

An optimal task assignment

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2. Classroom Assignment

- one day seminar
- 14 seminars
- 5 rooms
- 8:00 - 5:00pm
- no seminars during the lunch hour 12:00 - 1:00pm

Seminar	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Periods	2	8	4	1	3	6	7	2	1	5	6	3	8	2
			5	2	4			8	3	2		7	4	3
					5									4

Period	1	2	3	4	5	6	7	8
Room1	D	D		C	C	F		B
Room2	I	I	E	E	E		G	G
Room3		H	H		J	K	K	
Room4		N	N	N				M
Room5		A	L	L				

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3. Soft Drink Bottling

- single machine
- 4 flavours
- each flavour has its own filling time
- cleaning and changeover time between the bottling of successive flavours

aim: to minimise cycle time,
sufficient: to minimise the total changeover time

f_1	f_2	f_3	f_4	$f_1 - f_2 - f_3 - f_4 - f_1$	$f_3 - f_4 - f_2 - f_1 - f_3$
f_1	$\begin{pmatrix} - & 2 & 70 & 50 \end{pmatrix}$			$2+3+2+50 = 57$	$2+5+6+70 = 83$
f_2	$\begin{pmatrix} 6 & - & 3 & 4 \end{pmatrix}$				
f_3	$\begin{pmatrix} 8 & 3 & - & 2 \end{pmatrix}$	$f_2 - f_3 - f_4 - f_1 - f_2$	$f_4 - f_2 - f_3 - f_1 - f_4$	$3+2+50+2 = 57$	$4+3+8+50 = 66$
f_4	$\begin{pmatrix} 50 & 5 & 6 & - \end{pmatrix}$				

optimal: $f_1 - f_2 - f_4 - f_3 - f_1$
 $2+4+6+8 = 20$

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Terminology

- **Scheduling** is the allocation, subject to constraints, of resources to objects being placed in space-time, so that the total cost is minimised.
Schedule includes the spacial and temporal information.
- **Sequencing** is the construction, subject to constraints, of an order in which activities are to be carried out.
Sequence is an order in which activities are carried out.
- **Timetabling** is the allocation, subject to constraints, of resources to objects being placed in space-time, so that the set of objectives are satisfied as much as possible.
Timetable shows when particular events are to take place.
- **Rostering** is the placing, subject to constraints, of resources into slots in a pattern.
Roster is a list of people's names that shows which jobs they are to do and when.

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Classification of Scheduling Problems

machines $i=1, \dots, m$
jobs $j=1, \dots, n$
(i, j) processing step, or operation, of job j on machine i

Job data

Processing time p_{ij} - processing time of job j on machine i
Release date r_j - earliest time at which job j can start its processing
Due date d_j - committed shipping or completion date of job j
Weight w_j - importance of job j relative to the other jobs in the system

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Scheduling problem: $\alpha | \beta | \gamma$

α machine environment
 β job characteristics
 γ optimality criteria

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Machine characteristics

Single machine 1

Identical machines in parallel P_m

- m machines in parallel
- Job j requires a single operation and may be processed on any of the m machines
- If job j may be processed on any one machine belonging to a given subset M_j
 $P_m | M_j | \dots$
- Machines in parallel with different speeds Q_m
- Unrelated machines in parallel R_m
machines have different speeds for different jobs

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Flow shop F_m

- m machines in series
- all jobs have the same routing
- each job has to be processed on each one of the m machines (permutation)
first in first out (FIFO)
 $F_m | prmu | \dots$

Flexible flow shop FF_s

- s stages in series with a number of machines in parallel
- at each stage job j requires only one machine
- FIFO discipline is usually between stages

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Open shop O_m

- m machines
- each job has to be processed on each of the m machines
- scheduler determines the route for each job

Job shop J_m

- m machines
- each job has its own route
- job may visit a machine more than once (recirculation)
 $J_m | recrc | \dots$

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Job characteristics

Release date r_j - earliest time at which job j can start its processing

Sequence dependent setup times s_{jk} - setup time between jobs j and k
 s_{ijk} - setup time between jobs j and k depends on the machine

Preemptions $prmp$ - jobs can be interrupted during processing

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Precedence constraints $prec$ - one or more jobs may have to be completed before another job is allowed to start its processing

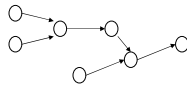
may be represented by an acyclic directed graph $G=(V,A)$

$V=\{1, \dots, n\}$ corresponds to the jobs

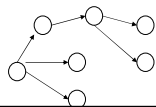
$(j, k) \in A$ iff j th job must be completed before k th

chains each job has at most one predecessor and one successor

intree each job has at most one successor



outree each job has at most one predecessor



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Breakdowns $brkdwn$ - machines are not continuously available

Machine eligibility restrictions M_j - M_j denotes the set of machines that can process job j

Permutation $prmu$ - in the flow shop environment the queues in front of each machine operates according to the FIFO discipline

Blocking $block$ - in the flow shop there is a limited buffer in between two successive machines, when the buffer is full the upstream machine is not allowed to release a completed job.

No wait $no-wait$ - jobs are not allowed to wait between two successive machines

Recirculation $recrc$ - in the job shop a job may visit a machine more than once

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Optimality criteria

We define for each job j :

C_{ij} **completion time** of the operation of job j on machine i

C_j time when job j exits the system

$L_j = C_j - d_j$ **lateness** of job j

$T_j = \max(C_j - d_j, 0)$ **tardiness** of job j

$U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$ unit penalty of job j

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Possible objective functions to be minimised:

Makespan $C_{max} = \max(C_1, \dots, C_n)$

Maximum lateness $L_{max} = \max(L_1, \dots, L_n)$

Total weighted completion time $\sum w_j C_j$ - weighted flow time

Total weighted tardiness $\sum w_j T_j$

Weighted number of tardy jobs $\sum w_j U_j$

Examples

Bicycle assembling: precedence constrained parallel machines
P3 | *prec* | C_{max}

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P and NP problems

- The efficiency of an algorithm for a given problem is measured by the maximum (worst-case) number of computational steps needed to obtain an optimal solution as a function of the size of the instance.
- Problems which have a known polynomial algorithm are said to be in class P. These are problems for which an algorithm is known to exist and it will stop on the correct output while effort is bounded by a polynomial function of the size of the problem.
- For NP (non-deterministic polynomial problems) no simple algorithm yields optimal solutions in a limited amount of computer time.

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Summary

- ☞ Scheduling is a decision making process with the goal of optimising one or more objectives
- ☞ Production scheduling problems are classified based on machine environment, job characteristics, and optimality criteria.

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