Design of Algorithms
Formative Coursework 2012-2013

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Abstract

This document details model solutions to the formative coursework for G54ALG in the academic year 2012–2013.

1 Verification Conditions

Applying the assignment axiom and conditional rule, the assertions are expanded as shown below.

$\{\ \ \mathsf{true}\ \ \}$

$\{\ \ 0 \le 0\ \wedge\ 0 = 0 \,\mathsf{mod}\, 9\ \ \}$

$P,r := 0,0\ ;$

$\{\ \ \textbf{Invariant:}\ \ 0 \le P\ \wedge\ r = P \,\mathsf{mod}\, 9\ \ \}$

$\mathsf{do\ true} \rightarrow$ $get.d\ \{\ 0 \le d \le 9\ \wedge\ 0 \le P\ \wedge\ r = P \,\mathsf{mod}\, 9\ \}\ ;$

$\{\ \ 0 \le 10{\times}P + d\ \wedge\ 0 \le r{+}d < 2{\times}9\ \wedge\ (r{+}d) \,\mathsf{mod}\, 9 = (10{\times}P + d) \,\mathsf{mod}\, 9\ \ \}$

$P\,,r := 10{\times}P + d\ ,\ r{+}d\ ;$

$\{\ \ 0 \le P\ \wedge\ 0 \le r < 2{\times}9\ \wedge\ r \,\mathsf{mod}\, 9 = P \,\mathsf{mod}\, 9\ \ \}$

$\mathsf{if}\ \ r < 9 \rightarrow$ $\{\ \ r < 9 \wedge 0 \le P \wedge 0 \le r < 2{\times}9 \wedge r \,\mathsf{mod}\, 9 = P \,\mathsf{mod}\, 9\ \ \}$

$\{\ \ 0 \le P\ \wedge\ r = P \,\mathsf{mod}\, 9\ \ \}$

skip

$\{\ \ 0 \le P\ \wedge\ r = P \,\mathsf{mod}\, 9\ \ \}$

$\Box\ \ r \ge 9 \rightarrow$ $\{\ \ r \ge 9 \wedge 0 \le P \wedge 0 \le r < 2{\times}9 \wedge r \,\mathsf{mod}\, 9 = P \,\mathsf{mod}\, 9\ \ \}$

$\{\ \ 0 \le P\ \wedge\ r - 9 = P \,\mathsf{mod}\, 9\ \ \}$

$$r := r-9$$

$$\{ \quad 0 \le P \ \wedge \ r = P \,\mathsf{mod}\, 9 \quad \}$$

fi

$$\{ \quad 0 \le P \ \wedge \ r = P \,\mathsf{mod}\, 9 \quad \}$$

od .

From this, we read off the verification condition for the initialisation:

$$[\ \mathsf{true} \Rightarrow 0 \le 0 \ \wedge \ 0 = 0 \,\mathsf{mod}\, 9 \ ] \quad .$$

and the verification conditions for the conditional correcness of the inner loop:

$$\begin{array}{ll} & [\ 0 \le d \le 9 \ \wedge \ 0 \le P \ \wedge \ r = P \,\mathsf{mod}\, 9 \\ \Rightarrow & 0 \le 10 \times P + d \ \wedge \ 0 \le r + d < 2 \times 9 \ \wedge \ (r+d) \,\mathsf{mod}\, 9 = (10 \times P + d) \,\mathsf{mod}\, 9 \end{array}$$

and

$$\begin{array}{ll} & [\ r < 9 \wedge 0 \le P \wedge 0 \le r < 2 \times 9 \wedge r \,\mathsf{mod}\, 9 = P \,\mathsf{mod}\, 9 \\ \Rightarrow & 0 \le P \ \wedge \ r = P \,\mathsf{mod}\, 9 \end{array}$$

and

$$\begin{array}{ll} & [\ r \ge 9 \wedge 0 \le P \wedge 0 \le r < 2 \times 9 \wedge r \,\mathsf{mod}\, 9 = P \,\mathsf{mod}\, 9 \\ \Rightarrow & 0 \le P \ \wedge \ r - 9 = P \,\mathsf{mod}\, 9 \end{array}$$

2 Reversing an Array

The array elements are reversed from the outside inwards. The two indices j and k delimit that part of the array still to be reversed. N is the length of the array.

$$\{ \quad 0 \le N \ \wedge \ \langle \forall i : 0 \le i < N : a[i] = a_0[i] \rangle \quad \}$$

$$j,k := 0, N-1$$

$\{$ **Invariant:**

$$0 \le j < N \ \wedge \ 0 \le k < N \ \wedge \ j = N-1-k$$

$$\wedge \quad \langle \forall i : 0 \le i < j \ \vee \ k \le i < N : a[i] = a_0[N-1-i] \rangle$$

$$\wedge \quad \langle \forall i : j \le i \le k : a[i] = a_0[i] \rangle$$

Bound function: $k-j$ $\}$

; do $j < k \ \rightarrow$ $\quad swap(j,k) \ ; \ j,k := j+1, k-1$

od

$$\{ \quad \langle \forall i : 0 \le i < N : a[i] = a_0[N-1-i] \rangle \quad \}$$

Note the use of two variables j and k. This preserves the symmetry between the top and bottom halves of the array and helps to avoid error in the calculation of the array indices.