1. Consider the following Pushdown Automaton $P$:

$$P = (Q = \{0, 1, 2\}, \Sigma = \{a, b, c\}, \Gamma = \{a, \#,\}, \delta, q_0 = 0, Z_0 = \#)$$

where the transition function $\delta$ is given by:

- $\delta(0, \varepsilon, \#) = \{(0, \varepsilon), (1, \#)\}$
- $\delta(0, a, \#) = \{(0, a\#)\}$
- $\delta(0, a, a) = \{(0, aa)\}$
- $\delta(0, \varepsilon, a) = \{(1, a)\}$
- $\delta(1, \varepsilon, \#) = \{(1, \varepsilon)\}$
- $\delta(1, b, \#) = \{(1, \#)\}$
- $\delta(1, b, a) = \{(1, a)\}$
- $\delta(1, c, a) = \{(2, c)\}$
- $\delta(2, \varepsilon, \#) = \{(2, c)\}$
- $\delta(2, c, a) = \{(2, c)\}$
- $\delta(\omega, \omega, \omega) = \emptyset$

Acceptance is by empty stack.

(a) Draw a transition diagram for $P$.

(b) For each of the following words, state whether they are accepted by $P$, and, for those that are, give a sequence of Instantaneous Descriptions leading to an accepting configuration.

i. $\varepsilon$

ii. $a$

iii. $ab$

iv. $ac$

v. $bb$

vi. $abc$

vii. $aabbc$

viii. $aabec$

ix. $abcecc$

Note: Just give the sequence of IDs separated by $\vdash$’s, a formal calculation with hints is not required.

(c) Give a set comprehension defining the language accepted by $P$.

2. Consider the Context-Free Grammar

$$G = (\{S, X, Y, Z\}, \{a, b, c, d\}, P, S)$$

where $P$ is given by:

- $S \rightarrow X \mid Y \mid cZ$
- $X \rightarrow aXb \mid \varepsilon$
- $Y \rightarrow bbc \mid abba$
- $Z \rightarrow cZdd \mid \varepsilon$
(a) For each of the following words, state whether they are in the language generated by $G$, and, for those that are, give a complete derivation sequence from the start symbol $S$.

i. $\varepsilon$
ii. $a$
iii. $c$
iv. $ab$
v. $ba$
vi. $bcc$
vii. $cd$
viii. $ccdd$
ix. $cabba$
x. $ccddd$
xi. $aaabbb$

(b) Give a set expression (using set comprehensions and operations on sets like union) denoting the language $L(G)$.

(c) Is it possible to construct a Regular Expression $R$ such that $L(R) = L(G)$? If so, do so. If not, give a brief justification of why not.

3. Consider the following Context-Free Grammar $Exp$:

\[
\begin{align*}
T & \rightarrow T + T \mid F \\
F & \rightarrow F * F \mid P \\
P & \rightarrow N \left(A\right) \mid (T) \mid I \\
N & \rightarrow f \mid g \mid h \\
A & \rightarrow T \mid \varepsilon \\
I & \rightarrow DI \mid D \\
D & \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
\end{align*}
\]

$T, F, P, N, A, I, D$ are nonterminals;
$+, *, f, g, h, (, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ are terminals;
$T$ is the start symbol.

(a) For each of the following strings of terminals and nonterminals:
- state whether it is in the language generated by $Exp$;
- state whether it is a sentential form of $Exp$;
- if it is a sentential form, state whether it is a left-sentential form;
- if it is a left-sentential form, give a complete leftmost derivation sequence from the start symbol $T$.

i. $\varepsilon$
ii. $g \left(13 + f \left(\right)\right)$
iii. $I * g \left(D + f + g \left(\right)\right)$
iv. $2 \left(f \left(17\right)\right) + g \left(4\right)$
v. $N \left(3 * F\right) * h \left(\right)$
vi. $33 + 7) * h \left(21 + 6 * f \left(13 * 542\right)\right)$
(b) **Bonus Exercise**

i. Modify the relevant productions of the grammar $Exp$ so that a function symbol (one of $f$, $g$, $h$) can be applied to zero, one, or more arguments, instead of just zero or one arguments. When there are two or more arguments, they should each be separated by a single comma. For example, it should be possible to derive words such as

$$f (7, g (), h (3 + 4))$$

ii. Explain your construction.

iii. Give a complete rightmost derivation sequence for the word $f(1, 2, 3)$ using your modified grammar.