Design of Algorithms
Assessed Coursework 2012-2013

School of Computer Science
University of Nottingham

October 31, 2012

1 Counting Intersections

The following program counts the number of times that the line

$$x, y \;::\; N{\times}x \;=\; M{\times}y$$

intersects a unit square between the points $(0,0)$ and (M,N), where M and N are strictly positive natural numbers.

$\{ \quad 0{<}M \;\wedge\; 0{<}N \quad \}$
$p,q,c \;:=\; 0,0,0\;;$
$\{$ **Invariant:** $\quad N{\times}p - N < M{\times}q \;\leq\; N{\times}p \;\vee\; M{\times}q - M < N{\times}p \;\leq\; M{\times}q \quad \}$
do $p{<}M \vee q{<}N \;\rightarrow$ if $\quad N{\times}p{<}M{\times}q \;\rightarrow\; p{:=}p{+}1$
$\Box \quad N{\times}p{=}M{\times}q \;\rightarrow\; p,q \;:=\; p{+}1\,,q{+}1$
$\Box \quad N{\times}p{>}M{\times}q \;\rightarrow\; q{:=}q{+}1$
fi ;
$c{:=}c{+}1$
od
$\{ \quad p{=}M \;\wedge\; q{=}N \quad \}$

(a) Construct verification conditions that check the correctness of the primitive tests and assignments with respect to the given assertions. Which verification condition(s) are invalid if one or more of M and N is not strictly positive? (10)
(b) Add conjuncts to the invariant and postcondition that assert the relation between c, p and q. (You do not need to construct the verification conditions.) (5)

2 Drawing an Ellipse

The equation of an *ellipse* is

$$x, y \quad :: \quad a \times x^2 + b \times y^2 \quad = \quad c \quad ,$$

where a , b and c are given constants.

Develop a Bresenham-like, "online" program to construct a drawing on a raster display of an ellipse in the quadrant

$$m, n \quad :: \quad 0 \leq m \ \wedge \ 0 \leq n \quad .$$

Split your program into two phases . The first part should plot (an integer approximation to) the ellipse for those values of m and n where the gradient is at least -1 . The second phase should plot (an integer approximation to) the ellipse for those values of m and n where the gradient is at most -1 . You should assume that a , b and c are strictly positive integers.

You must provide full details of the design following the steps used in the lecture notes for drawing a straight line and a circle. Make clear how the termination condition for the first phase of the plot is determined.

Marks will be awarded as follows: phase 1 (excluding termination condition), 20 marks; termination condition for phase 1, 10 marks; phase 2, 5 marks.

3 Submitting Your Solution

Your solutions should be submitted to the School Office by ***3pm on Friday, 7th December***. If you fail to meet this deadline, standard University rules for late submission apply unless documented evidence is provided of extenuating circumstances that hinder submission. Marks awarded will be made known in advance of the January examination period; note, however, that all marks are *provisional* until ratified by the Examination Board in June 2013.

Clear, well-written solutions will be rewarded. Conversely, sloppy, badly-spelt or poorly-written solutions will be penalised. Allowance for incorrect grammar will be made for those whose native language is not English, but every effort should be made to ensure that, for example, spelling is correct.

Write your name clearly on the cover page. Number each page, and write the number of pages on the cover page. Fasten all pages together in a secure manner. The last page should end with the words "THE END". Anything written after these words will not be marked. (This is to ensure that no pages are lost.)