# School for Computer Science and Information Technology

Machines and their languages (G51MAL) Spring 2004

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**8th Coursework – Mock exam**

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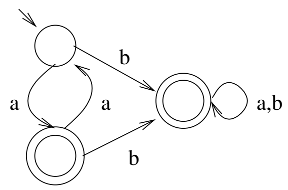
**No Deadline**

**Question 1: (Compulsory)** The following questions are multiple choice. There is at least one correct choice but there may be several. To get all the marks you have to list all the correct answers and none of the wrong ones.

a. Given the following Deterministic Finite Automaton (DFA) $A_1$ over $\Sigma_1 = \{\mathtt{a}, \mathtt{b}\}$.




Which of the following statements about $A_1$ are correct?

(i) $\mathtt{aabb} \in L(A_1)$

(ii) $\mathtt{bbaa} \in L(A_1)$

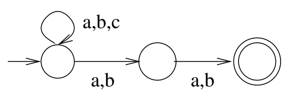
(iii) $A_1$ accepts precisely the words where the symbol a occurs before the symbol b.

(iv) $A_1$ accepts precisely the words with an odd number of as or at least one b.

(v) $A_1$ accepts precisely those words where the last symbol is the symbol b.

(5)

b. Given the following Nondeterministic Finite Automaton (NFA) $A_2$ over $\Sigma_2 = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.




Which of the following statements about $A_2$ are correct?

(i) $\mathtt{abc} \in L(A_2)$

(ii) $\mathtt{cba} \in L(A_2)$

(iii) $A_2$ accepts precisely the words which contain the sequence ab.

(iv) $A_2$ accepts precisely the words where neither of the last two symbols is the symbol c.

(v) $A_2$ accepts the empty word.

(5)

c. Which of the following is equivalent to "$L$ is regular", where $L$ is a language?

(i) $L$ is recognized by a Pushdown Automaton (PDA).
(ii) $L$ doesn't contain any palindromes.
(iii) $L$ is recognized by a computer program using only fixed finite memory.
(iv) $L$ is deterministic.
(v) $L$ is recognized by a Nondeterministic Finite Automaton (NFA).

(5)

d. Given the following Context-Free Grammar (CFG)

$$G = (\{S, T\}, \{\texttt{(}, \texttt{)}, \texttt{a}, \texttt{b}\}, S, P)$$

with productions $P$:

$$S \to \texttt{(}T\texttt{)} \mid \texttt{a}$$
$$T \to \texttt{(}S\texttt{)} \mid \texttt{b}$$

Which of the following are in the language of $G$ (i.e. are elements of $L(G)$)?

(i) `((a))`
(ii) `(b))`
(iii) `a(b)`
(iv) `(a)`
(v) `b`

(5)

e. Which of the following propositions are correct?

(i) The empty word is not contained in any language.
(ii) There are languages which can be recognized by a Nondeterministic Finite Automaton (NFA) but not by a Deterministic Finite Automaton (DFA).
(iii) There are languages which can be recognized by a Pushdown Automaton (PDA) but not by a Turing Machine (TM).
(iv) A Turing Machine may fail to terminate.
(v) A language is a set of words.

(5)

**Question 2**

a. Given the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ we define the language $L \subseteq \Sigma^*$ as:

$$L = \{\mathtt{a}^i \mathtt{b}^j \mathtt{c}^k \mid i + j \equiv k \mod 2\}$$

i.e. a word in the language is of the form

$$\underbrace{\mathtt{a}\ldots\mathtt{a}}_{i}\underbrace{\mathtt{b}\ldots\mathtt{b}}_{j}\underbrace{\mathtt{c}\ldots\mathtt{c}}_{k}$$

such that $i + j$ is even if and only if $k$ is even; e.g. in the language are

$$\epsilon, \mathtt{aa}, \mathtt{ab}, \mathtt{abcc}, \mathtt{aabccc}$$

and not in the language are

$$\mathtt{abc}, \mathtt{aabcc}, \mathtt{c}, \mathtt{cb}, \mathtt{cba}$$

Construct a Deterministic Finite Automaton (DFA) which accepts $L$. The automaton should be presented using a transition diagram where the initial and final states are marked as usual.

(12)

b. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and let $\#_\mathtt{a}, \#_\mathtt{b} \in \Sigma^* \to \mathbb{N}$ be functions which count the numbers of as and bs respectively. E.g. $\#_\mathtt{a}(\mathtt{aababa}) = 4$ and $\#_\mathtt{b}(\mathtt{aababa}) = 2$. We use these functions to define the following languages:

$$\begin{aligned} L_1 &= \{w \mid \#_\mathtt{a}(w) \equiv \#_\mathtt{b}(w) \mod 2\} \\ L_2 &= \{w \mid \#_\mathtt{a}(w) = \#_\mathtt{b}(w)\} \\ L_3 &= \{w \mid \#_\mathtt{a}(w) > 0 \iff \#_\mathtt{b}(w) > 0\} \end{aligned}$$

(i) Which of the languages $L_1, L_2, L_3$ are regular?

(5)

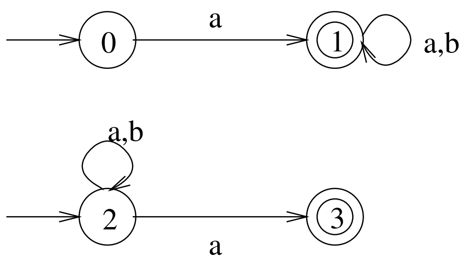
(ii) Provide evidence for your answer to (i) by either

- Constructing a DFA, NFA or a Regular Expression defining the language, or
- Using the pumping lemma to show that the language in question is not regular.

(8)

**Question 3**

a. Give the following Nondeterministic Finite Automaton (NFA) $A$ over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$:




Apply the subset construction to derive a Deterministic Finite Automaton (DFA) $D(A)$ which recognizes the same language. You should restrict yourself to the reachable part of $D(A)$, i.e. states which are not reachable from the initial state should be left out.

(10)

b. Is the automaton constructed in part a. minimal? I.e. can you find a Deterministic Finite Automaton (DFA) which accepts the same language but uses fewer states?

(5)

c. Give Regular Expressions for the following languages over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$:

(i) Words where all `a`s occur before any `b`s.

(ii) Words that do contain the symbol `c`.

(iii) Words that contain an odd number of symbols.

(iv) Words that end with the sequence `aa`.

(v) Words that do not end with the sequence `aa`.

(10)

**Question 4**

The syntax of formulas of propositional logic over the atomic propositions $p, q, r$ is given by the following rules:

- $p, q, r$ are formulas.
- $\text{true}, \text{false}$ are formulas.
- If $A$ is a formula then $\neg A$,$(A)$ are formulas.
- If $A, B$ are formulas then $A \wedge B, A \vee B$ are formulas.

To avoid ambiguity we also introduce the following conventions:

- $\neg$ binds stronger than $\wedge$ and $\vee$
- $\wedge$ binds stronger than $\vee$

a. Construct a Context-free Grammar (CFG) over

$$\Sigma = \{p, q, r, (, ), \wedge, \vee, \neg, \text{true}, \text{false}\}$$

whose language is the set of syntactically correct formulas. Design your grammar such that the conventions about binding strength are reflected by the parse trees.

(15)

b. Draw parse trees for $p \wedge q \vee r$ and $p \wedge (q \vee r)$.

(5)

c. Answer in one or two sentences the question: *How would you go about implementing a parser for this language?*

(5)

**Question 5**

Given the following Pushdown Automaton (PDA) $P$

$$P = (Q = \{q_0, q_1\}, \Sigma = \{\mathtt{a}, \mathtt{b}\}, \Gamma = \{a, b, \#\}, \delta, q_0, Z_0 = \#, F = \{q_0\})$$

where $\delta$ is given by the following equations:

$$\begin{array}{lcl}
\delta(q_0, \mathtt{a}, \#) & = & \{(q_1, a\#)\} \\
\delta(q_0, \mathtt{b}, \#) & = & \{(q_1, b\#)\} \\
\delta(q_1, \mathtt{a}, b) & = & \{(q_1, \epsilon)\} \\
\delta(q_1, \mathtt{a}, a) & = & \{(q_1, aa)\} \\
\delta(q_1, \mathtt{b}, a) & = & \{(q_1, \epsilon)\} \\
\delta(q_1, \mathtt{b}, b) & = & \{(q_1, bb)\} \\
\delta(q_1, \epsilon, \#) & = & \{(q_0, \#)\} \\
\delta(q, w, z) & = & \{\} \qquad \text{everywhere else}
\end{array}$$

a. Construct sequences of Instantaneous Descriptions (IDs) for the words

$$\mathtt{bba}, \mathtt{baab}, \epsilon$$

Which of these words are accepted (using acceptance by final state)?

(12)

b. Describe the language accepted by $P$ in one sentence.

(5)

c. What does it mean for a PDA to be deterministic? Is $P$ deterministic?

(8)

**Question 6**

Write a short essay addressing each of the questions below. Try to be as concise as possible.

a. What is a Turing Machine (TM)? Given an informal description.

(5)

b. What is the difference between:

- A TM *accepts* a language, and
- A TM *decides* a language.

(5)

c. What is the *halting problem*? Can it be considered as a language?

(5)

d. How do we show that the *halting problem* is undecidable?

(10)