e_j(n') \leq e_j(n))\) then
     return true
2. for \(\sigma \in \{\sigma \in DA(s(n)) | \text{cost}(\sigma) \leq e(n)\}\) do
   1. strat &- true
   2. for \(s' \in \text{out}(s(n), \sigma)\) do
     1. strat &- strat \(\land\) BOX±STRATEGY\((\text{node}(n, \sigma, s'), \langle A^b \rangle \Box \psi)\)
   if strat then
     return true
3. return false
Complexity

- The model-checking problem for RB±ATL is EXPSPACE-hard.
- Special cases have lower complexity:
  - One resource: PSPACE
  - No production (RB-ATL): PTIME in formula and transition system, exponential in the number of resources.
There are many open problems in both areas

- other tractable cases of resource reasoning
- modelling combinations of reasoning and acting in resource logics
- accounting for the costs of observation and communication in dynamic epistemic logic
- etc.
Infinite bound versions

Since the infinite resource bound version of RB-ATL modalities correspond to the standard ATL modalities, we write

- $\langle A^\infty \rangle \bigcirc \phi$ as $\langle A \rangle \bigcirc \phi$
- $\langle A^\infty \rangle \phi U \psi$ as $\langle A \rangle \phi U \psi$
- $\langle A^\infty \rangle \Box \phi$ as $\langle A \rangle \Box \phi$
Auxiliary functions: $\text{split}(b)$

$s\text{plit}(b)$ is a function that takes a resource bound $b$ and returns the set of all pairs $(d, d') \in \mathbb{N}_\infty \times \mathbb{N}_\infty$ such that:

1. $d + d' = b$
2. $d_i = d'_i = \infty$ for all $i \in \{1, \ldots, r\}$ where $b_i = \infty$
3. $d$ has at least one non-0 value

The set of all pairs $(d, d')$ is partially ordered in increasing order of $d'$ (i.e., if $d'_1 < d'_2$, then $(d_1, d'_1)$ precedes $(d_2, d'_2)$)
Auxiliary functions: $Sub^+(\phi_0)$

$Sub^+(\phi_0)$ includes all subformulas of $\phi_0$, $Sub(\phi_0)$, and in addition:

- if $\langle\langle A^b \rangle\rangle \square \psi \in Sub(\phi_0)$, then $\langle\langle A^{d'} \rangle\rangle \square \psi \in Sub^+(\phi_0)$ for all $d'$ such that $(d, d') \in split(b)$

- if $\langle\langle A^b \rangle\rangle \psi_1 \cup \psi_2 \in Sub(\phi_0)$, then $\langle\langle A^{d'} \rangle\rangle \psi_1 \cup \psi_2 \in Sub^+(\phi_0)$ for all $d'$ such that $(d, d') \in split(b)$

$Sub^+(\phi_0)$ is partially ordered in increasing order of complexity and of resource bounds (e.g., if $b' \leq b$, $\langle\langle A^{b'} \rangle\rangle \square \psi$ precedes $\langle\langle A^b \rangle\rangle \square \psi$)
$Pre(A, \rho, b)$ is a function which takes a coalition $A$, a set $\rho \subseteq S$ and a bound $b$, and returns the set of states $s$ in which $A$ has a joint action $\sigma_A$ with $cost(s, \sigma_A) \leq b$ such that $out(s, \sigma_A) \subseteq \rho$
UNTIL-STRATEGY (RB-ATL)

function UNTIL-STRATEGY(M, ⟨⟨A^b⟩⟩ψ_1 U ψ_2)
    case φ' = ⟨⟨A^b⟩⟩ψ_1 U ψ_2 where ∀i b_i ∈ {0, ∞}:
        ρ ← [false]_M; τ ← [ψ_2]_M
        while τ ⊈ ρ do
            ρ ← ρ ∪ τ; τ ← Pre(A, ρ, b) ∩ [ψ_1]_M
        return ρ
    case φ' = ⟨⟨A^b⟩⟩ψ_1 U ψ_2 where ∃i b_i ∉ {0, ∞}:
        ρ ← [false]_M; τ ← [false]_M
        foreach d' ∈ {d' | (d, d') ∈ split(b)} do
            τ ← Pre(A, [⟨⟨A^{d'}⟩⟩ψ_1 U ψ_2]_M, d) ∩ [ψ_1]_M
            while τ ⊈ ρ do
                ρ ← ρ ∪ τ; τ ← Pre(A, ρ, 0 ∞ b) ∩ [ψ_1]_M
        return ρ
BOX-STRATEGY (RB-ATL)

function BOX-STRATEGY($M, \left\langle A^b \right\rangle \Box \psi$)

**case** $\phi' = \left\langle A^b \right\rangle \Box \psi$ where $\forall i b_i \in \{0, \infty\}$:

$\rho \leftarrow [true]_M; \quad \tau \leftarrow [\psi]_M$

**while** $\rho \not\subseteq \tau$ **do**

$\rho \leftarrow \tau; \quad \tau \leftarrow Pre(A, \rho, b) \cap [\psi]_M$

**return** $\rho$

**case** $\phi' = \left\langle A^b \right\rangle \Box \psi$ where $\exists i b_i \not\in \{0, \infty\}$:

$\rho \leftarrow [false]_M; \quad \tau \leftarrow [false]_M$

**foreach** $d' \in \{d' \mid (d, d') \in split(b)\}$ **do**

$\tau \leftarrow Pre(A, [\left\langle A^{d'} \right\rangle \Box \psi]_M, d) \cap [\psi]_M$

**while** $\tau \not\subseteq \rho$ **do**

$\rho \leftarrow \rho \cup \tau; \quad \tau \leftarrow Pre(A, \rho, \bar{0} \bowtie b) \cap [\psi]_M$

**return** $\rho$