G52MAL Machines and Their Languages Lecture 13 Pushdown Automata (PDA)

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Where Are We? (1)

- We have encountered the *regular languages* and two ways to describe them:
 - Through *automata*: a mechanical procedure for deciding whether or not a word belongs to the language.
 - Through *regular expressions*: a direct description of the words in a language.
- We have learned that not all languages are regular. In particular, the class of *context-free languages* includes the regular languages but is strictly larger.

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Where Are We? (2)

- We have learned *one* way to describe context-free languages:
 - Through *context-free grammars*: a direct description of the words in a language.

QUESTION: Can the context-free languages also be described through some form of automaton?

In other words: is there a mechanical procedure for determining language membership for the context-free languages, and if so, what is the "simplest" such machine?

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Pushdown Automata (PDA)

The answer is YES! Pushdown Automata (PDA).

- PDA \approx FA + *unlimited stack*. I.e., the amount of memory that can be used
- to determine language membership is no longer finite.
- LIFO stack access:
 - Suffices for describing the context-free languages (CFL).
 - Is not enough to describe languages more general than the CFLs (as we will see).

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