Quantum Computing

Part 2

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How to build your own quantum computer

in theory
Quantum memory: the qubit

\[ \alpha |0\rangle + \beta |1\rangle \]

\[ \alpha, \beta \in \mathbb{C} \]

\[ |\alpha|^2 + |\beta|^2 = 1 \]

superposition of 2 probability amplitudes given as complex numbers

subset of a 2-dimensional complex vectorspace

The Bloch sphere

Examples:

\[ |0\rangle \quad |1\rangle \]

\[ \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \quad \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \]

base states

superpositions
Activities

* Refresh your knowledge about complex numbers (e.g. by reading the wikipedia entry). Use this to implement a small library of operations on complex numbers including

\[ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

* Lookup vector spaces in wikipedia. Show that finite vectors of complex numbers form a vector space.

* Study the mathematics of the Bloch sphere and implement a program which displays any qubit (given as a pair of complex number) as a point on the Bloch sphere using polar coordinates (or using 3D graphics).
Operations on qubits: Measurement

\[ \alpha |0\rangle + \beta |1\rangle \]

\[ |\alpha|^2 \rightarrow |0\rangle \]
\[ |\beta|^2 \rightarrow |1\rangle \]

* probabilistic
* irreversible

Example

\[ \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \]

\[ \frac{1}{2} \rightarrow |0\rangle \]
\[ \frac{1}{2} \rightarrow |1\rangle \]
Operations on qubits: Unitaries

- deterministic
- reversible
- correspond to rotations of the Bloch sphere

\[
\begin{bmatrix}
  u_{00} & u_{01} \\
  u_{10} & u_{11}
\end{bmatrix}
\]

linear independent + norm preserving

\[\alpha |0\rangle + \beta |1\rangle\]
\[\mapsto (\alpha u_{00} + \beta u_{10}) |0\rangle + (\alpha u_{01} + \beta u_{11}) |1\rangle\]

Examples

\[
\begin{bmatrix}
  0 & 1 \\
  1 & 0
\end{bmatrix}
\]
\[
\frac{1}{\sqrt{2}} \begin{bmatrix}
  1 & 1 \\
  1 & -1
\end{bmatrix}
\]

negation (X)  Hadamard (H)
Activities

- Negation and Hadamard are their own inverses. Find some unitary operators for which this is not the case.
- Study the Pauli matrices (see wikipedia). What rotations of the Bloch sphere do they correspond to?
Two qubits (and more)

\[ \alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle \]

\[ |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1 \]

Examples:

\[ \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle) \]

\[ = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \]

\[ = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \]

* Tensor product of qubits
* Subset of a 4-dimensional vectorspace
* How many dimensions do we get for 3 qubits?

entangled state (Bell state)

separable state
Measurements on 2 qubits

\( \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \)

1st qubit

2nd qubit
Measurements on 2 qubits

\[ \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \]

1st qubit

2nd qubit
Unitaries on several qubits

cnot

\[ \begin{array}{cc|cc}
  a & b & c & d \\
  \hline
  0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 1 \\
  1 & 0 & 1 & 1 \\
  1 & 1 & 1 & 0 \\
\end{array} \]

cond-U

Toffolli
Activities

* Study the implementation of reversible addition using reversible classical circuits. What is the role of the so called ancilla bits.

* Study the behaviour of the following circuit. What is its behaviour for classical inputs?