1. Let $L_1$ and $L_2$ be two languages over the alphabet $\Sigma = \{a, b, c\}$, defined as follows:

$L_1 = \{a, ab\}$  
$L_2 = \{\varepsilon, bb, bbc\}$

Enumerate the words in the following languages:

(a) $L_3 = L_1 \cup L_2$
(b) $L_4 = L_2 L_1$
(c) $L_5 = L_4 \emptyset L_3$
(d) $L_6 = L_4^* \cap L_2^*$

2. Given $\Sigma = \{a, b, c\}$ which of the following equations for $L_1, L_2 \in \mathcal{P}(\Sigma^*)$ are universally true:

(a) $L_1 L_2 = L_2 L_1$
(b) $L_1 \Sigma^* = L_1$
(c) $L_1 L_1 = L_1$
(d) $L_1^* L_1^* = L_1^*$
(e) $(L_1 L_2)^* = L_1^* L_2^*$

Either give a counterexample or give a short explanation why you think the equation is true.

3. Consider the following DFA $A$ over the alphabet $\Sigma = \{b, c\}$:

$A = (\{0, 1, 2\}, \Sigma, \delta, 0, \{2\})$

where

$\delta(0, b) = 1$
$\delta(0, c) = 0$
$\delta(1, b) = 2$
$\delta(1, c) = 0$
$\delta(2, b) = 2$
$\delta(2, c) = 2$
(a) Draw a transition table for $A$.
(b) Draw a transition diagram for $A$.
(c) Which of the following words belong to $L(A)$:
   i. $\varepsilon$
   ii. $cb$
   iii. $cbbeb$
   iv. $bcbcbbeb$
(d) Explicitly calculate $\hat{\delta}(0, bcbb)$. Clearly show each step of the calculation.
(e) Describe the language $L(A)$ in plain English.

4. **Bonus Exercise**

Recall the DFA $D_1$ from Lecture 3:

Encode $D_1$ in Haskell by giving definitions for the following data types and functions:

```
data Q = ...
data Σ = ...
type Word = ...
q₀ :: Q
final :: Q → Bool
δ :: (Q, Σ) → Q
\hat{δ} :: (Q, Word) → Q
accept :: Word → Bool
```

Note that for syntactical reasons, $Σ^*$ has been renamed `Word`.