An Introduction to Type Theory

Practical 2

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Lego exercises

Load `basic.l` to have access to the basic definitions.

1. Prove

   ```
   injs : {m,n:Nat}(Id (su m) (su n))->(Id m n)
   ```

   in LEGO.

Lego exercises

2. Use prec to define the following functions:

exp2 `exp2 : Nat → Nat`

such that $\exp 2\, n = 2^n$

mult `mult : Nat → Nat → Nat`

such that $\mathrm{mult}\, m\ n = mn$

half `half : Nat → Nat`

such that $2(\mathrm{half}\, n) \leq n$ and $2(\mathrm{half}\, n) + 1 > n$

Lego exercises

3. Use ind to prove the following properties:

```
exp2lem : {n:Nat}Id (exp2 (su n)) (mult n2 (exp n))
halflem : {n:Nat}Id (half (mult n2 n)) n
```

You will have to prove auxilliary lemmas.