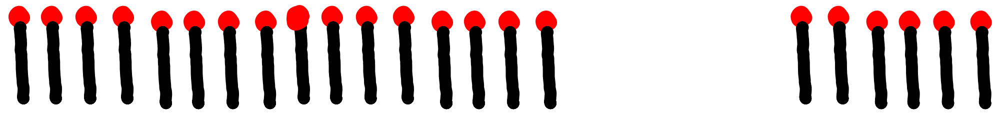
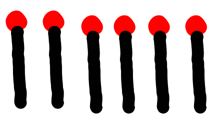
Game Sum

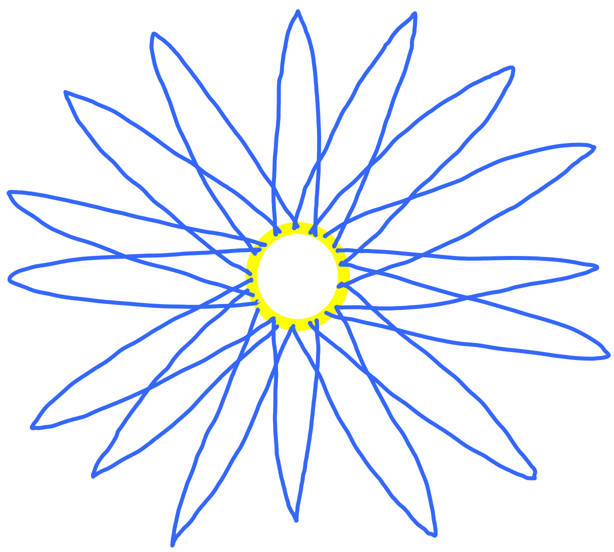
Note Title 02/11/2005



A game with two piles of matches is the "sum" of two single-pile matchstick games.

Another "sum" of two games:





Move: choose either remove some matches
or remove a petal

The *sum* of two games is a game defined as follows:

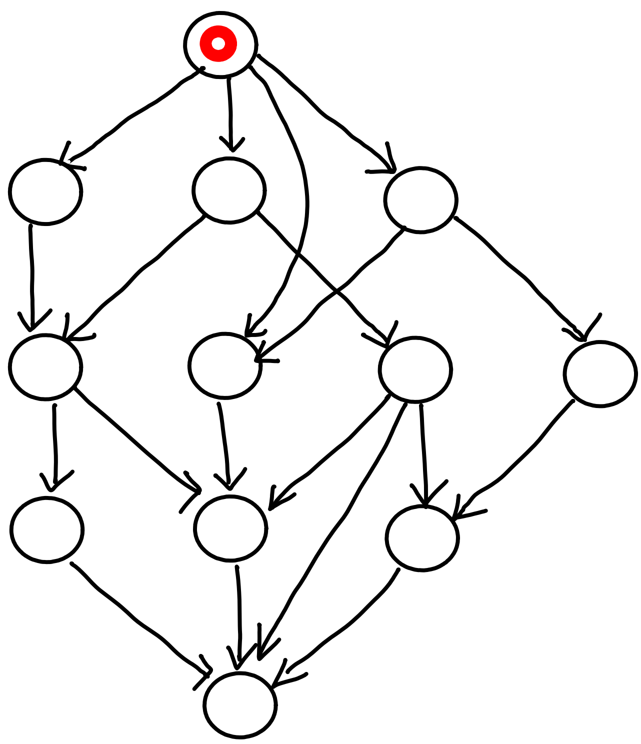
Call the two games the *left* game and the *right* game.

A *position* in the sum game is an ordered pair (ℓ, r) of positions, where ℓ is a position in the left game and r is a position in the right game.

A *move* $(\ell, r) \mapsto (\ell', r')$ in the sum game satisfies

either : $\ell \mapsto \ell'$ is a move in the left game, and $r = r'$,

or : $\ell = \ell'$ and $r \mapsto r'$ is a move in the right game.

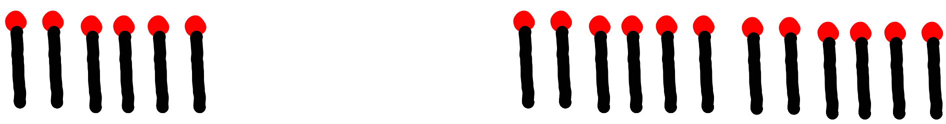


How to win.

Identify a property of positions
(the *strategy invariant*) such that

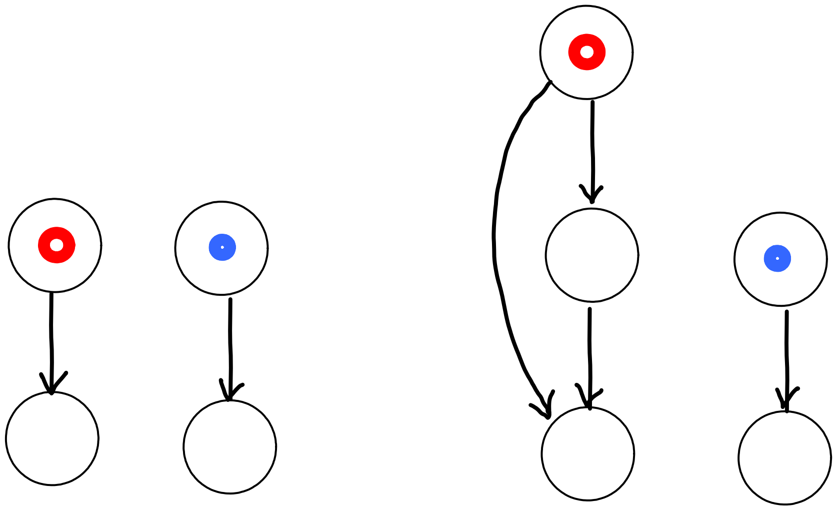
- all end positions satisfy the property
- *every* move from a position satisfying the property *falsifies* the property.
- for every position that does not satisfy the property *there is* a move that *truthifies* the property.

m matches n matches

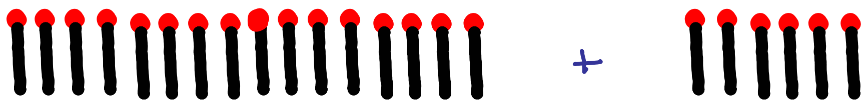


Move in left or right game: remove any +ve no. of matches.

Strategy:



Game 1 Game 2



Left game : remove 1 thru M matches

mex no. of m matches $=$ m mod (M+1)

Right game : remove 1 thru N matches

mex no. of n matches $=$ n mod (N+1)

Strategy invariant : m mod (M+1) $=$ n mod (N+1)

Winning strategy :

if m mod (M+1) > n mod (N+1) → remove m mod (M+1) − n mod (N+1) matches from left pile

□ m mod (M+1) < n mod (N+1) → remove n mod (N+1) − m mod (M+1) matches from right pile

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Key: loser
winner

$\{ m \bmod (M+1) = n \bmod (N+1) \}$

if $m \neq 0 \rightarrow$ decrease m by at most M

$\square$ $n \neq 0 \rightarrow$ decrease n by at most N

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$\{ m \bmod (M+1) \neq n \bmod (N+1) \}$

if $m \bmod (M+1) > n \bmod (N+1) \rightarrow$ remove $m \bmod (M+1) - n \bmod (N+1)$ matches from left pile

$\square$ $m \bmod (M+1) < n \bmod (N+1) \rightarrow$ remove $n \bmod (N+1) - m \bmod (M+1)$ matches from right pile

fi

$\{ m \bmod (M+1) = n \bmod (N+1) \}$