Quantifiers

$$\langle \oplus k : R : T \rangle$$

Key:

$\langle \; \rangle$	delimit scope of dummy
$\oplus$	quantifier
k	dummy
R	range
T	term

Rules

Dummy Renaming

$$\langle \oplus k : R : T \rangle \; = \; \langle \oplus j : R[k := j] : T[k := j] \rangle$$

provided no capture/release of dummies (bound variables) occurs.

Nesting

$$\langle \oplus k, js : R \wedge S : T \rangle$$
$$= \quad \langle \oplus k : R : \langle \oplus js : S : T \rangle\rangle$$

provided no capture/release of dummies occurs.

Range Part

Empty Range

$$\langle \oplus k : \mathit{False} : T \rangle \quad = \quad 1_{\oplus}$$

where $1_{\oplus}$ is the unit of $\oplus$.

One-point

$$\langle \bigoplus k : k = e : T \rangle \quad = \quad T[k := e]$$

Splitting

$$\langle \bigoplus k : P : T \rangle \oplus \langle \bigoplus k : Q : T \rangle$$

$$= \langle \bigoplus k : P \vee Q : T \rangle \oplus \langle \bigoplus k : P \wedge Q : T \rangle \quad .$$

Trading

$$\langle \oplus k : P \wedge Q : T \rangle$$

$$= \quad \langle \oplus k : P : \text{if } Q \rightarrow T \;\square\; \neg Q \rightarrow 1_{\oplus} \text{ fi} \rangle$$

Term Part

Rearranging

$$\langle \oplus k : R : T_0 \oplus T_1 \rangle$$

$$= \quad \langle \oplus k : R : T_0 \rangle \oplus \langle \oplus k : R : T_1 \rangle$$

Distributivity

If f is such that

$$f.1_{\oplus} = 1_{\otimes}$$

$$f.(x \oplus y) = f.x \otimes f.y$$

then

$$f.\langle \oplus k : R : T \rangle = \langle \otimes k : R : f.T \rangle$$

Example. Prove that the sum of a finite set of integers is even equivales the number of odd integers in the set is even.

Example. Suppose a and b are two permutations of the same bag of numbers. Show that, if the number of numbers is odd, the product of the differences $a_k - b_k$ is even.

E.g.

a	1	3	3	9	10	10	12	18	20	26	26
b	26	26	20	12	10	18	9	3	3	1	10
a - b	-25	-23	-17	-3	0	-8	3	15	17	25	16