Recent Developments on Combinatorial Optimisation Problems

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Dr Khin Lwin, Teesside University, UK
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Campuses, The University of Nottingham
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ASAP Group, The University of Nottingham
Main Algorithms & Applications

• Methodologies
  – Meta-heuristics
    • Evolutionary algorithms, Local search
    • Hyper-heuristics
    • Hybrids
  – Exact approaches
    • Constraint programming
    • Integer / linear programming
  – Hybridisations
Main Algorithms & Applications

• Applications
  – Personnel/workforce scheduling
  – Portfolio optimisation
  – Telecommunication network routing
  – Vehicle routing in logistics
  – Timetabling
  – ...

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HYPER-HEURISTICS
Recent Algorithms on Combinatorial Optimisation Problems

Background

- **Search space**
  - All possible solutions

- **Design of algorithms**
  - Problem specific information hard coded
  - Parameters fine tuned for different problems (or instances)

Meta-heuristics

Operates upon

Potential solutions
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Background

meta-heuristics

operates upon

potential solutions

hyper-heuristic

operates upon

heuristics

operates upon

potential solutions
All the term *hyper-heuristic* says is: “*Operate on a search space of heuristics*” or “*Heuristics that choose heuristics*”

- Most meta-heuristics operate directly on problems
- Hyper-heuristics operate on heuristics, which are then applied on the actual problems  
  — *automatically* work well on *different* problems
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Background

• Research challenges
  – Automate heuristic design
    • Now made by human experts
    • Not cheap!
  – How general we could make hyper-heuristics
    • No free lunch theorem\cite{WOL97}

\textbf{Background}

Recent Algorithms on Combinatorial Optimisation Problems

## Applications

### December

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Total Penalty: 176
Unassigned Shifts: 0

### Optimal Solution:

```
M3
M2
M1
```

```
3,1
2,1
1,1
1,2
3,2
2,3
1,3
3,3
```

Optimal solution:
Algorithms

• Low level heuristics
  – Constructive: Construct solutions step by step
    • Graph colouring heuristics, etc.
  – Improvement: Initial solutions improved iteratively
    • Different improvement strategies

• High level heuristics
  – Genetic Algorithms, Tabu Search, Simulated Annealing, Genetic Programming, etc.
  – Case Based Reasoning, Multi-objective techniques, Fuzzy Techniques, choice function, etc.
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HYPER-HEURISTICS
- A GRAPH BASED HYPER-HEURISTIC
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Background

• Constructive heuristics in scheduling
  – Job shop scheduling: dispatching rules
  – Timetabling: graph heuristics
  – Bin packing: 2D/3D packing heuristics
  – Simple and fast

In complex scheduling problems, using only the basic constructive heuristics often produce unacceptable solutions

• Automated hybridisation / combination of simple heuristics
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An example – Timetabling Problems

• Important activities in all universities
• A general timetabling problem
  – A set of events
  – A set of timeslots
  – A set of rooms
  – Schedule the events to timeslots
    • No events for students at the same time
    • Spread students’ events
    • ...

...
An example – Timetabling Problems

• Timetabling problems
  – Assign a set of events into a number of time slots, minimising violations of soft constraints

• Hard constraints
  – Conflicted events in different time slots
  – Room capacity to hold the events, etc.

• Soft constraints
  – Spread out events over time slots / at least n events or no event on a day
  – No event scheduled on specific time slots, etc.
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An example – Timetabling Problems

• Timetabling problems
  –Exact methods
    • IP/MILP
  –Constructive heuristics
    • Graph heuristics
    • Constraint satisfaction, etc
  –Meta-heuristics
    • Local search based algorithms
    • Population based algorithms
    • Hybridisations, etc
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Framework

• The high level framework
  – Any meta-heuristics or learning/search methodology
• The low level graph heuristics: order events by how difficult to schedule them
  – Saturation Degree: least available slots
  – Colour Degree: most conflicted with those scheduled
  – Largest Degree: most conflicted with the others
  – Largest Weighted Degree: LD + students involved
  – Largest Enrolment: students enrolled
• Hyper-heuristics: Heuristics to choose heuristics
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Framework

events

heuristic list

order of events

slots

Framework

The University of Nottingham

ASAP research group
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Framework

| events | e2 | e4 | e5 | e6 | e7 | e8 | e10 | e11 | e12 | ...
|--------|----|----|----|----|----|----|-----|-----|-----|-----|

| Heuristic list | SD | SD | LD | CD | LE | SD | SD | LW | SD | LD | CD | RO | ...
|-----------------|----|----|----|----|----|----|----|----|----|----|----|----|-----|

| order of events | e6 | e17 | e28 | e19 | e10 | e31 | e12 | e5 | e22 | e32 | e27 | e19 | ...
|-----------------|----|-----|-----|----|-----|-----|-----|----|-----|-----|-----|-----|-----|

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Framework

High level search

Events

| e2 | e4 | e5 | e7 | e8 | ... |

Heuristic list

| SD | SD | LD | CD | LE | SD | SD | LW | SD | LD | CD | RO | ...

Order of events

| e5 | e32 | e19 | e22 | e13 | e31 | e12 | e7 | e2 | e15 | e27 | e12 | ...

Slots

| e1 | e3 | e6 | e19 | e26 | e25 | e28 | e17 | e10 | e5 | e13 | e32 | e19 | e13 |
Research issues

• Which high/low level search heuristics?

• Search in two search spaces

• Heuristic hybridisations

• Landscape analysis on heuristic spaces

• Extensions on the framework and other problems
Research issues

• High level search methods
  – Iterated Local Search
  – Tabu Search
  – Steepest Descent
  – Variable Neighbourhood Search

  – Objective function
    • heuristic lists $\rightarrow$ penalties (costs of timetables constructed)
  – “Walks” are allowed
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**Research issues**

- High level search methods
  - Similar performance within the same framework (same total no. of evaluations, same initials, etc)
  - ILS and VNS are slightly better
  - Results are comparable to best approaches on both course and exam benchmark problems

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Research issues

• Low level heuristics
  – Different subsets of graph heuristics (SD+LD, SD+LWD, SD+LE, SD+LWD+CD, etc)
  – With a limited computational time
    • SD + LWD performed the best
  – With more graph heuristics
    • Longer time given, the better the results
    • \( h/l \) \((l: \text{length of the sequence}, \ h: \text{number of graph heuristics})\)
  – Random ordering also contributes the performance
Research issues

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GHH: search is upon heuristics, not solutions – not all the solutions in solution space are reachable?
## Research issues

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<th>Representation</th>
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Research issues

• Hybridisation in the framework with simple greedy search
  – High level search in heuristic space: a, b, c, ...
  – Greedy search in solution space: b -> d, ...

• Coverage of the solution space
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Research issues

• Hybridisation in the framework with simple greedy search
  – Results greatly improved!

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Recent Algorithms on Combinatorial Optimisation Problems

Research issues

• Hybridisation in the framework with simple greedy search
  – Hybrid GHH vs. Memetic Algorithms
• Diversification vs. intensification

![Diagram](image-url)
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Research issues

• Search in two search spaces
  – Diversification of the high level search in the framework in the heuristic space
  – Intensification by the local search in the solution space

• Role of high level search methods
  *To explore diversified solutions in the solution space by searching in the high level heuristic space*
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Hybridisations

• How to (adaptively) hybridise heuristics?
  Knowledge / lesson learnt from the offline heuristic hybridisations?
I – Random (SD+LWD, SD+LE, SD+LD)
  A large collection of different heuristic sequences
  Systematically produce heuristic sequences
  Full coverage of different amount of hybridisations
II – Analyze the best/worst 5% heuristic sequences
  Rates of hybridisation at different positions of heuristic sequences
  Trends of hybridizations in the best sequences

<table>
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Hybridisations

• Results of analysis
  – Hybridising SD with LWD obtained better results compared with LE or LD
  – In the best 5% sequences
    • Higher percentage at early stage
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Hybridisations

- Adaptive online heuristic hybridization
  - Focus on early stage of heuristic hybridization
  - Rate of LWD hybridisation adaptively adjusted
Landscape analysis

• Understanding the structure of heuristic search spaces, i.e. heuristic sequences vs. solutions
• Fitness landscape analysis on constructive hyper-heuristics
  – Fitness distance correlation ($fdc$) of local optima to the global optimum
  – One-flip of global optimum
  – Correlation length
• Although rugged, the encouraging feature of a globally big valley structure
• A high level of neutrality and positional bias
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Landscape analysis

[Graph showing cost versus Hamming distance with points indicating local optima for sta83 I and hec82 I]
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Landscape analysis

The University of Nottingham
OTHER RESEARCH TOPICS
## Nurse Rostering Problems

### Nurse Rostering web site at
**http://www.asap.cs.nott.ac.uk/projects/nmhpr/data**

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### Too few resting time (10)

- Circled cells indicate issues.
- Unassigned Shifts: 0
- Total Penalty: 176

### Too few consecutive night shifts (5)

- Circled cells indicate issues.
- Unassigned Shifts: 0
- Total Penalty: 176

### Too few consecutive late shifts (5)

- Circled cells indicate issues.
- Unassigned Shifts: 0
- Total Penalty: 176
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Nurse Rostering Problems

- Hybrid variable neighbourhood search
  - HARMONY™, ORTEC, The Netherlands
- Constraint programming
- Sequence based adaptive approach
Portfolio Optimisation

- Allocation of capital of budget to selected assets, aiming to minimise risk and maximise return
- Markowitz’s modern portfolio theory
  - Mean-Variance model
  - Efficient frontier
  - Risk vs. return

Matlab™
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**Freight Transport Routing**

- Design of routing plan, starting from a depot, to serve all customers within a network
- **Constraints**
  - Capacity
  - Time window
  - Pick-up vs. drop
- **Objectives**
  - Cost
  - Empty load
  - ...
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Multicast Routing

• Finding the multicast tree serving all terminals with the minimal cost while satisfying delay bound
• Multiple objectives
  – Maximal end-to-end delay
  – Maximal link utilisation
  – Average delay
Questions?

Thank you!

– More details at: http://www.cs.nott.ac.uk/~rxq/publications.htm
Recent Algorithms on Combinatorial Optimisation Problems

Selected References

• More references at http://www.asap.cs.nott.ac.uk/?q=bibliography