School of Computer Science, University of Nottingham

G52MAL Machines and their Languages, Spring 2012

Thorsten Altenkirch

Solutions to Exercises, Set 4

7 March 2012

1. We begin the table-filling algorithm by drawing the table of all possible pairings. We then apply the basis of the algorithm which states that all accepting states are distinguishable from all non-accepting states. Thus we mark all such pairs as distinguishable in the table, and list the remaining pairs.

	0	1	2	3	4
5				×	×
4	×	×	×		
3	×	×	×		
2					
1					

$(0,1)$	$(0,2)$	$(0,5)$	$(1,2)$	$(1,5)$	$(2,5)$	$(3,4)$

We now perform the inductive step of considering each pair in turn. First we look at 0 and 1. We know that for an input symbol a, we go to the same pair of states 0 and 1. Thus for the symbol a they are indistinguishable. However, for the symbol b, we go to the pair of states 1 and 2. We do not yet know whether 1 and 2 are distinguishable, so we note that whether 0 and 1 are distinguishable depends on whether 1 and 2 are distinguishable.

$(0,1)$	$(0,2)$	$(0,5)$	$(1,2)$	$(1,5)$	$(2,5)$	$(3,4)$
			$(0,1)$			

Next we consider 0 and 2. For the input symbol a, we go to the pair $(0,3)$. We know that 0 and 3 are distinguishable, so by induction we know that 0 and 2 are distinguishable. We mark this in the table, and cross off the $(0,2)$ pair.

	0	1	2	3	4
5				×	×
4	×	×	×		
3	×	×	×		
2	×				
1					

$(0,1)$	~~$(0,2)$~~	$(0,5)$	$(1,2)$	$(1,5)$	$(2,5)$	$(3,4)$
			$(0,1)$			

Next is 0 and 5. For an a they stay in 0 and 5, which is cyclic and tells us that they are indistinguishable for a. For a b they both go to state 1, so are indistinguishable for b. Thus we conclude that 0 and 5 are indistinguishable, and thus *do not* mark them in the table.

The next pair is 1 and 2. For an a we go to $(1,3)$, which we already know are distinguishable. Thus we mark 1 and 2 as distinguishable, and cross off the $(1,2)$ pair. The pair $(1,2)$ has a pair underneath it, and thus this implication can be discharged and we can also cross off $(0,1)$.

	0	1	2	3	4
5				×	×
4	×	×	×		
3	×	×	×		
2	×	×			
1	×				

~~$(0,1)$~~	~~$(0,2)$~~	$(0,5)$	~~$(1,2)$~~	$(1,5)$	$(2,5)$	$(3,4)$
			$(0,1)$			

We next consider 1 and 5. For a b they go to 1 and 2, which we have just determined are distinguishable, and thus $(1, 5)$ can be crossed off.

	0	1	2	3	4
5		$\times$		$\times$	$\times$
4	$\times$	$\times$	$\times$		
3	$\times$	$\times$	$\times$		
2	$\times$	$\times$			
1	$\times$				

~~$(0,1)$~~	~~$(0,2)$~~	$(0,5)$	~~$(1,2)$~~	~~$(1,5)$~~	$(2,5)$	$(3,4)$
			$(0,1)$			

Next is 2 and 5, which can be distinguished by an a (going to $(3, 5)$).

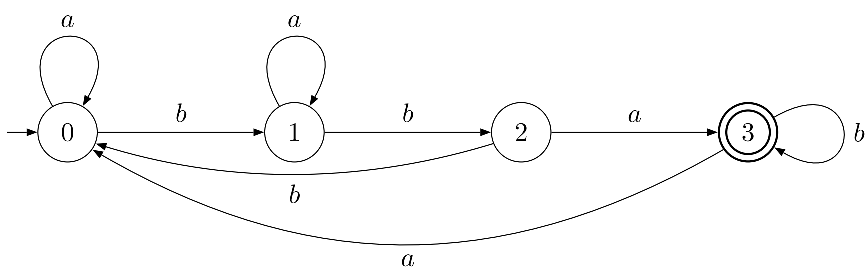
	0	1	2	3	4
5		$\times$	$\times$	$\times$	$\times$
4	$\times$	$\times$	$\times$		
3	$\times$	$\times$	$\times$		
2	$\times$	$\times$			
1	$\times$				

~~$(0,1)$~~	~~$(0,2)$~~	$(0,5)$	~~$(1,2)$~~	~~$(1,5)$~~	~~$(2,5)$~~	$(3,4)$
			$(0,1)$			

Finally, 3 and 4 cannot be distinguished by a b, but could potentially be by an a if 0 and 5 are distinguishable. Thus we write $(3, 4)$ underneath $(0, 5)$.

~~$(0,1)$~~	~~$(0,2)$~~	$(0,5)$	~~$(1,2)$~~	~~$(1,5)$~~	~~$(2,5)$~~	$(3,4)$
		$(3,4)$	$(0,1)$			

However, we have now finished, and neither $(3, 4)$ nor $(0, 5)$ have been found to be distinguishable. Thus they must be equivalent, and can be merged.



2.

3. (a) L_1 s regular and given by $L_1 = L((00)^*)$.
   (b) Is not regular. We use the pumping lemma: let k be the pumping number. We know that $w = 0^k10^k1 \in L_2$. Clearly, $y = 0^l$ and by the pumping lemma $0^{k-l}10^k$ should be in the language but it is not of the form ww. hence L_2 is not regular.
   (c) L_3 is regular and is given by $L_3 = L((00) * (11) * +0(00)^*1(11)^*)$.
   (d) L_4 is not regular. (This was proven in the lecture).
   (e) L_5 is regular and given by $L_5 = L((11 + 111)^*)$.

4. **Bonus Exercise**

   Let k be the pumping number. $w = 0^k1^{k+k!} \in L$. Using the pumping lemma we know that $y = 0^j$ for $j \leqslant k$ and for all i $0^{k+ji}1^{k+K!}$ should be in L. Now we choose $\frac{i=k!}{j}$. This is an integer since $j < k$ and hence $k!$ is divisible by j. However, $\frac{k+ji=k+(k!}{j)j=k+k!}$ but $0^{k+k!}1^{k+k!}$ is not in L. Hence L is not regular.