Mathematics for Computer Scientists

G51MCS

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Today I'm going to illustrate these topics with some puzzles.

Logic: Cats and Gorillas

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- Cats with whiskers always wear heron suits.

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- No sociable cat has blunt claws.

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Can you then logically conclude that:

No cat with blunt claws will play with a gorilla?

From Ian Stewart, *Professor Stewart's Cabinet of Mathematical Curiosities*

Logic: Knights and Knaves

There is an island where all inhabitants are either knights, who always tell the truth, or knaves, who always lie.

From Raymond Smullyan, *What is the name of this book?*

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There is an island where all inhabitants are either knights, who always tell the truth, or knaves, who always lie.

First puzzle: You meet two inhabitants, Anna and Benjamin. Anna tells you '*We are both knaves*'. Can you deduce who's a knight and who's a knave?

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Logic: Knights and Knaves

There is an island where all inhabitants are either knights, who always tell the truth, or knaves, who always lie.

Second puzzle: You meet two other inhabitants, Carl and Dora. Carl tells you '*If Dora is a knave, and only in that case, I am also a knave*'. Dora tells you '*We are of different kind*'. Who's a knight and who's a knave?

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Third puzzle: Two inhabitants, Frank and Elisa, are standing at a fork in the road. You know that one of them is a knight and the other a knave, but you don't know which. You also know that one road leads to Death, and the other leads to Freedom. By asking one yes/no question, can you determine the road to Freedom?

From Raymond Smullyan, *What is the name of this book?*

Arithmetic: The Monkey and the Coconuts

Five men and a monkey were shipwrecked on a desert island, and they spent the first day gathering coconuts for food. They piled them all up together and then went to sleep for the night.

The Monkey and the Coconuts

But when they were all asleep one man woke up and decided to take his share. He divided the coconuts in five piles. He had one coconut left and he gave it to the monkey. He took his pile and put the rest all back together.

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But when they were all asleep one man woke up and decided to take his share. He divided the coconuts in five piles. He had one coconut left and he gave it to the monkey. He took his pile and put the rest all back together.
Later a second man woke up and acted in exactly the same way: He divided the coconuts into five piles; there was one coconut left that he tossed to the monkey; he took his pile and put the rest all back together.

The Monkey and the Coconuts

One after the other all five men did the same thing and every time there was one extra coconut given to the monkey.
The next morning they divided what coconuts were left into five parts. Once again there was one coconut left that was given to the monkey.

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How many coconuts were there in the beginning?

From Martin Gardner, *The Colossal Book of Mathematics*

Combinatorics: Handshakes at the Smiths' house

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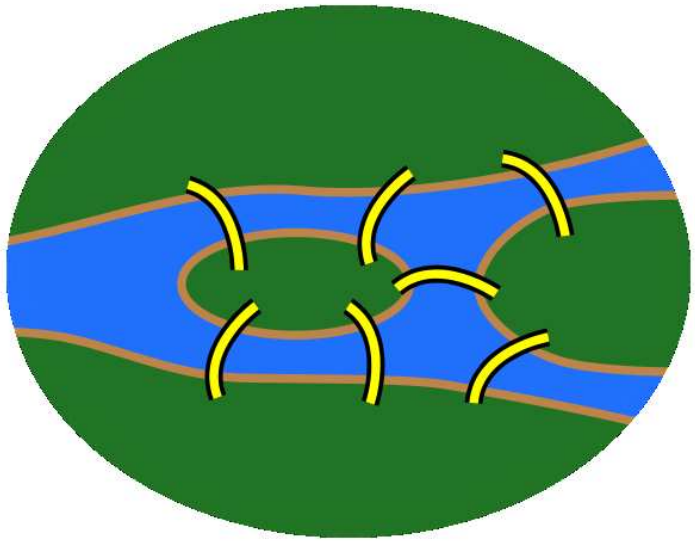
At the end of the party, Mr Smith observes: '*If you don't count me, there are no two people who shook hands the same number of times*'.

How many times did Mrs Smith shake hands?

Graphs: the Bridges of Königsberg

The city of Königsberg was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges:



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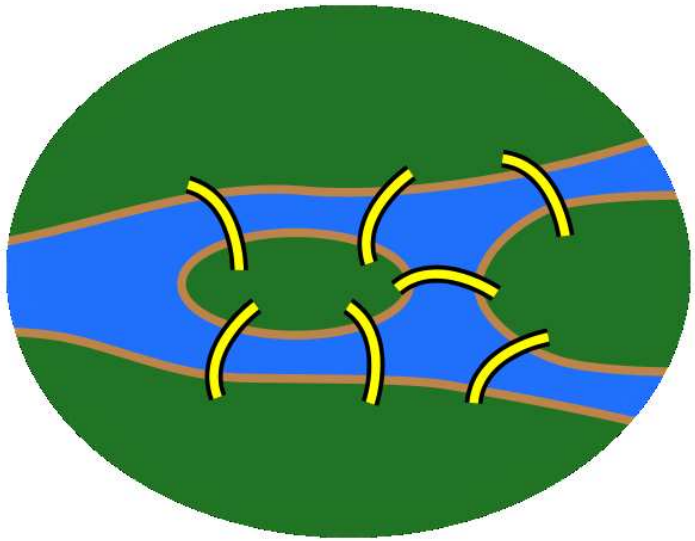

Can you find a walk through the city, starting and and arriving at the same place, that crosses each bridge once and only once?