

Testing using Dependent Types

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TYPES'06

$$\text{reverse} (\text{reverse } xs) = xs$$

reverse (*reverse xs*) = *xs*

OK

Constrained Input

for ranked binary trees

$$\textit{empty} : \textit{RTree } X$$

$$\textit{node} : X \rightarrow \textit{Nat} \rightarrow \textit{RTree } X \rightarrow \textit{RTree } X \rightarrow \textit{RTree } X$$

define what it is to be heap

$$\textit{IsHeap } h = \textit{Leftist } h \wedge \textit{WellRanked } h \wedge \textit{HOP } h$$

implement element insertion

$$\textit{insHeap} : X \rightarrow \textit{RTree } X \rightarrow \textit{RTree } X$$

preserves heap property

$$\textit{IsHeap } h \rightarrow \textit{IsHeap } (\textit{insHeap } x h)$$

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tricky

Constrained Input, contd.

Idea: Use a proof search tool for a dependent functional programming language to perform testing.

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Heap enumeration

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(empty, ...)

...
(node 0 (s 0) (node (s ?) (s 0) empty empty) empty, ...)

...

Higher-Order Terms

Testing may involve witnessing a function

$$\exists f. \exists x. R x (f x)$$

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Function synthesis

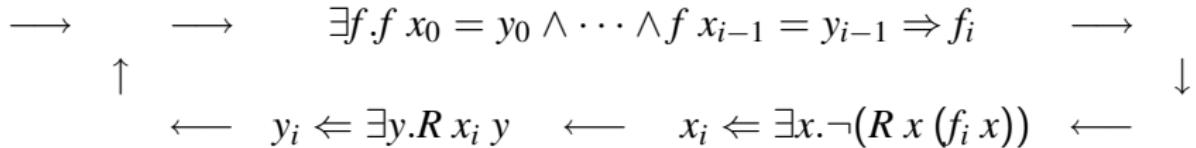
$$\exists f. \forall x. R f (f x)$$

$$?_f : A \rightarrow B, \quad ? : (x : A) \rightarrow R x (?_f x)$$

Function Synthesis

Instead of co-evolving function and proof for the specification:

- ▶ Given $\exists f. \forall x. R x (f x)$
- ▶ Produce candidate functions, f_i
- ▶ Accumulate input/output pairs, (x_i, y_i)



Function Synthesis, contd.

List sorting

- ▶ **Cheat:** Correct search depth given, no iterated deepening.

$$f : (X : S_{<}) \rightarrow [|X|] \rightarrow [|X|]$$

$$\forall X : S_{<}. \forall xs : [|X|]. \text{Sorted } (f \ xs) \wedge \text{Perm } xs \ (f \ xs)$$

$$x_i = [0], [1, 0], [0, 1], [1, 0, 1], [0, 0, 1], [1, 1, 0]$$

$$f_6 = \lambda X \ x \rightarrow \text{elimList } x \ (\lambda y \rightarrow y)$$

$$(\lambda y \ z \ w \ y' \rightarrow w \ ($$

$$\text{elimList } y' \ (y :: [])$$

$$(\lambda a \ b \ c \rightarrow \text{elimBool } (X.(<) \ a \ y) \ (a :: c) \ (y :: a :: b))$$

$$)) []$$

Alternatives when Writing Properties

Boolean function

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- ▶ Decomposing properties, $M(\vec{?}) \rightsquigarrow M_1(\vec{?}) \wedge M_2(\vec{?})$, checked in parallel
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bool. fcn. $\textit{perm} : (X : S_=) \rightarrow [|X|] \rightarrow [|X|] \rightarrow \textit{Bool}$
 $\textit{True} (\textit{perm} X xs ys)$

ind. def. $P_0 : \textit{Perm} [] []$

$P_1 : \textit{Delete} x ys zs \rightarrow \textit{Perm} xs zs \rightarrow \textit{Perm} (x :: xs) ys$

$D_0 : \textit{Delete} x (x :: ys) ys$

$D_1 : \textit{Delete} x ys zs \rightarrow \textit{Delete} x (y :: ys) (y :: zs)$

$\textit{Perm} xs ys$

Proof Search Tool

- ▶ Meta variables and explicit substitutions
- ▶ Sequent calculus style treatment of implication and tuple elimination
- ▶ Delayed instantiation, equality constraints
- ▶ Prolog like search algorithm

Summary

- ▶ Idea: Perform testing using a proof search tool for a dependently typed functional language
- ▶ Instead of “generate, then test” – incremental construction of terms guided by input constraints and specification
- ▶ Hopes: Less need for customized generators and randomness
- ▶ Problems which are not completely testable could be attacked.
- ▶ Ability to construct simple higher-order terms

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Future

- ▶ Measure efficiency for standard testing problems
- ▶ Explore the usefulness of handling non-testable testing situations