Design of Algorithms
Formative Coursework 2012-2013

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Abstract

This document details the formative coursework for G54ALG in the academic year 2012–2013.

1 Casting Out Nines

The program below is a so-called *online* algorithm for computing the remainder of a decimal number after dividing by 9.

The program inputs the digits of the number one-by-one. So, if the number is 12345, it inputs the digits 1, 2, 3 etc. in turn. This is done in the statement $get.d$. As the digits are input, it computes the number P entered thus far, and its remainder r. Thus, for our example, P would be successively 0 (its initial value), 1, 12, 123, etc. and r would be 0, 1, 3, 6, etc.

$\{ \quad \mathsf{true} \quad \}$
$P,r := 0,0 \;;$
$\{ \quad \textbf{Invariant:} \quad 0 \leq P \;\wedge\; r = P \bmod 9 \quad \}$
$\mathsf{do}\ \mathsf{true} \;\rightarrow\; get.d \;\{\; 0 \leq d \leq 9 \;\} \;\;;$
$P,r := 10 \times P + d \,,\, r + d \;\;;$
$\{ \quad 0 \leq P \quad \wedge \quad 0 \leq r < 2 \times 9 \quad \wedge \quad r \bmod 9 = P \bmod 9 \quad \}$
$\mathsf{if} \quad r < 9 \;\rightarrow\; \mathsf{skip}$
$\Box \quad r \geq 9 \;\rightarrow\; r := r - 9$
fi
$\mathsf{od} \quad .$

The outer loop does not terminate. A test could be added to check when entry of the number has been completed, but that will be ignored here.

Construct verification conditions whose validity establish the conditional correctness of the algorithm. For the outer loop construct only the conditions that verify that the invariant is maintained. You may treat the statement *get.d* as a skip but with the effect that the property $0 \leq d \leq 9$ is established.

2 Reversing an Array

Develop an algorithm to perform an in-situ reversal of the elements of an array. For example, if the array stores the values $2\,,\ 7\,,\ 5\,,\ 1$ then after the reversal it should store $1\,,\ 5\,,\ 7\,,\ 2\,.$ "In-situ" means that the reversal has to be effected by repeated execution of a procedure *swap* that has two parameters; if i and j are correctly within the array bounds, *swap*(i,j) swaps the values indexed by i and j. Use variable A to name the array and N to name its length. The type of the array elements is irrelevant.

Add code that will check whether or not the array is a palindrome. (A palindrome is an array that equals its reverse. For example, $1\,,2\,,3\,,2\,,1$ is a palindrome.)

Your algorithm must be fully and precisely documented by precondition, postcondition and an invariant and bound function for any loops in the program.

3 Submitting Your Solution

Your solutions should be submitted to the School Office by ***Friday, 2nd November***. Feedback is only guaranteed on work submitted before this deadline.

Clear, well-written solutions will be rewarded. Conversely, sloppy, badly-spelt or poorly-written solutions will be penalised. Allowance for incorrect grammar will be made for those whose native language is not English, but every effort should be made to ensure that, for example, spelling is correct.

Write your name clearly on the cover page. Number each page, and write the number of pages on the cover page. Fasten all pages together in a secure manner. The last page should end with the words "THE END". Anything written after these words will not be marked. (This is to ensure that no pages are lost.)