1. (a) Transition diagram for $A$

Transition diagram for $B$

(b) Determine which of the following words belong to $L(A), L(B)$:

i. $\epsilon \in L(A)$  $\epsilon \notin L(B)$
ii. $aabb \in L(A)$  $aabb \notin L(B)$
iii. $aaab \notin L(A)$  $aaab \in L(B)$
iv. $bbb \in L(A)$  $bbb \notin L(B)$

(c)

$\hat{\delta}_A(0, bab) = \hat{\delta}_A(\hat{\delta}_A(0, b), ab)$  definition of $\hat{\delta}$

$= \hat{\delta}_A(2, ab)$  because $\hat{\delta}_A(0, b) = 2$

$= \hat{\delta}_A(\hat{\delta}_A(2, a), b)$  definition of $\hat{\delta}$

$= \hat{\delta}_A(3, b)$  because $\hat{\delta}_A(2, a) = 3$

$= \hat{\delta}_A(\hat{\delta}_A(3, b), \epsilon)$  definition of $\hat{\delta}$

$= \hat{\delta}_A(1, \epsilon)$  because $\hat{\delta}_A(3, b) = 1$

$= 1$  definition of $\hat{\delta}$
\[ \delta_B(0, \text{bab}) = \delta_B(\delta_B(0, b), ab) \text{ definition of } \delta \]
\[ = \delta_B(0, ab) \text{ because } \delta_B(0, b) = 0 \]
\[ = \delta_B(\delta_B(0, a), b) \text{ definition of } \delta \]
\[ = \delta_B(1, b) \text{ because } \delta_B(0, a) = 1 \]
\[ = \delta_B(\delta_B(1, b), \epsilon) \text{ definition of } \delta \]
\[ = \delta_B(3, \epsilon) \text{ because } \delta_B(1, b) = 3 \]
\[ = 3 \text{ definition of } \delta \]

(d) \( L(A) \) contains all the words such that the number of a’s and b’s have a different remainder when divided by 2. Writing \( \#(x, w) \) for the number of x’s in w we can express this by:

\[ L(A) = \{ w \mid \#(a, w) \not\equiv \#(b, w) \text{mod2} \} \]

\( L(B) \) contains all words such that the letter before the last one is a.

\[ L(B) = \{ wax \mid w \in \{a, b\}^*, x \in \{a, b\} \} \]