# Induction

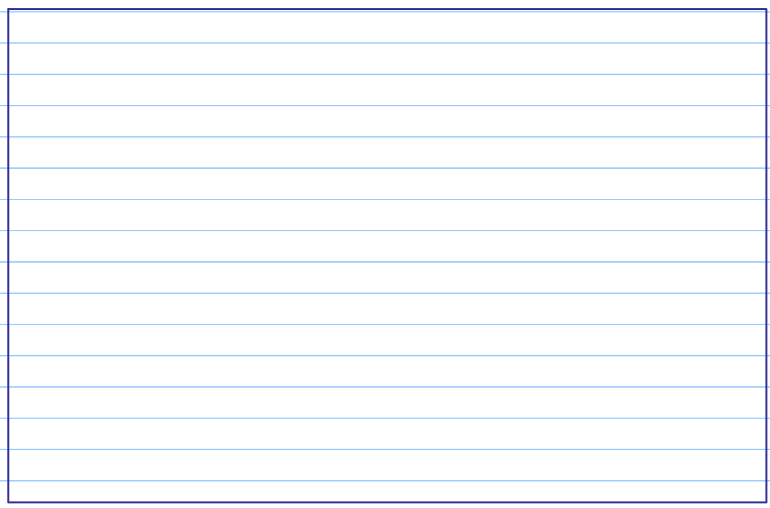


Used to solve a *class* of problems
where each *instance* has a *size* (a natural number).

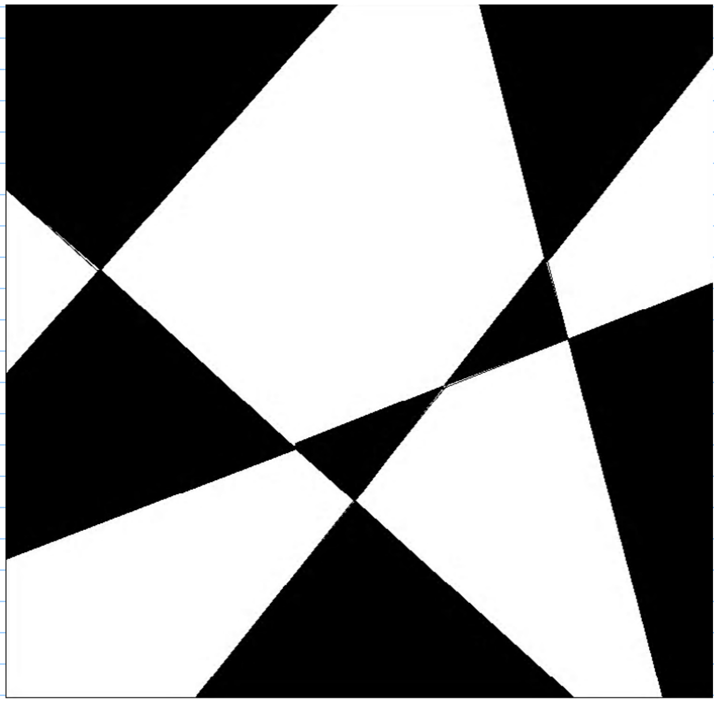
- Construct a solution to instances of size 0.

- Assuming that it is possible to construct solutions to instances of size $n$, construct a solution to instances of size $n+1$.

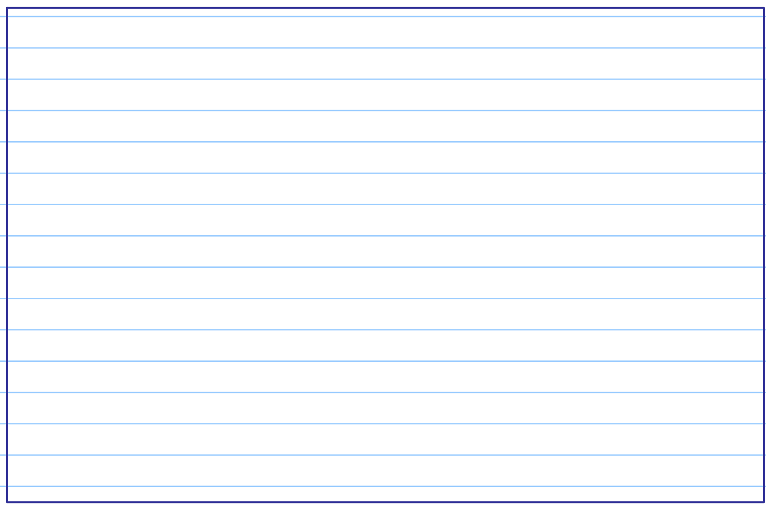
## Cutting the Plane



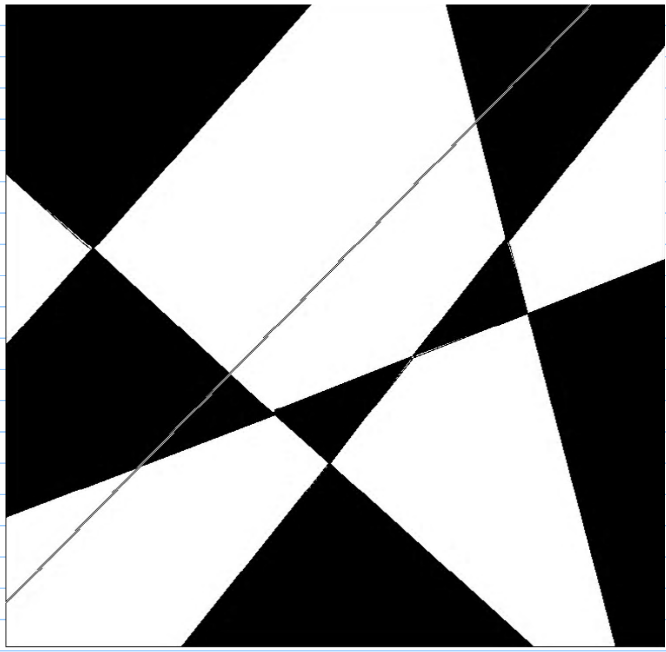
Example



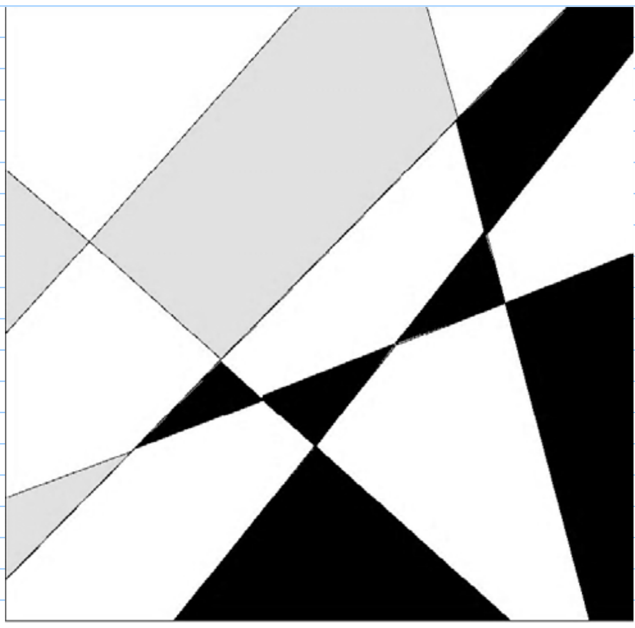
Basis: 0 lines.



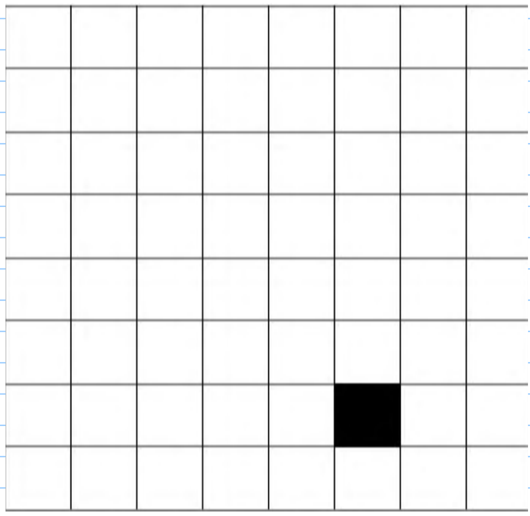
# Induction step: n+1 lines

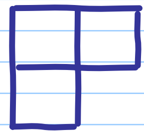


# Induction step (continued)

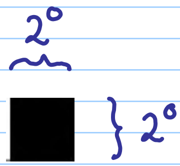


# Triomino Problem

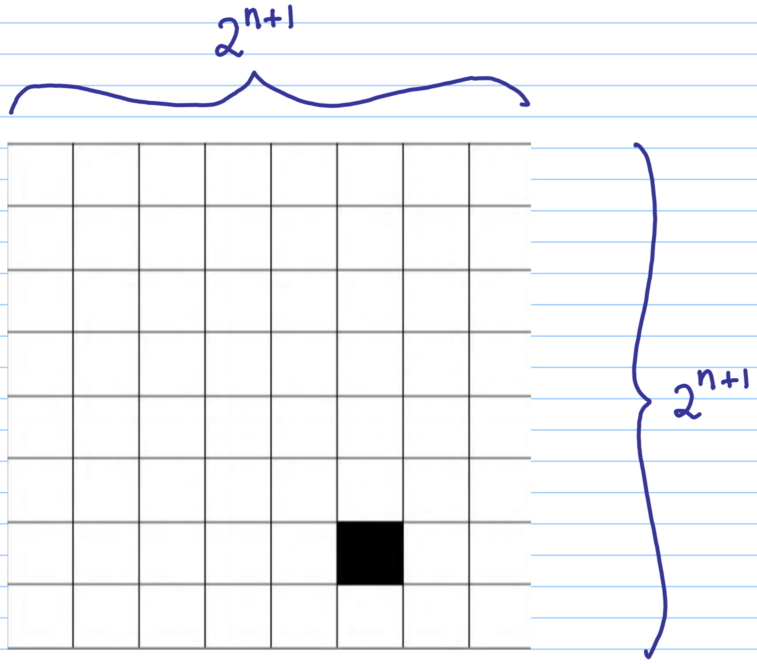


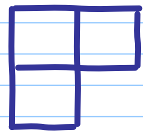


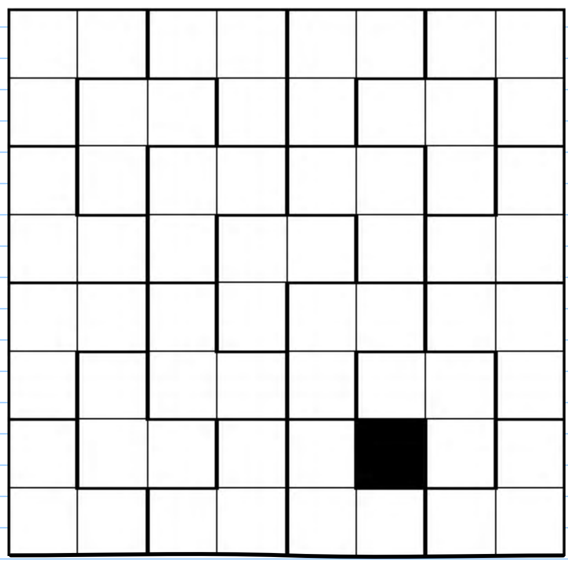
# Basis: $n = 0$



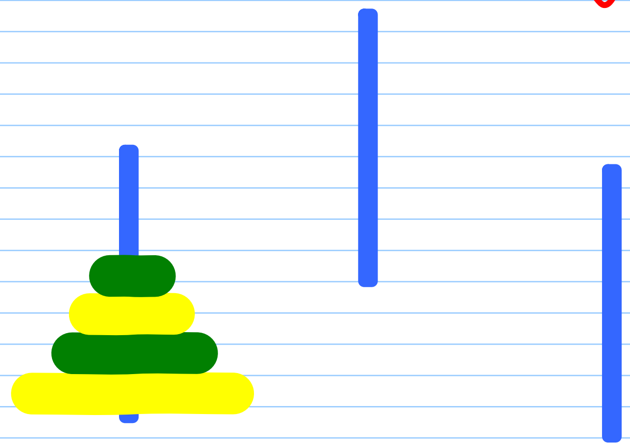
# Induction step: $n+1$





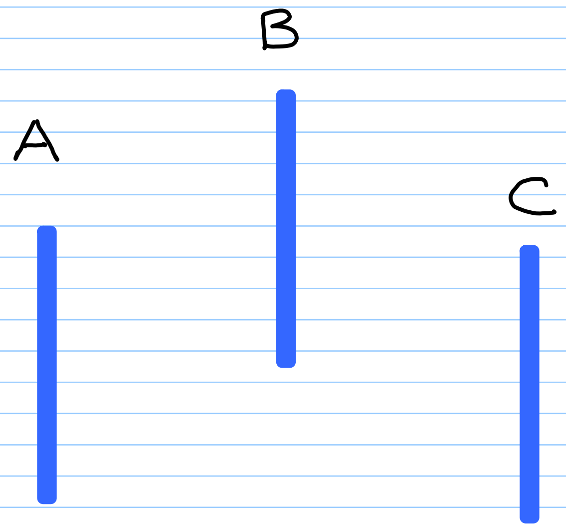
# Tower of Hanoi
# State Transition Diagram



Move n disks from one pole to the next
- one at a time
- so that a larger disk is never on top of a smaller disk.

State: sequence of poles

Eg. ABACA



Basis: 0 disks

Induction step: $n+1$ disks

NB: disk $n+1$ (largest) can only be moved to an *empty* pole.

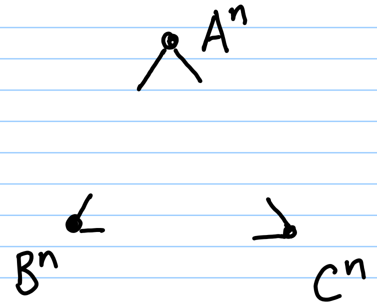
$A^n B$ —— $A^n C$

$B^n C$ —— $B^n A$

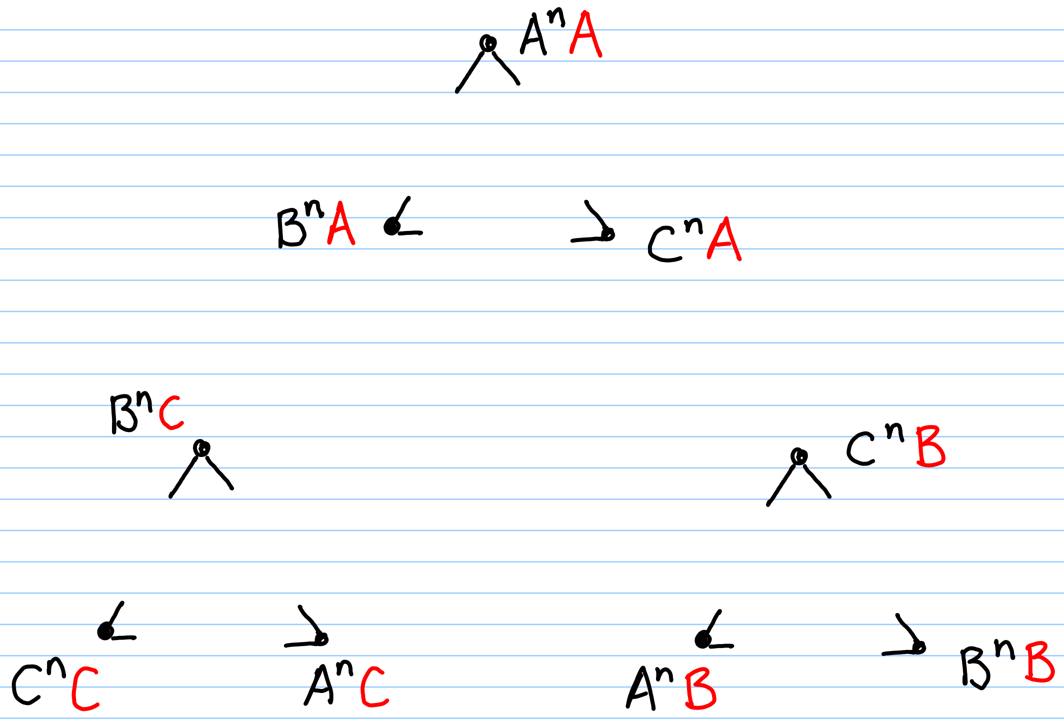
$C^n A$ —— $C^n B$

Hypothesis: assume transition diagram for $n$ disks.

1. identify states $A^n$, $B^n$, $C^n$.



2. Make 3 copies, each with a different position for the (n+1) th disk.




3. Add the transitions for disk n+1.